1ST SEM ./COMMON / 2022(W)

Th-3 Engineering Mathematics-I

Full Marks: 80 Time- 3 Hrs

Answer any five Questions including Q No.1& 2
Figures in the right hand margin indicates marks

1. Answer **All** questions 2 x 10

a. Find the value of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$.

b. Find x and y when $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

c. Find the minimum and maximum value of $5 \sin x + 12 \cos x$.

d. Find $\tan\left(\frac{\pi}{4} + 2 \cot^{-1} 3\right)$.

e. Determine the ratio in which the line segment joining $(2, -3)$ and $(5, 6)$ is divided by x –axis.

f. Find the perpendicular distance from the point $(2, 1)$ to the straight line $12x - 5y + 9 = 0$.

g. Find the equation of the circle which touches the x –axis and whose centre is at the point $(3, 4)$.

h. Find image of the point $(1, -2, 4)$ with respect to YZ–plane.

i. Find the direction cosines of a straight line whose direction ratios are 1, 2, 3.

j. Find the centre and radius of the sphere
$3x^2 + 3y^2 + 3z^2 - 12x - 6y + 9z + 1 = 0$.

2. Answer **Any Six** Questions 6 x 5

a. Without expanding prove that

$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc\,(a-b)(b-c)(c-a)$$

b. Solve the following equations by Matrix Method,
$x + 2y = 3$ and $3x + y = 4$

c. Prove that $\sin 10^\circ . \sin 30^\circ . \sin 50^\circ . \sin 70^\circ = \frac{1}{16}$

d. Find the equation of the straight line which passes through the point $(3, 4)$ and sum of its intercepts on the axes is 14.

e. Find the equation of plane passing through the point $(2, -2, -1)$ and parallel to the plane $2x + y - 3z - 2 = 0$.

f. Find the equation of the sphere whose centre at $(3, 1, -2)$ and the sphere passing through the point $(1, 1, 2)$.

g If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

3 a. Solve the following equations by Cramer's Rule, 5
$2x - 3y + 5 = 0$ and $5y - 3x - 8 = 0$

b. Find the equation of the plane passing through the interaction of planes $2x + 3y - 4z + 1 = 0$ and $3x - y + z + 2 = 0$, and passing through the point $(3, 2, 1)$. 5

4 a. Find the equation of the circle which passes through the points $(1, -2)$ and $(4, -3)$ and has its centre lies on the line $3x + 4y = 7$. 7

b. If the point (x, y), $(1, -2)$ and $(3, -4)$ are collinear, prove that $x + y + 1 = 0$. 3

5 a. Find the equation of the sphere passing through $(1, 2, -3)$ and $(3, -1, 2)$ and centre lying on $X-$axis. 5

b. If $A + B + C = \pi$, 5
Prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$.

6 a. In a ΔABC if $m\angle A = 90^\circ$, prove that $\tan^{-1} \frac{b}{a+c} + \tan^{-1} \frac{c}{a+b} = \frac{\pi}{4}$, 5
where a, b and c are the sides of the triangle.

b. Verify that $[AB]^T = B^T A^T$, 5
where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$.

7 a. Find the equation of a straight line parallel to the line $2x + 3y + 11 = 0$ and sum of its intercepts on the axes is 15. 6

b. If $A + B = 45^\circ$, show that $(1 + \tan A)(1 + \tan B) = 2$. 4

1st SEMESTER/COMMON/2021(W)(NEW)

Th3 ENGINEERING MATHEMATICS - I

Full Marks: 80 Time- 3 Hrs

Answer any five Questions including Q No.1& 2

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1. Answer all questions 2 x 10

a. Find M_{23} *and* C_{32} of the determinant $\begin{bmatrix} 4 & 3 & 8 \\ 6 & 7 & 5 \\ 9 & 0 & 6 \end{bmatrix}$.

b. Find k for which the following lines are perpendicular to each other 2x+3y-1=0 and kx-4y+2=0.

c. Find $\sin(\tan^{-1} x + \cot^{-1} x)$.

d. Find the centre and radius of the sphere $(x-2)(x+2) + y^2 + (z-3)(z+3) = 0$

e. If $[3 \quad 4 \quad 2] \times B = [2 \quad 1 \quad 0 \quad 3 \quad 6]$. *Find order of* B

f. What is ASTC Rule in Trigonometry?

g. If the equation $3x^2 - \frac{k}{2}y^2 - 6x + 9y - 3 = 0$ represents a circle, find k.

h. A line makes angle α, β, γ with X,Y,Z axes, then find $sin^2\alpha + sin^2\beta + sin^2\gamma$.

i. Find the multiplicative inverse of the matrix $\begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$.

j. Find the intercepts cut off by the plane 2x+3y-z=6 on the axes.

2. Answer **Any Six** Questions 5X6

a. Find the angle between two lines whose direction ratios are$\langle 1,2,1 \rangle$ *and* $\langle 2,-3,4 \rangle$.

b. Find the equation of the circle whose diameter is the portion of the line 3x+4y-12=0 intercepted between the coordinate axes.

c. Prove without expanding

$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a).$$

d. Find the maximum and minimum value of the following $6\cos x - 8\sin x - 3$

e. Find the equation of the line which passes through (-3, 7) and makes intercepts on the axes equal in magnitude but opposite in sign.

f. In a triangle ABC if $m\angle A = 90^0$,prove that
$\tan^{-1}\frac{b}{a+c} + \tan^{-1}\frac{c}{a+b} = \frac{\pi}{4}$, where a, b, c are sides of the triangle.

g. If $A=\begin{pmatrix}3 & 2\\ 2 & 3\end{pmatrix}$, *evaluate* $A^2 - 6A + 8I$, where I is the Identity matrix of the given order.

Answer any **three** questions

3 a. Find the equation of the line passing through intersection of the lines $x + 3y - 7 = 0$ *and* $3x - y - 11 = 0$ and centroid of the triangle whose vertices are the points (3,-1), (1, 3) and (2, 4). 5

b. Evaluate $\sin 18^0$. 5

4 a. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to the planes x+2y+2z=5 and 3x+3y+2z=8. 7

b Solve by Cramer's Rule
2x+3y=1 and –x+y=-3 3

5 If $A + B + C = \pi$, *prove that*
$sin^2A + sin^2B + sin^2C = 2 + 2\cos A\cos B\cos C$ 10

6 Find the equation of a sphere whose centre lies on the plane $x + y + z = 0$ and which passes through the points (1,-3,4),(1,-5,2) and (1,-3,0) . 10

7 a. Evaluate $\tan^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ 6

b. Find the value of 'a' so that the points (1, 4),(2,7),(3,a) are collinear. 4

1ST SEMESTER/ COMMON/ 2020(W)(NEW)

TH-3 ENGINEERING MATHEMATICS-I

Time- 3 Hrs

Full Marks: 80

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2 x 10

1. Answer **All** questions

a. Find the value of $\frac{\sin 15+\cos 15}{\cos 15-\sin 15}$

b. Find the value of $\tan^{-1}\left(2\cos\frac{\pi}{3}\right)$

c. The maximum value of $\begin{vmatrix} sin^2x & sinxcosx \\ -cosx & sinx \end{vmatrix}$

d. Find the value of k if the lines $2x-3y+7=0$ and $x-ky+2=0$ are perpendicular to each other.

e. If $A=\begin{pmatrix} 2 & 4 \\ 3 & 13 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 5 \\ 2 & -2 \end{pmatrix}$, then find the value of $A-2B$

f. Find centre and radius of sphere $x^2+y^2+z^2-2x-2y-2z-1=0$

g. If the distance between the points $(-1,-1,z)$ and $(1,-1,1)$ is 2 , then find the value of z

h. Find the image of the point $(3,-1,5)$ with respect to XY - Plane

i. Find the direction cosines of a line whose direction ratios are $(1,1,1)$

j. Find the Value of $\sin 70\,(4cos^2 20-3)$

6 x 5

2. Answer **Any Six** Questions

a. Solve by Cramer's rule 2x-3y=7 and 3x-2y=3

b. Find the equation of circle having centre at $(2,3)$ and circle passes through the point $(1,2)$.

c. Prove that $sin20\ sin40\ sin60\ sin80 = \frac{3}{16}$

d. Find angle between the planes $2x+y-3z+2=0$ and $3x-y+2z+3=0$

e. Find Inverse of the matrix $\begin{pmatrix} 2 & 1 & -2 \\ 1 & 2 & 1 \\ 3 & 6 & 4 \end{pmatrix}$

f. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$ then prove that $x+y+z=xyz$

g Find the equation of line passing through the point $(2, -4)$ and parallel to the line $4x + y - 3 = 0$ 10

3 Prove that without expanding $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

4 a Find the equation of line passing through intersection of lines $2x - y - 1 = 0$ and $3x - 4y + 6 = 0$ and parallel to the line $x + y - 2 = 0$ 5

b Find the value of $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\frac{3}{\sqrt{10}}$ 5

5 Find the ratio and co-ordinate in which the line segment joining the points $(1, 3, -1)$ and $(2, 6, -2)$ is divided by ZX-Plane 10

6 Solve by matrix method
$x - y + z = 4$, $2x + y - 3z = 0$, $x + y + z = 2$ 10

7 Find the equation of plane passing through the points $(2, -3, 1)$ and $(-1, 1, -7)$ and perpendicular to the plane $x - 2y + 5z + 1 = 0$ 10