2[ND] SEM. / COMMON /2025(S)

TH3 MATHEMATICS-II

Time: 3 Hrs

Full Marks= 70

Answer Any Four Questions including Q No.1& 2
Figures in the right hand margin indicates marks

1. Answer **All** Questions. 2 x 10

a. Define a unit matrix. Give an example of a unit matrix of order 3.

b. Solve $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix} = 5$.

c. Evaluate $\int_0^1 \frac{dx}{1+x^2}$.

d. Evaluate $\int \sqrt{1+\cos 2x}\, dx$.

e. Find the centre and radius of the circle $2x^2 + 2y^2 + 4x + 8y + 2 = 0$.

f. Find the perpendicular distance from the point (2,1) to the straight line $12x - 5y + 9 = 0$.

g. For what value of 'α', the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = \alpha\hat{\imath} + \hat{\jmath} + 5\hat{k}$ are perpendicular to each other.

h. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ then calculate $\vec{a} \times \vec{b}$.

i. Find the order and degree of the equation $3\frac{d^2y}{dx^2} = \left\{2 + \left(\frac{dy}{dx}\right)^2\right\}^{5/3}$

j. Solve $xdx - ydy = 0$

2. Answer **Any Six** Questions. 5 x 6

a. Solve $2x - y = 2$, $3x + y = 13$ by Cramer's rule.

b. Find the inverse of a matrix $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.

c. Find the scalar and vector projection of $2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on $\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

d. Find the area enclosed by the circle $x^2 + y^2 = a^2$.

e. If Latus rectum of an ellipse is half of its minor axis then calculate its eccentricity 'e'.

f. Evaluate $\int e^x \sin x\, dx$.

g. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}$.

Answer **Any Two** Questions.

3 a. Prove that $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$ 5

b. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx$. 5

4 a. Evaluate $\int \frac{3x+1}{(x+1)(x-2)} dx$. 5

b. Find the equation of straight line passing through point $(-4, 2)$ and parallel to the line $4x - 3y - 10 = 0$. 5

5 a. Find the equation of the circle which passes through the points $(0,0)$, $(3,0)$ and $(0,4)$. 5

b. Solve $\frac{dy}{dx} = (e^x + 1)y$ 5

6 a. Verify that $(AB)^T = B^T A^T$ where 5

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \ \& \ B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}.$$

b. Find the area of triangle whose two sides are represented by the vectors $\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ *and* $\hat{\imath} + \hat{\jmath} + 2\hat{k}$. 5