

2ND SEM. / COMMON /2023(S) NEW

TH- 3 ENGINEERING MATHEMATICS - II

Full Marks: 80

Time- 3 Hrs

Answer any five Questions including Q No.1& 2
Figures in the right hand margin indicates marks

1. Answer All questions 2 x 10

- a. Evaluate $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$
- b. if $f(x) = mx + c$, $f(0) = f'(0) = 1$
then find the value of $f(1)$
- c. Determine order and Degree of $2 \frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 + 5}$
- d. Integrate $\int \frac{\cos x}{1 + \sin x} dx$
- e. Find the unit vector in the direction of the vector $2\hat{i} - \hat{j} + 2\hat{k}$
- f. Find the derivative of $\sqrt{2x^2 + 3x + 5}$
- g. Evaluate $\int_0^3 [x] dx$
- h. Solve $\frac{dy}{dx} = \frac{e^{2x} + 1}{e^x}$
- i. If $Z = \log(x^2 - y^2)$, then find $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$
- j. if $x = 2t^2$ and $y = 4t$, then find $\frac{dy}{dx}$ at $t = 1$

2. Answer Any Six Questions 6 x 5

- a. Differentiate $x^{\sin x}$
- b. Integrate $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$
- c. Test the continuity of the function
$$F(x) = \begin{cases} \frac{|x|}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases} \text{ at } x = 0$$
- d. prove that $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
- e. Find Scalar and Vector projection of $\vec{a}$ on $\vec{b}$,
where $\vec{a} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} + 3\hat{k}$

- f. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \sqrt{\tan x}}$
- g. Solve $(1 + x^2)dy + (1 + y^2)dx = 0$
- 3 a) Evaluate $\lim_{x \rightarrow 0} \frac{e^{4x} - e^{3x}}{e^{3x} - e^{2x}}$ 5
- b) Find $\frac{dy}{dx}$ if $x^y y^x = 1$ 5
- 4 a) Find the area of parallelogram whose adjacent sides are the vectors $\hat{i} - 3\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ 5
- b) If $y = \tan^{-1} x$ then show that $(1 + x^2)y_2 + 2xy_1 = 0$ 5
- 5 a) Solve $x \log x \frac{dy}{dx} + y = 2 \log x$ 5
- b) Integrate $\int x \tan^{-1} x dx$ 5
- 6 a) Differentiate $5^{\ln \sin x}$ 5
- b) Integrate $\int e^{\cos^2 x} \sin 2x dx$ 5
- 7 a) Evaluate $\lim_{x \rightarrow 0} \frac{\log(x+1)}{\sqrt{x+1}-1}$ 5
- b) Find the area of the circle $x^2 + y^2 = 16$ 5

2nd SEMESTER/COMMON TO ALL BRANCHES/2023(S)

ENGINEERING MATHEMATICS-II (TH-3)

Full Marks – 80

Time: 3 Hours

**Answer any five Questions including Q No. 1 & 2.
The figures in the right-hand margin indicate marks.**

1) Answer all questions.

2 × 10 = 20

- Define equal vectors.
- Find the unit vector in the direction of $\vec{a} + \vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$
- Calculate $\lim_{x \rightarrow 0} (1 + 3x)^{\frac{1}{x}}$.
- Find the scalar product of $3\hat{i} - 4\hat{j}$ and $-2\hat{i} + \hat{j}$.
- Define Modulus function and represent it graphically.
- Evaluate $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$.
- Differentiate $\sqrt{\sin \sqrt{x}}$ with respect to x .
- Write the fundamental theorem of integral calculus.
- Evaluate $\int_0^1 \frac{dx}{1+x^2}$.
- Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

2) Answer any six questions.

5 × 6 = 30

- Find the value of p for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} + p\hat{j} + 3\hat{k}$ are perpendicular to each other.
- Evaluate $\lim_{x \rightarrow 0} \frac{x^3 - 8}{x^5 - 32}$.
- Differentiate a^x with respect to x^a .
- Find f_{xx} and f_{xy} if $f(x, y) = x^3 + y^3 + 3xy$.
- Find the value of $\int_{-1}^1 (|x| + [x]) dx$.
- Evaluate $\int \cos^5 x dx$.
- If sum of two unit vector is a unit vector. Then show that the magnitude of their difference is $\sqrt{3}$.

$$3 \times 10 = 30$$

- 3) Calculate the area of the triangle ABC (by vector method) where $A(1,1,2)$, $B(2,2,3)$ and $C(3,-1,-1)$.
- 4) a) For any vector $\vec{a}$ and $\vec{b}$. Prove that $(\vec{a} \times \vec{b})^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$.
- b) If $|\vec{a}| = 2$, $|\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 8$. Then find the value of $\vec{a} \cdot \vec{b}$.
- 5) Determine the values of a and b for which $f(x)$ is continuous at $x = 1$
- $$f(x) = \begin{cases} ax^2 + b & x < 1 \\ 1 & x = 1 \\ 2ax & x > 1 \end{cases}$$
- 6) If $y = e^{ax} \sin bx$, then prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.
- 7) a) Prove that $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$.
- b) Evaluate $\int \frac{e}{e^{2x} + 9} dx$.

2nd. SEM. /COMMON/ 2022(S)
TH-3 -ENGINEERING MATHEMATICS -II

Time- 3 Hrs

Full Marks: 80

Answer any five Questions including Q No.1& 2
 Figures in the right hand margin indicates marks

2 x 10

1. Answer **All** questions
1. a. Define Modulus Function and represent it graphically.
- b. Evaluate $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$
- c. Differentiate $\sec^{-1} \left(\frac{\sqrt{a^2+x^2}}{a} \right)$ with respect to x .
- d. Define unit vector and find the unit vector of the given vector $2\hat{i} + 3\hat{j} + 6\hat{k}$.
- e. Evaluate the integral $\int (e^{5 \ln x} - e^{4 \ln x}) dx$.
- f. Define Homogeneous Function and State Euler's Theorem.
- g. Find the value of α so that $\vec{a} = \hat{i} + \hat{j} + \alpha\hat{k}$, $\vec{b} = 4\hat{i} - 3\hat{k}$ are perpendicular to each other.
- h. Find the order and degree of the following differential equation

$$\frac{d^2y}{dx^2} = \frac{3y + \frac{dy}{dx}}{\sqrt{\frac{d^2y}{dx^2}}}$$
- i. Find the value of $\int_{-2}^2 |x| dx$.
- j. If $y = t^2$ and $x = t^3$ find $\frac{dy}{dx}$ at $t = 1$.

6 x 5

2. Answer **Any six** questions:

- a. If $f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$, then find the value of 'a' and 'b'.
- b. Find $\frac{dy}{dx}$ if $y = (\ln x)^{\tan x}$.
- c. Determine the area within the curve $y^2 = 4ax$ and the x-axis, the ordinate $x=4$.
- d. Evaluate $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx$.
- e. Solve $(1 + x^2)dy + (1 + y^2)dx = 0$.

- f. Find the scalar and vector projections of the vector $2\hat{i} - 3\hat{j} - 6\hat{k}$ on the line joining the points $(3,4,-2)$ and $(5,6,-3)$.
- g. Find $\frac{dy}{dx}$ if $x = \frac{2t}{1+t^2}$, $y = \frac{2t}{1-t^2}$.
- 3 i. If $\sqrt{1-x^6} + \sqrt{1-y^6} = k(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$ 7
- ii. Evaluate $\lim_{x \rightarrow 0} \frac{1-\cos^3 x}{x \sin 2x}$. 3
- 4 i. If $u = \tan^{-1}(x^2 + y^2 + z^2)$, show that $xu_x + yu_y + zu_z = \sin 2u$ 7
- ii. If sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$. 3
- 5 i. Evaluate $\int \frac{2x+11}{\sqrt{x^2+10x+29}} dx$. 6
- ii. If $y = \tan^{-1} x$, prove that $(1+x^2)y_2 + 2xy_1 = 0$ 4
- 6 i. Solve the following differential equation $(1+y^2)dx = (\tan^{-1} y - x)dy$ 7
- ii. Find the derivative of $y = e^x$ by first principle. 3
- 7 i. In a triangle AOB , angle $AOB = 90^\circ$. If P, Q are the points of trisection of $\overline{AB}$, prove that $OP^2 + OQ^2 = \frac{5}{9} AB^2$ by vector method. 6
- ii. Evaluate $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx$. 4

BST201 ENGINEERING MATHEMATICS-II

Full Marks: 80

Time- 3 Hrs

Answer any five Questions including Q No.1& 2
 Figures in the right hand margin indicates marks

1. Answer All questions

2 x 10

- Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2+2}{x^3+5}$
- Find the derivative of $\log x$ with respect to x^2
- Evaluate $\int_0^1 \frac{dx}{1+x^2}$
- If $y = e^{\cos x^2}$, then find y_1
- Find order and degree of the differential equation

$$2 \frac{d^2 y}{dx^2} = \left\{ 5 + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{3}{5}}$$

- Integrate $\int \log x \, dx$
- Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $z = \log(x^2 - y^2)$
- Find Image of the point $(2, -3, 1)$ with respect to XY- plane
- Find centre and Radius of the Sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$
- Integrate $\int e^x \{ \cot x + \log \sin x \} dx$

2. Answer Any Six Questions

6 x 5

- Find angle between the planes $x + 2y + 2z - 7 = 0$ and $2x - y + z - 6 = 0$
- Evaluate $\lim_{x \rightarrow 0} \frac{\log(x+1)}{\sqrt{x+1}-1}$
- Solve $\frac{dy}{dz} = (y^2 + 1)(z^2 + 1)$
- Find the value of $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot x}$
- Find $\frac{dy}{dx}$, if $y = x^{\sin x}$
- Integrate $\int x \sin^{-1} x \, dx$

- g Find $\frac{dy}{dx}$, if $x = t + \sin t$, $y = 1 + \cos t$ at $t = \frac{\pi}{4}$ 10
- 3 Determine extremum value and extremum points of the function 10
 $y = 2x^3 - 15x^2 - 36x + 18$
- 4 Integrate $\int e^{2x} \cos 3x \, dx$ 5
- 5 a) Solve $\frac{dy}{dx} + y \sec x = \tan x$ 5
- b) Find $\frac{dy}{dx}$, if $x^2y + xy^2 + 1 = 0$ 5
- 6 a) Test the Continuity of the function 5
 $f(x) = \begin{cases} 3x - 2 & \text{when } x \leq 0 \\ x + 1 & \text{when } x > 0 \end{cases}$ at $x = 0$
- b) If $y = e^{\tan^{-1} x}$, Then prove that $(1 + x^2)y_2 + (2x - 1)y_1 = 0$
- 7 a) Find co ordinate of foot of perpendicular drawn from (1,2,3) on line 5
 joining the points $(-2,3,4)$ and $(2, -1,6)$
- b) Prove that $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + k$, where $K =$ integrating constants 5

Answer any five questions including Q. Nos. 1 & 2

Figures in the right-hand margin indicate marks

2 × 10

1. Answer all questions :

(a) Evaluate :

$$\lim_{x \rightarrow 1} \left(\frac{\frac{1}{x^2} - \frac{1}{4}}{x - 2} \right)$$

(b) If $u = t^2$ and $v = \sin t^2$, then find $\frac{dv}{du}$.(c) If $f(x, y) = e^{-xy}$, then find $y \cdot \frac{\partial f}{\partial y}$.(d) Find derivative of $\sqrt{x}$ w.r.t. x^2 .

(e) Examine the existence of

$$\lim_{x \rightarrow \frac{5}{2}} [x]$$

(f) If $y = c_1 e^x + c_2 e^{-x}$, then find $\frac{d^2 y}{dx^2}$.

(g) Evaluate

$$\int e^{(5x+3)} \cdot dx$$

(h) The two forces act on a particle at a point. Find their resultant if they are $(4\hat{i} + \hat{j} - 3\hat{k})$ and $(3\hat{i} + \hat{j} - \hat{k})$.

(i) Solve

$$\frac{dy}{dx} = \frac{x}{y}$$

(j) Find the derivative of $\sin^{-1}(3x)$.

5 × 6

2. Answer any six questions :

$$(a) \text{ If } f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases} \text{ at } x = 0.$$

Show that $\lim_{x \rightarrow 0} f(x)$ does not exist.

(Turn Over)

(b) Evaluate

$$\lim_{x \rightarrow 0} \left(\frac{x - x \cos 2x}{\sin^3 2x} \right).$$

(c) If $y = \tan^{-1} x$, prove that

$$(1+x^2) y_2 + 2xy_1 = 0.$$

(d) If $f(x, y) = \frac{2x-3y}{x^2+y^2}$, find $f_x(1, 2)$ and $f_y(1, 2)$.

(e) Solve the differential equation,

$$x(1+y^2) dx + y(1+x^2) dy = 0.$$

(f) Evaluate

$$\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx.$$

(g) Find the area bounded by the curve $xy = c^2$, the x -axis and $x = 2, x = 3$.

(h) Evaluate

$$\int_0^{\pi/2} \frac{dx}{1 + \cot x}.$$

3. Find the value of 'a' if

$$\lim_{x \rightarrow 2} \frac{\log_e(2x-3)}{a(x-2)} = 1.$$

10

4. Differentiate, $\tan^{-1}(\sec x + \tan x)$.

10

5. Evaluate

$$\int \log(1+x^2) dx.$$

10

6. If $y = (\sin^{-1} x)^2$, show that

$$(1-x^2) y_2 - xy_1 - 2 = 0.$$

10

7. Find sine of the angle between the vectors $\vec{a}$ and $\vec{b}$ where

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}.$$

10

II Marks: 80

Th. 3-ENGINEERING MATHEMATICS-II

Time : 3 Hours

Answer any Five Questions including Q No. 1 & 2
Figures in the right hand margin indicates marks

Answer ALL the Questions:

2 X 10

Evaluate $\lim_{x \rightarrow 0} \frac{\sin px}{\sin qx}$

Evaluate $\lim_{n \rightarrow \infty} \frac{\sqrt{n} - 1}{\sqrt{n} + 1}$

Find the derivative of $\sqrt{ax^2 + bx + c}$ with respect to x , where a, b, c are constants.

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = \cos^{-1}\left(\frac{x}{y}\right)$

Integrate $\int \sqrt{1 + \cos 2x} \, dx$

Integrate $\int \frac{\sec^2 x}{1 + \tan x} \, dx$

Find order and degree of the differential equation

$$3 \frac{d^2 y}{dx^2} = \left\{ 2 + \left(\frac{dy}{dx} \right)^2 \right\}^{\frac{5}{3}}$$

h) Find the unit vector in the direction of the vector $3\hat{i} + \hat{j} + \hat{k}$

i) Solve $\frac{dy}{dx} = (x^2 + 1)(y^2 + 1)$

j) Find slope of the curve $y = \log x$ at $x = 1$

5x6

Q.2 Answer any SIX Questions:

a) Evaluate $\lim_{x \rightarrow 1} \frac{2^{x-1} - 1}{\sqrt{x} - 1}$

b) Test the continuity of the function $f(x) = \begin{cases} (1+2x)^{\frac{1}{2}}, & \text{if } x \neq 0 \\ e^2, & \text{if } x = 0 \end{cases}$ at $x = 0$

c) Differentiate $5^{\cos x^2}$ with respect to x

d) Find $\frac{dy}{dx}$ if $x = 2\cos^3 t$ and $y = 2\sin^3 t$

e) Prove that $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$. Where c is integrating constant.

f) Solve $\frac{dy}{dz} = \frac{\sqrt{1-y^2}}{\sqrt{1-z^2}}$

g) Find Scalar and Vector Projection of $\vec{a}$ on $\vec{b}$, Where $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

No3 Integrate $\int e^{3x} \cos 2x dx$

No4 a) Differentiate $(\log x)^{\tan x}$

b) If $y = \sin(\sin x)$ prove that

$$\frac{d^2 y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$$

No.5 a) Integrate $\int e^{\cos^2 x} \sin 2x dx$

b) Determine the area of parallelogram, whose adjacent sides are the vector $2\hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$

No.6 a) Find the value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

b) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin 2x}$

No7 Solve $\frac{dy}{dx} + y \tan x = \sec x$

(4)

(c) Solve

$$(1+x^2) \frac{dy}{dx} + 2xy = x^3.$$

7

6. (a) If the direction ratios of a line are $\langle 1, 1, 2 \rangle$, find direction cosines of the line.

2

- (b) Find the ratio in which the line segment through $(1, 3, -1)$ and $(2, 6, -2)$ is divided by zx -plane.

5

- (c) Find the equation of the plane passing through the points $(2, 2, 1)$ and $(9, 3, 6)$ and perpendicular to the plane $2x + 6y + 6z + 9 = 0$.

7

7. (a) Find the centre and radius of the sphere

$$x^2 + y^2 + z^2 - 4x + 2y - 2z - 10 = 0$$

2

- (b) Find $\frac{dy}{dx}$ when $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$

5

- (c) Integrate :

$$\int e^{2x} \sin x \, dx$$

7

ENGG. MATHEMATICS-II

(Code : BST-201)

Full Marks : 70

Time : 3 hours

Answer any five questions

Figures in the right-hand margin indicate marks

1. (a) Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan 5x}{\tan 7x}$$

2

- (b) Evaluate

$$\lim_{x \rightarrow a} \frac{\sqrt{x-b} - \sqrt{a-b}}{x^2 - a^2}, (a > b)$$

5

$$(c) \text{ If } f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$$

7

is continuous at $x = 1$, find 'a' and 'b'.

(2)

2. (a) Find the derivative of $\log x$ with respect to $\sqrt{x}$.

2

- (b) Find $\frac{dy}{dx}$, if $x^y = y^x$

5

- (c) Differentiate

$$\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$$

7

3. (a) Integrate

$$\int \frac{1}{2-5x} dx$$

2

- (b) Integrate

$$\int \sin^{20} x \cdot \cos^3 x \, dx$$

5

- (c) Determine the maximum and minimum value of the function

$$f(x) = 2x^3 - 3x^2 - 12x + 6$$

7

(3)

4. (a) Determine the slope of the curve

$$y = \log x \quad \text{at} \quad x = \frac{1}{2}$$

2

- (b) If $y = e^{\sin^{-1} x}$, prove that

$$(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = m^2 y$$

5

- (c) If $u = x^2 y + y^2 z + z^2 x$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$$

7

5. (a) Find the order and degree of the differential equation.

$$\frac{d^3 y}{dx^3} = \sqrt{x + \left(\frac{dy}{dx} \right)^5}$$

2

- (b) Evaluate

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$$

5

(4)

Total Pages—4

II—Sem/COMMON/2017(W)
(Old)

ENGG MATHEMATICS-II

(Theory—1)

Full Marks : 80

Time : 3 hours

Answer any five questions

Figures in the right-hand margin indicate marks

1. (a) Evaluate : $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x}$ 2

(b) Evaluate : $\lim_{x \rightarrow 0} \frac{\sqrt{3-2x} - \sqrt{3}}{x}$ 6

(c) Examine the continuity of the function $f(x)$ at $x = 0$ defined by

$$f(x) = \begin{cases} (1+2x)^{\frac{1}{x}} & \text{if } x \neq 0 \\ e^2 & \text{if } x = 0 \end{cases} \quad \text{at } x = 0. \quad 8$$

(c) Find the equation of the plane passing through the line of intersection of the planes $2x + 3y - 4z + 1 = 0$ and $3x - y + z + 2 = 0$ and passing through the point (3, 2, 1). 8

7. (a) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0$. 2

(b) A bag contains 7 white, 5 black and 4 red balls. If two balls are drawn at random find the probability that one is red and the other is black. 6

(c) Find the median of the distribution of weights of 150 persons from the data given below : 8

Weight : 30-40 40-50 50-60 60-70 70-80 80-90

No. of

Persons : 18 37 45 27 15 8

(2)

2. (a) Find the derivative of $\sin x$ w.r.t. $\cos x$. 2(b) Find $\frac{dy}{dx}$ if $y = (\log x)^{\tan x}$. 6(c) If $y = e^{m \sin^{-1} x}$, prove that
 $(1 - x^2)y_2 - xy_1 - m^2y = 0$. 83. (a) Integrate $\int \frac{1}{2-3x} dx$. 2(b) Integrate $\int x \tan^{-1} x dx$. 6(c) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$. 8

4. (a) Determine the order and degree of the differential equation.

$$\frac{d^2 y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad 2$$

(Continued)

(3)

(b) Find the maximum and minimum values of the function $y = x^3 - 6x^2 + 9x - 4$. 6(c) Solve: $\frac{dy}{dx} + y \sec x = \tan x$. 85. (a) Evaluate $\int_0^3 [x] dx$. 2(b) If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$. Prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u \quad 6$$

(c) Find the total area of the circle $x^2 + y^2 = 16$. 8

6. (a) Find the probability of getting an even number in throwing a die once. 2

(b) Find the ratio in which the line segment through $(1, 3, -1)$ and $(2, 6, -2)$ is divided by ZX-Plane. 6

(Turn Over)

(4)

4. If

$$f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$$

is continuous at $x = 1$, then find a and b . 10

5. Solve: $(1+x^2)\frac{dy}{dx} + 2xy = x^3$. 10

6. Find the equation of the sphere which passes through the points $(0,0,0)$, $(0,1,0)$, $(1,0,0)$ and $(0,0,1)$. 10

7. If $x = \sin t$, $y = \sin(pt)$, then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0. \quad 10$$

ENGG. MATH-II

(BST - 201)

Full Marks : 80

Time : 3 hours

Answer any five questions including Q.Nos. 1 & 2.

The figures in the right-hand margin indicate marks.

1. Answer all questions :

2 × 10

(a) Evaluate $\lim_{x \rightarrow 1} \left(\frac{x^2 - 2x + 1}{x^2 - x} \right)$.

(b) For what values of K

$$f(x) = \begin{cases} \frac{\sin 2x}{2x} & \text{if } x \neq 0 \\ K & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$?

(c) If $z = \sin\left(\frac{x}{y}\right)$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(d) Find $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$.

(Turn Over)

(2)

- (e) Find the order and degree of the differential equation

$$\left(\frac{dy}{dx}\right)^2 + y^3 = \frac{d^2y}{dx^2}$$

- (f) Find the equation of the sphere with centre (3,2,5) and radius is y.

(g) Integrate $\int(\sqrt{1-\cos 2x})dx$.

- (h) Find the derivative of $\cos^{-1}x$ w.r.t. $\tan^{-1}x$.

- (i) Find the image of the point (6, 3, -4) w.r.t. XY plane.

- (j) If the direction cosines of a line are

$$\left(\frac{2}{7}, \frac{3}{7}, \frac{K}{7}\right). \text{ What is the value of } K?$$

2. Answer any six questions :

5 × 6

- (a) Find $\frac{dy}{dx}$ if $x = 3\cos t - 2\cos^3 t$ and $y = 3\sin t - 2\sin^3 t$

(3)

(b) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x$.

(c) Integrate $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$.

- (d) Find the maximum and minimum values of the function

$$Y = 2x^3 - 15x^2 - 36x + 18$$

(e) Solve $\frac{dy}{dz} = \frac{\sqrt{1-y^2}}{\sqrt{1-z^2}}$.

- (f) Find the angle between the planes

$$x + 3y - 5z + 1 = 0 \text{ and}$$

$$2x + y - z + 3 = 0$$

- (g) Determine the area bounded by the curve

$$y^2 = x, x = 0, y = 1.$$

(h) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$.

3. Differentiate $y = \cos^2 \left(\cot^{-1} \sqrt{\frac{1+x}{1-x}} \right)$.

10