LECTURE NOTES

SUBJECT - ENGINEERING MECHANICS

CODE – Th- 4 (b)

Preapared by

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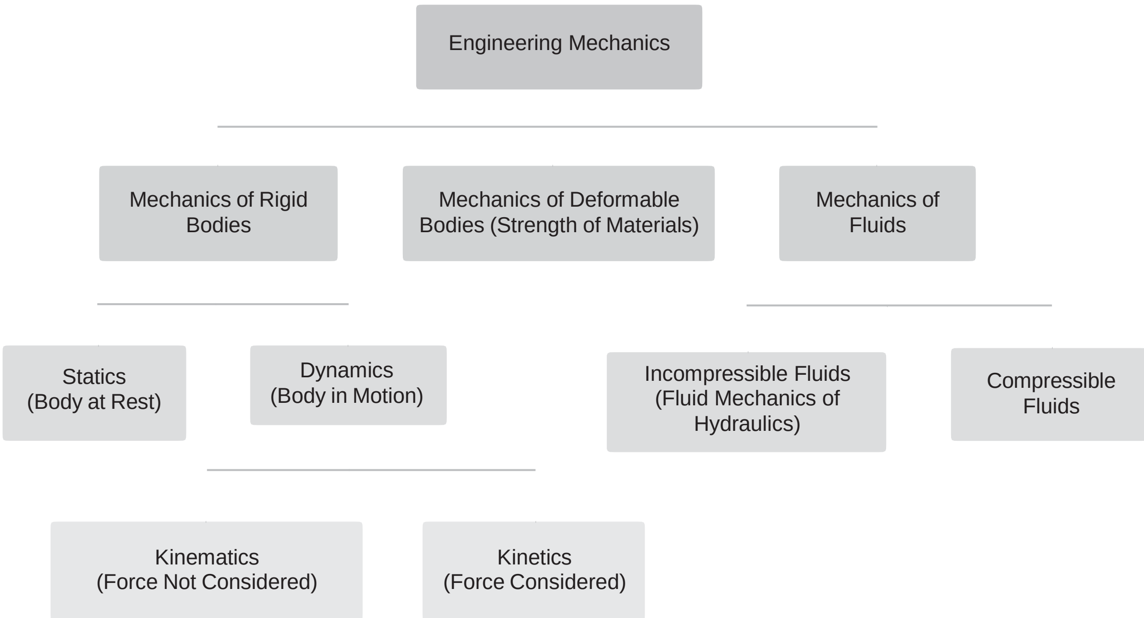
CHAPTER - 1 Basics of Mechanics and Force System



FUNDAMENTAL CONCEPTS`

Space : It is a region, which extends in all direction and contains everything in it. Examples : Sun,moon, star etc. In space position of body is located with respect to a reference system. The position of anaircraft in space found with respect to earth.

Time : It is measure of succession of events. The time is measured in second(s) and other related units. An event can be describe by position of point.

Mass : It is an indication of the quantity of matter present in a system. The more mass means morematter.

Flexible body : A body, which deform, under the action of applied force, is call flexible body.

Rigid body : A body, which does not deform, under the action of applied forces, is call rigid body.

SCALAR AND VECTOR

The physical quantities in mechanics can be Express mathematically as follows :

Scalar Quantity : Quantities, which described by their magnitude known as scalar quantity. Examples are mass, length, time, volume, temperature etc.

Vector Quantity : Quantities, which described by their magnitude and direction (both) known asB vector quantity. Examples are velocity, force, acceleration, momentum etc.

A vector quantity can be represented by straight line with an arrow head. The length of straight line represents the magnitude while direction of line represents the direction of vector and arrow head indicate the sense of direction.

UNITS OF MEASUREMENT [SI UNITS]

Fundamental units : Length, Mass and Time are the basic fundamental quantities and unit of these quantities are known as fundamental units.

Derived units : Units of other than fundamental quantities may be derive from the basic units referred as derived units. Examples: (1) Area is result of multiplication of two lengths quantity as L^2. (2) Velocity is length divided by time as $\frac{L}{T}$. (3) Force is product of mass & acceleration as kg . m/sec^2 [N].

SI units : By international agreement in in 1960, the international system of units known as S.I. Unit is accepted and used all over the worldwide. The symbols and notation of SI units and their derivatives are standardize to avoid any possibility of confusion.

Fundamental SI units

Sr. No.	Fundamental Quantity	Name of SI unit	Symbol
1	Length	Meter	m
2	Mass	Kilogram	kg
3	Time	Second	s
4	Electrical current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous intensity	Candela	cd

FORCE

Introduction

what is force ? Suppose you are driving the nail in a wall. Naturally you are required to push thenail in the wall. Then what is this "push" ? It is the force. Now consider another situation in which drumis rolling down and you want to stop it. Then naturally you will apply some resistance to its motion. Thisresistance is nothing but the force. Hence force is an external agent which tends to changes the state ofbody at rest or in motion.

Unit of force in SI system

The force is measure in Newton (N). A Newton is defined as a force, which can produce an acceleration of 1 meter per second^2 in a body of 1 kg mass. The larger units of force are –

1 Kilo Newton (kN) = 1000 Newton = 10^3 N

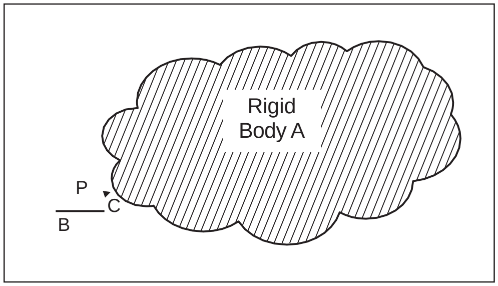
1 Mega Newton (MN) = 1000 × 1000 Newton = 10^6 N

1 Giga Newton (GN) = 1000 × 1000 × 1000 Newton = 10^9 N

Characteristics of force

As you know, force is a vector quantity, it means, it can identify by magnitude as well as direction. To represent force completely it is required following four elements, which are known as characteristics of force.

(A) Magnitude (B) Direction (C) Sense - Type of force - and (D) Point of application.



Characteristics of force

Figure show a rigid body A on which force P act at point C, which is point of application,whileline BC show direction of force P with magnitude shown as P above the line and arrow head at point Cshow sense (type of force).

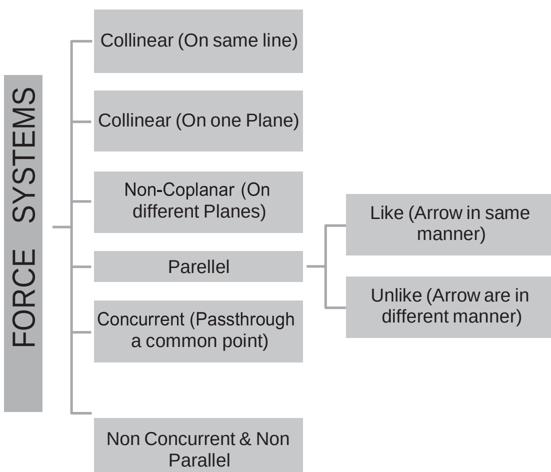
Effect of Force

The force when applied, following effects may happens on rigid body.

i. Change its state of rest OR motion.
ii. Accelerate OR retard its motion.
iii. Change its shape and size.
iv. Turn OR rotate it.
v. Keep it in equilibrium.

Force system and Classification

When several forces or group of force act simultaneously on a body, the body is said to be under the action of force system. These force systems are further classify according to the line of action and arrangements of the forces as shown below.



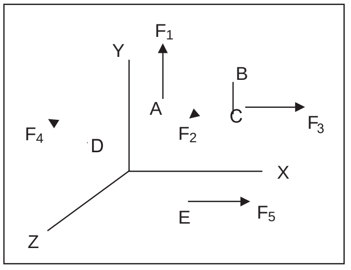
Collinear force system

The line of action of all forces lies along the same straight line as shown in fig then that force system is known as collinear force system.

Collinear force system

Coplanar force system

All the forces in this system are lie in one plane, system is known as coplanar force system. Forces F_1, F_2 and F_3 only of force system acting on plan XY i.e., on one (same) plane shown in fig is example of coplanar force system.



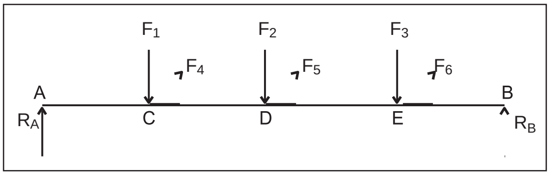
Coplanar force system and non-coplanar force system

Non-coplanar force system

All the force in the system are not lie in the same plane but act on different planes as shown in fig acting on planes XY, YZ and ZX. (Forces F_1, F_2, F_3, F_4 and F_5)

Parallel force system

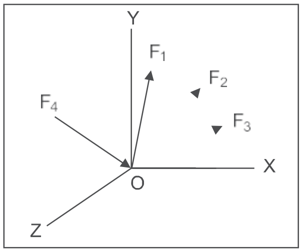
The line of action means direction of all the forces are parallel to each other and do not intersect. This system is further sub classified as, Like parallel forces and Unlike parallel forces. If forces acting in the same direction, and are parallel to each other, are known as like parallel forces, where as if they are acting in opposite direction, and are parallel to each other, are known as unlike parallel forces as shown in fig



Parallel force system

Concurrent force system

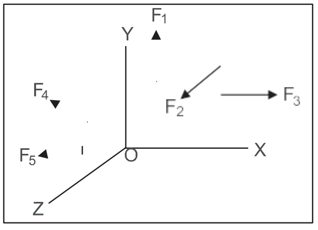
All the forces have different direction but their line of action (Direction) passes through a single common point. Such force system known as concurrent force system. The point, where the line of action of all the forces meet is known as point of concurrency of the force system.



Concurrent force system

Non-concurrent and Non-parallel force system

If the forces of force systems are not lie in same line and not pass-through common point as well as line of action are not parallel to each other, it means, force system which not satisfy the condition of parallel, concurrent and linear force system, then such system is known as Non-concurrent & Non-parallel force system. If all the forces lie on same plane, it's known as coplanar non-concurrent non-parallel force system and if all the force lies on different planes, then is non coplanar non-concurrent non-parallel force system.



Non-concurrent non-parallel force system

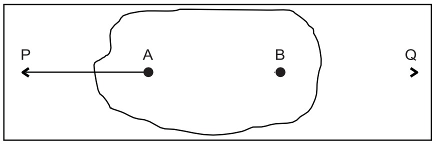
Principles of static for force

Following law or principles for force are required to study the coplanar concurrent force system.

(B) Equilibrium law (B) Principle of superposition (C) Principle of transmissibility.

(A) Equilibrium law of force

Two forces can be in equilibrium only, if they are equal in magnitude, opposite in direction and collinear in action.



Equilibrium law of force

(B) Principle of superposition of force

The action of a given system of forces on a body will not change, if we add or subtract from it another system of the forces in equilibrium.

(c) Principle of transmissibility of force

The point of application of force may be transmitted along its line of action without changing the effect of forces on the body.

COMPOSITION OF FORCE (RESULTANT FORCE)

If number of forces in force system are applied on a body, then we can replace it in a single force, which produce the same effect as force system, then this replaced single force is known as resultant force and the process by which the resultant force is found out is known as composition of forces. There are two methods for finding out resultant force. (I) Analytical method and (II) Graphical method. We have study the graphical methods in practical.

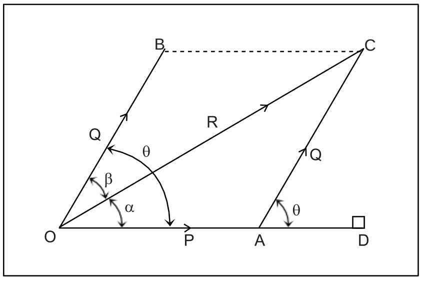
Analytical methods for concurrent force system`

The resultant force of a given force system can be find out by following three methods: (a)Law of parallelogram of force (B) Law of triangle of force (C) Method of resolution of forces

Law of parallelogram of forces

This method is use to find resultant of two coplanar concurrent forces acts on a body. Law of parallelogram of force state as below.

Two forces acting simultaneously on a body, if represent in magnitude and direction by two adjacentsides of a parallelogram, then diagonal of parallelogram from the point of intersection of two forces representthe resultant force in magnitude as well as in direction.



Let consider force P and Q acting on a body at a point O, with angle θ between two forcesP and Q. The resultant force R of two forces P & Q can be mathematically represent by:

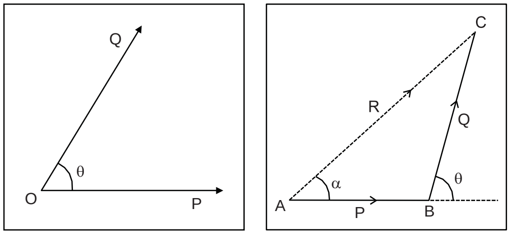
Magnitude of resultant is obtained by $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Direction of resultant (σ) is obtained by $\tan\sigma = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

Law of triangle of force

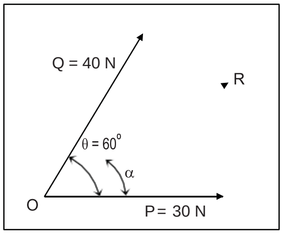
When only two and two forces are acting on common point, we can apply this law to find out resultant force of force system. It States "If two forces acting at a point are represented in magnitude and direction by two sides of a triangle taken in order; the third side of the triangle taken in opposite order represent the resultant force of two forces in magnitude and direction."

Let consider force P and Q acting on a body at a point O, with angle θ between two forcesP and Q. This method is generally use as graphical method, which we have to study in practical.



Law of triangle of force

Example 1. *Find the resultant force of two forces 30 N and 40 N acting at a point with an angle of 60º*



Solution:

P = 30 N, Q = 40 N & Angle (between P and Q) θ = 60º.

Put this value in equations, we get magnitude as well as direction of resultant force.

(i) Magnitude $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$R = \sqrt{(30)^2 + (40)^2 + 2 \times 30 \times 40 \times \cos 60} = \sqrt{3700}$$

Hence, magnitude of resultant R = 60.83 N **(Answer)**

(ii) Direction $\tan\sigma = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$$= \frac{40 \times \sin 60}{30 + (40 \times \cos 60)} = \frac{34.64}{50}$$

Hence, σ = 34.71º from direction force P.

(Answer)

Example 2 *Two forces equal to P and 2P respectively act on a particle. When the first force is increasedby 120 Newton and the second force is doubled, the direction of resultant force remains the same in bothcases. Determine the value of force P.*

Solution :

Case (i) $P_1 = P$; $Q_1 = 2P$; $\theta_1 = \theta$; $\sigma_1 = \sigma$

Case (ii) $P_2 = P+120$; $Q_2 = 4P$; $\theta_2 = \theta$; $\sigma_2 = \sigma$

Apply Condition of direction of resultant force remains same in both case.

For Case (i) $$\tan\sigma_1 = \frac{Q_1 \sin\theta_1}{P_1 + (Q_1 \cos\theta_1)} = \frac{2P\sin\theta}{P + (2P\cos\theta)} = \tan\sigma$$

Case (ii) $$\tan\sigma_2 = \frac{Q_2 \sin\theta_2}{P_2 + (Q_2 \cos\theta_2)} = \frac{4P\sin\theta}{(P+120)\,(4P\cos\theta)} = \tan\sigma$$

Equating both cases; $$\frac{2P\sin\theta}{P + (2P\cos\theta)} = \frac{4P\sin\theta}{(P+120)\,(4P\cos\theta)}$$

$$\frac{2P\sin\theta}{4P\sin\theta} = \frac{P + (2P\cos\theta)}{(P+120)\,(4P\cos\theta)}$$

$$\frac{1}{2} = \frac{P + (2P\cos\theta)}{(P+120)\,(4P\cos\theta)}$$

$$(P+120) + (4P\cos\theta) = 2P + (4P\cos\theta)$$

$$P + 120 = 2P$$

$P = 120$ N **(Answer)**

Method of Resolution

Step-1: If necessary, rearrange all the forces in either pull or push form and gives notation F_1, F_2... & so on in anticlockwise manner from positive X axis. Also compute angle of all forces with the positive X axis in anticlockwise manner.

Step-2: Find algebraic sum of horizontal component of all the force with relevant sign and give notation as ΣH. [+ve as: $\rightarrow$ and -ve as: $\rightarrow$]

Step-3: Find algebraic sum of vertical component of all the force with relevant sign and give notation as ΣV. [+ve as: $\rightarrow$ and -ve as: $\rightarrow$]

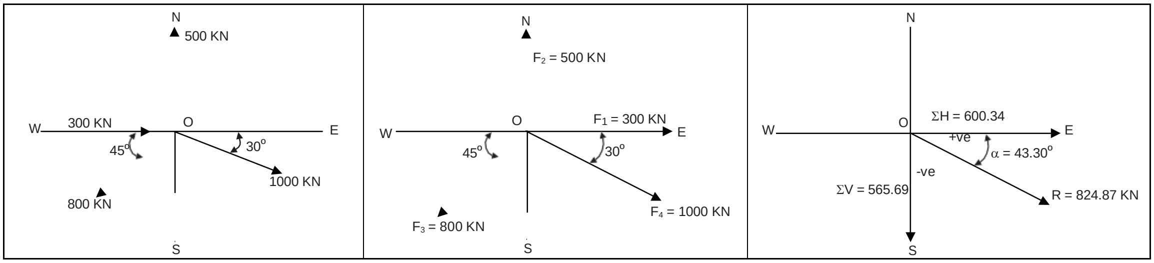
Step-4: Find the magnitude of resultant force R by equation. $R^2 = \Sigma H^2 + \Sigma V^2$

Step-5: Find angle (σ) of resultant force with horizontal by equation. $\tan\sigma = \dfrac{\Sigma V}{\Sigma H}$

Example 3 *A system of four coplanar concurrent forces are acting at a point as given below. Find the magnitude and direction of resultant force.*

(i) 500 kN acting due North *(iii) 800 kN acting South-West*

(ii) 1000 kN acting 30° South of East *(iv) 300 kN acting from West*



(a) Data (b) All forces in same sense (c) Resultant force

Step-1: In This case, 300 kN force acting as push force on the point & all others are pull, so this 300 kN force should re-arranged as pull force by extending line of action as shownfig (b).

Step-2 & 3: Now to Find out Horizontal and vertical components of all the forces it is easy way to workout in tabular form as shown below.

Sr. No.	Magnitude of Forces (kN)	Angle θ with respect to +X axis	Horizontal Component $F_x = F\cos\theta$ (kN)	Vertical Component $F_y = F\sin\theta$ (kN)
1	$F_1 = 300$	0°	300 cos 0° = 300.00	300 sin 0° = 0.00
2	$F_2 = 500$	90°	500 cos 90° = 0.00	500 sin 90° = 500.00
3	$F_3 = 800$	180+45 = 225°	800 cos 225° = –565.69	800 sin 225° = –565.69
4	$F_4 = 1000$	360 – 30 = 330°	1000 cos 330° = 866.03	1000 sin 330° = –500.00
		Algebraic Sum	$\Sigma H = +600.34 \rightarrow$ kN	$\Sigma V = -565.69 \downarrow$ kN

Step-4: Find magnitude of resultant force by equation:

$$R^2 = \Sigma H^2 + \Sigma V^2 = (600.34)^2 + (-565.69)^2 = 680413.2917$$

R = 824.87 kN **(Answer)**

Step-5: Find the angle σ of resultant force R from ΣH sign to ΣV sign

$$\tan\sigma = \frac{\Sigma V}{} = \frac{-565.69}{} = -0.9423$$

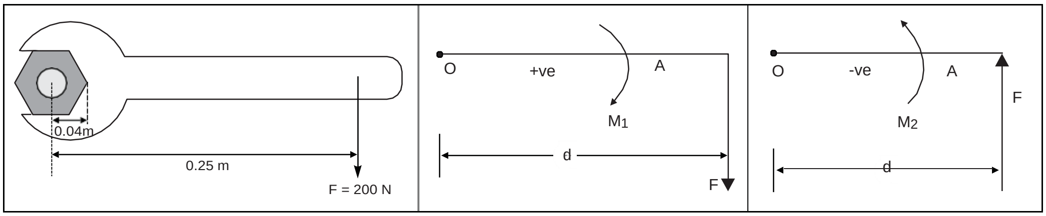
ΣH   600.34

$\sigma = \tan^{-1}(-0.9423)$ $\therefore\ \sigma = 43.30^\circ$ from ΣH → (East) towards ΣV ↓ (South)

or $\sigma = (360^\circ - 43.30^\circ) = 316.70^\circ$ from +X axis in anticlockwise manner **(Answer)**

Moment of force

When a force acts on a body, the body moves or tends to move in the direction of the force. However, if the force acts on the body at some distance through an arm; it produces moment on the body resulting in rotation. For example, the spanner used to tighten or open the Bolt



(a) Clockwise (b) Anticlockwise

Here force F is applied at a distance d from center point O of bolt which producing moment & hence rotation of the bolt. The moment of force about the point is given by product of force F and perpendicular distance (Arm) of the line of action of the force from the given point O as d.

Mathematically; Moment = M = Force (F)X Perpendicular distance (d)

Therefore, $M = F \cdot d$

Unit of moment of force involves two quantities; namely Force and Distance. Depending upon unit of force and distance, the moment can be expressed in Newton · metre (N·m) or kilonewton · meter (kN·m)

Types of moment : Force F on spanner has a turning effect on Bolt in clockwise direction as shown in figure (a). Now if you change the sense of force as upward, it will have turning effect in anticlockwise direction as shown in figure (b). Hence moments are of the two types and gives sign convention as follow:

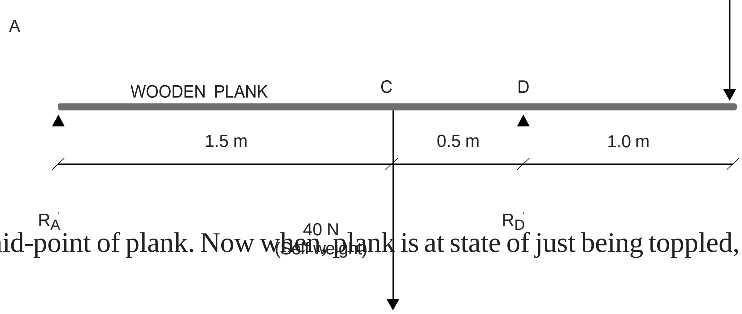
(i) clockwise moment as positive (+ve) and (ii) anticlockwise moment as negative (–ve)

Varignon's principle or Principle of moments

It states the moment of force about any point is equal to the sum of moments of the components of the force about the same point.

Example 4. *A Uniform wooden plank AB of length 3m has a weight of 40 N. It is supported at end A and at point D which is 1m from other end B. Determine the maximum weight W that can be places at end B,so that Plank does not topple.*

Solution



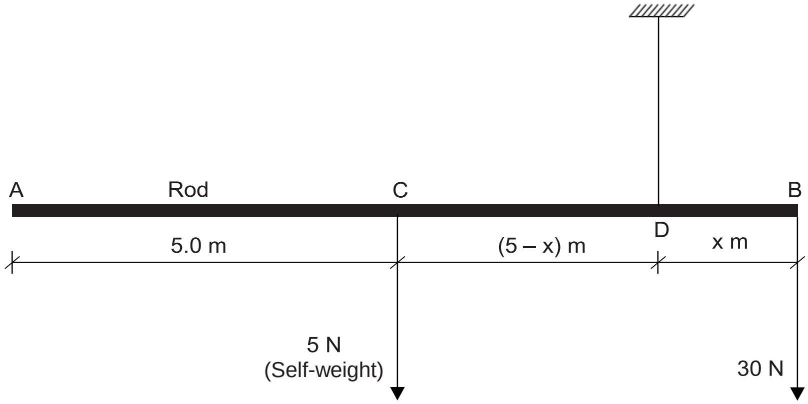
Self-weight act at mid-point of plank. Now when plank is at state of just being toppled, the reactionR_A at point A is zero.

Hence taking moment about point D, we get

$(\circlearrowright) W \times 1 = 40 \times 0.5 \ (\circlearrowleft)$

$\therefore$ W = 20 N **(Answer)**

Example . *A Uniform rod of 10m length has a self-weight of 5 N. The rod carries a weight of 30 N hung from one of its ends. From what point the rod be suspended so that it remains horizontal?*



Let us take the rod be suspended at point D, which is at distance x from its end B where weight of 30 N is hung. To remains rod to horizontal, moment at point D should be zero. By considering $\circlearrowright$ clockwise moment as +ve, take moment about point D, we get

$(30 \times x) - [5 \times (5 - x)] = 0$

$\therefore (30 \times x) - 25 + (5 \times x) = 0$

$\therefore (35x) = 25$

$\therefore X = 25/35 = 0.714$ m **(Answer)**

The rod is suspended to remain horizontal at point D at distance 0.714m from end B where weight of 30 N is hung.

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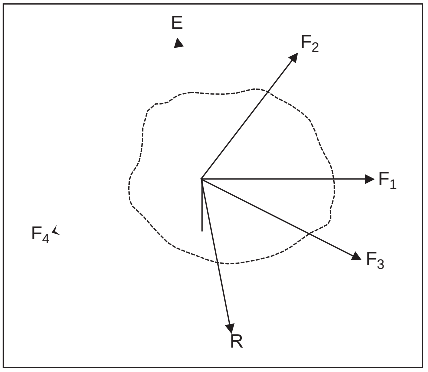
CH-2 EQUILIBRIUM

EQUILIBRIUM & EQUILIBRANT



If the resultant of all the forces and resultant moments of all the forces on the body is zero, then the body is say to be in equilibrium. In such condition, the body may at rest or moving with constant velocity. Now if resultant force on the body is not zero, then to bring the body in equilibrium, we have to apply the force, which is known as equilibrant force. The equilibrant force is equal, opposite & collinear with the resultant force of force system acting on the body.

Condition of Equilibrium

Coplanar force system as we have already study in unit-I are following.

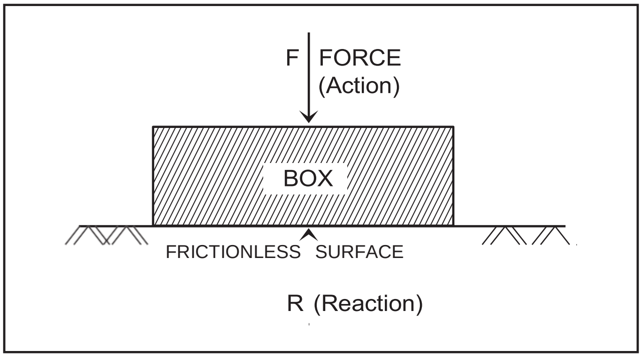
(a) Collinear (b) Concurrent (c) Parallel (d) Non concurrent non parallel

For all above force system, we can say that the body is in equilibrium, when total effect on body is zero. Mathematically (i) $\Sigma H = 0$ (ii) $\Sigma V = 0$ and (iii) $\Sigma M = 0$. These are analytical condition for equilibrium.

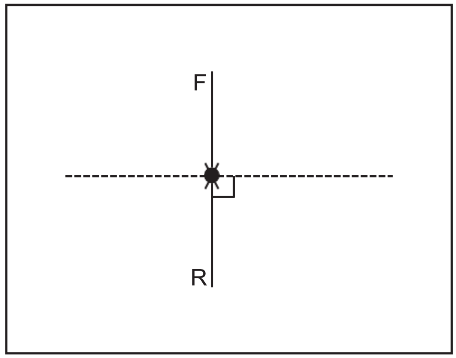
Graphical condition of equilibrium is: Force polygon must be close means closing side of polygon is zero.

Free body and Free body diagram

For equilibrium of the body or structure, a diagram of the body is drawn as isolated from its surrounding, removing its supports & holding devices. The forces acting on it are shown clearly showing magnitude, direction and location of all external forces including weight, applied forces, reactions and dimensions & angles. The body may be shown as a point when the forces acting on it are concurrent. The diagram so created is known as the free body diagram & said body as free body. In constructing the free body diagrams, it is necessary to know the kind of the forces offered by the supports.



(b) Space diagram

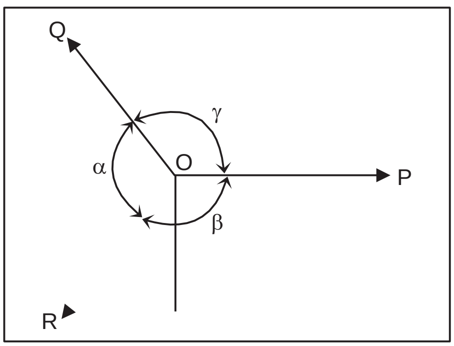


(b) Free-body diagram

LAMI'S THEOREM

It states : *If three coplanar concurrent forces acting on the body are in equilibrium then each force is proportional to the sine of angle between the other two forces.*

Let P, Q & R be three forces acting on the body as shown in fig. (a). Since body is in equilibrium, they can be represented by sides of triangle ABC as shown in fig. (b). Applying sine rule for triangle ABC;



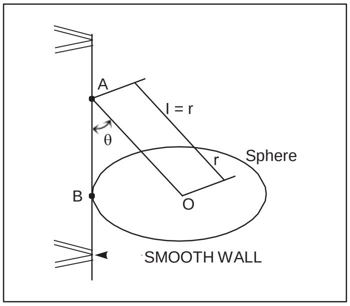
Lami's theorem

$$\frac{AB}{\sin(\pi-\alpha)} = \frac{BC}{\sin(\pi-\beta)} = \frac{CA}{\sin(\pi-\gamma)}$$

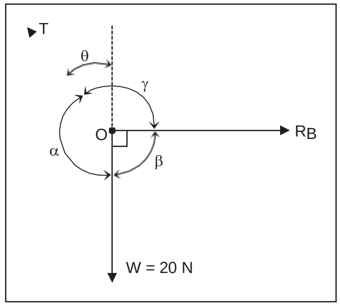
$$\therefore \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Example 1. *A smooth sphere of radius r 150 mm and weight W 20 N is hung by string whose length equal the radius of sphere with contact to smooth vertical wall. Find inclination and tension in string as well as reaction of wall.*

Solution:



(a) Space diagram



(b) Free body diagram

(i) Draw space diagram as shown in fig. (a), from given data.

In triangle ABO,

$$\sin\theta = \frac{OB}{OA} = \frac{r}{2r} = 0.5$$

$\therefore$ $\theta = 30°$ **(Answer)**

(ii) Now apply lami's theorem for free body diagram as shown in fig. (b), We get

$$\frac{R_B}{\sin\alpha} = \frac{T}{\sin\beta} = \frac{W}{\sin\gamma}$$

Here $\sigma = 180 - \theta = 150$; $\beta = 90$ and $\gamma = 90 + \theta = 120$ and W= 20N.

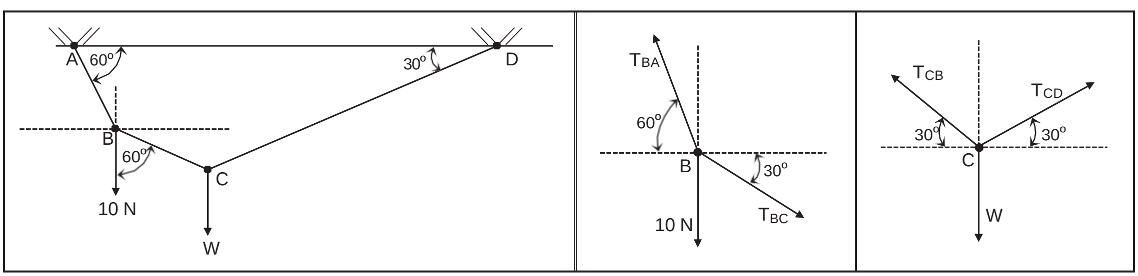
Putting values in Lami's equation, we get

$$\frac{R_B}{\sin 150^o} = \frac{T}{\sin 90^o} = \frac{W}{\sin 120^o}$$

$\therefore$ $R_B = 11.55$ N **(Answer)**

and $\therefore$ T = 23.10 N **(Answer)**

Example 2. *Find the value of W if a light weight chain ABCD is suspended as shown in below fig. 2.7(a).*



(a) Data (b) FBD for point B (c) FBD for point C

Solution:

Free body diagram for point B and Point C are shown in fig. (b) and (c) respectively.

(a) Applying Lami's theorem at point B [fig. (b)]

$$\frac{T_{BC}}{\sin(90+60)^{o}} = \frac{10}{\sin(180-60+30)^{o}}$$

$$\therefore\ T_{BC} = \frac{\cancel{10 \times \sin 150^{o}}}{\sin 150^{o}} = 10\text{ N}$$

(b) Applying Lami's theorem at point C [fig. (c)]

Considering $T_{BC} = T_{CB}$

$$\frac{W}{\sin(150-30-30)^{o}} = \frac{T_{CB}}{\sin(90+30)^{o}} = \frac{T_{BC}}{\sin(120)^{o}}$$

$$\therefore\ W = \frac{T_{BC} \times \sin(120)^{o}}{\sin(120)^{o}}$$

$\therefore$ W = 10 N **(Answer)**

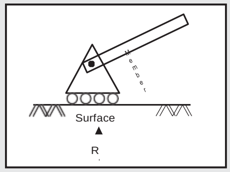
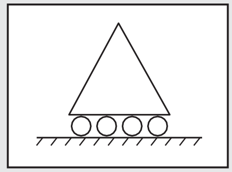
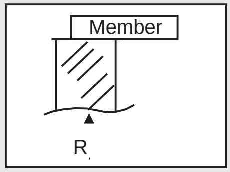
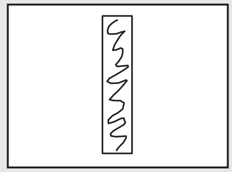
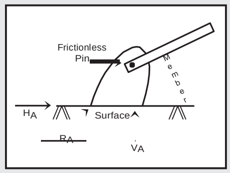
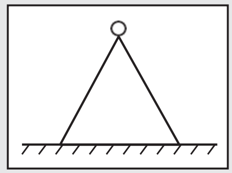
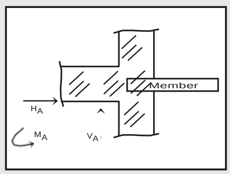
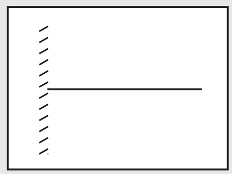
TYPES OF SUPPORTS, LOADING & BEAM

Beam is a structural element which is taken as specimen for studying the effects of loads, on the structure. It's carrying transverse load. The function of beam is to carry loads. It rest on supports which can offer reaction to keep system in equilibrium. Beam are classified according to their type of supports.

Types of supports

Structure or their components can be supported on different types of supports which can be classified depending upon the reaction offered by them as following.

Types of Supports

Sr. No.	Name of Support	Description for reaction	Diagram with reaction	Symbol & Nos. of reaction
1	Roller	It provides the resistance to movement in the direction perpendicular to supporting surface. Ex. Skating roller		 (01)
2	Simple	It Supports without any type of joint or connection & hence reaction is always acting along the direction of support.		 (01)
3	Hinge	It provides resistance to movement in any directionby offering inclined reaction.Ex. Door hinge		 (02)
4	Fixed	It provides resistance to rotation & it effectively held in position & restrained against rotation. Ex. Nail in the wall		 (03)

Types of loading

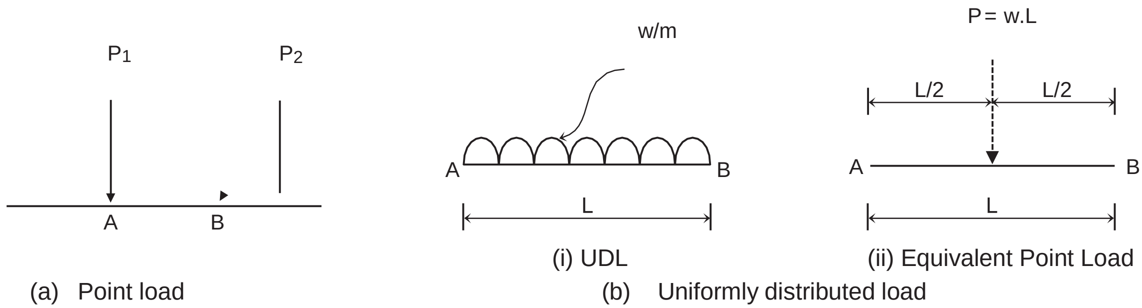
Loads which act on structural components can be external or due to self-weight of body. These load act as forces on structure. Following are important types of loading. (A) Concentrated or Point load (B) Uniformly distributed load (C) Uniformly varying load (D) Moment (E) Couple.

(A) Concentrated or Point load

Load concentrated on a very small length compared to length of beam is known as concentrated or point load. It is practically assumed to be acting through a point. Example of point load is car standing on ground..

(B) Uniformly distributed load (UDL)

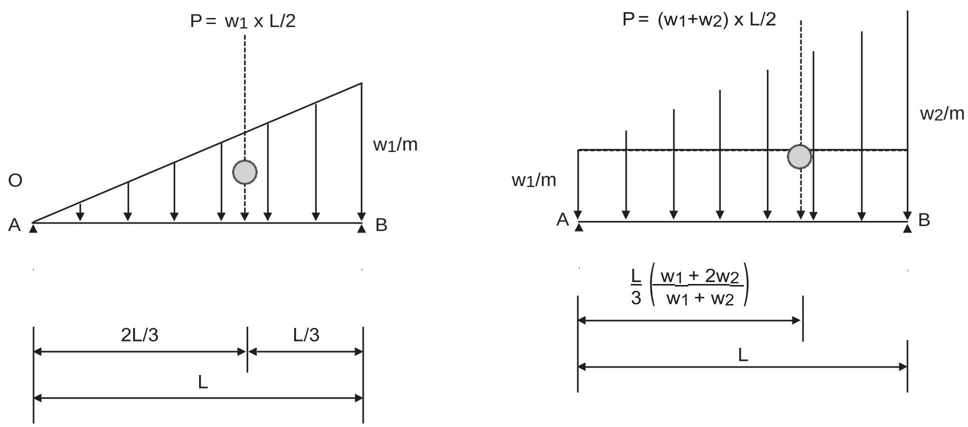
Load uniformly spread over the length of a beam is known as uniformly distributed load (UDL). In this type of loading, Weight of load per unit length is known as intensity of load; and is same along the length which denoted by w with units N/cm or kN/cm or N/m or KN/m. A truck loaded with sand of equal height & compound wall transferring load on the ground & person sleeping on bed are examples of UDL. For analysis, total load is taken as (w × l) acting as point load P at mid-point of length of UDL as equivalent value of UDL to find support reaction of beam as shown in fig. b(ii).



Types of load

(C) Uniformly varying load (UVL)

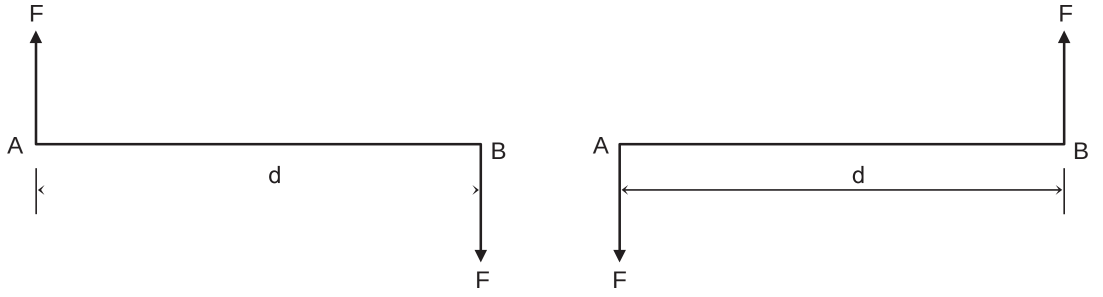
If the intensity of load is not same along the length but if uniformly increasing or decreasing from one end to another is known as uniformly varying load. If intensity increase from 0 to any value w_1 at the other end, then UVL known as triangular load and if intensity increase or decrease from w_1 value at one end to w_2 value at other end then UVL known as trapezoidal load as shown in fig (c).



Types of Load - Uniformly Varying Load (UVL)

A truck loaded with sand with top surface as inclined is the Example of UVL. In this type of load, total load is to be acting at C.G. of load diagram & value is total area of load diagram as shown in fig

The plane in which the forces constituting the couple act is called plane of the couple & perpendicular distance between the lines of action of force constituting couple is called arm 'd' of the couple as shown in fig.(d). Moment of couple is multiplication of force F and arm d.



(i) Clockwise (ii) Anti-Clockwise

Types of Load - Couple

Type of couple : According to rotation of the body due to couple, it classified as clockwise couple & anticlockwise couple as shown in fig (i) & (ii) respectively.

Types of Beam

Beam are broadly classified in to two groups.

(A) Statically determinate beam & (B) Statically indeterminate beam.

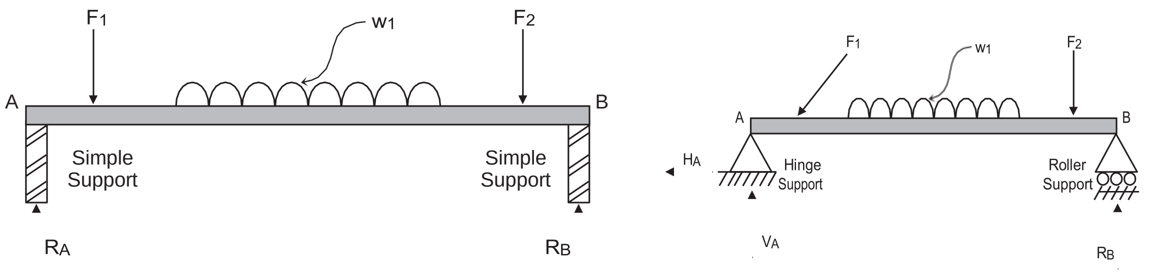
Analysis of statically indeterminate beams is not in scope for you at this stage.

(A) Statically determinate beams

A beam is said to be statically determinate beam if the number of unknown reactions are not more than the number of equilibrium conditions. There are three equations from equilibrium condition which are (i) $\sum H = 0$ (ii) $\sum V = 0$ (iii) $\sum M = 0$. Hence according to types of supports of beam maximum three unknown reaction can be solved.

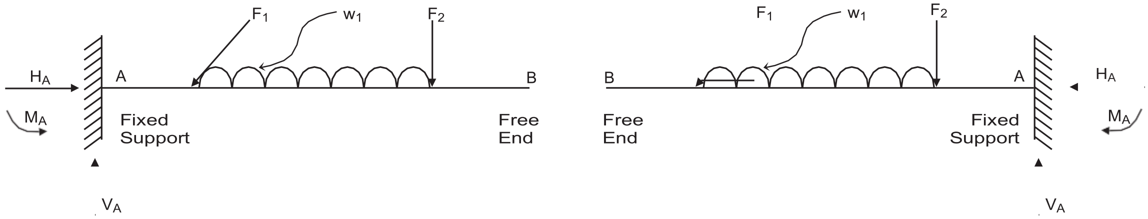
Following are statically determinate beams.

Simply supported beam : It is supported on two simple supports at each end of the beam.In this case supports offers only reaction force and not moment. Usually one supportis hinge & other is roller or both support is simple support. Nos. of unknown supports reaction are not more than 3 in any case as shown in below fig.



Simply supported beam

(i) Cantilever beam : In this beam, one end is fixed support and other end is free i.e., no support. In practice such beams are used when it is not possible to provide support at one end of the ends of the beam. Nos. of unknowns are not more than 3 in this beams as shown in below fig. Fixed support may be on left end or right end, as shown in fig. (a) & (a) respectively.

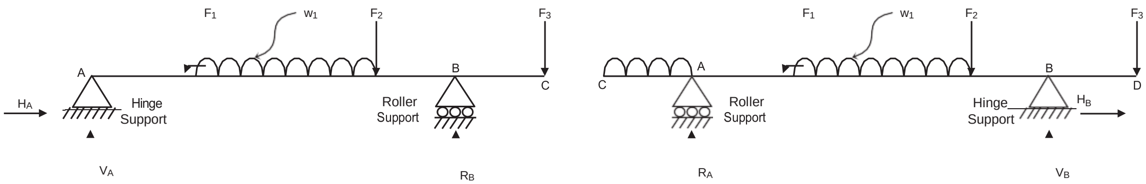


(a) Left hand end fixed support

(b) Right hand end fixed support

Cantilever beam

(ii) Overhang beam : If the one portion or two portions of the simply supported beam are extended beyond the support, then it's known as overhang beam. Depending upon the overhang, they are classified as single overhanging beam or double overhanging beam as shown in below fig. We can say that its special type of simply supported beam.

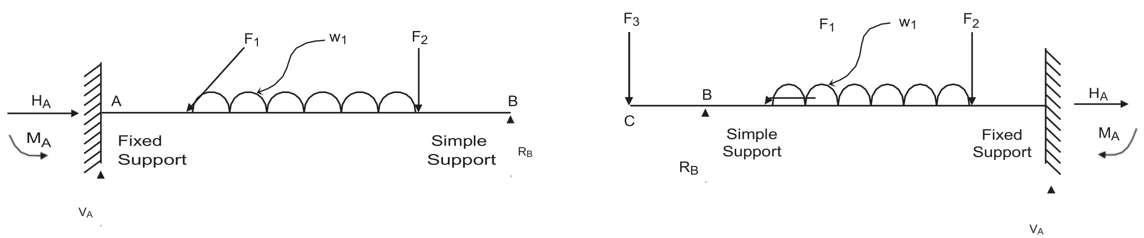


Overhang beam

(B) Statically indeterminate beams

If nos. of unknown reaction are more than the equilibrium condition then such type of beam is known as statically indeterminate beam. Following are different statically indeterminate beams.

(i) Propped cantilever beam : In this beam one end is fixed support and other end is simple support with overhang or no overhang as shown in fig.

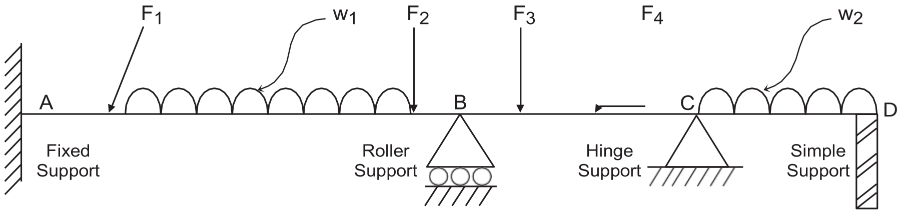


(a) Without overhang

(b) With overhang

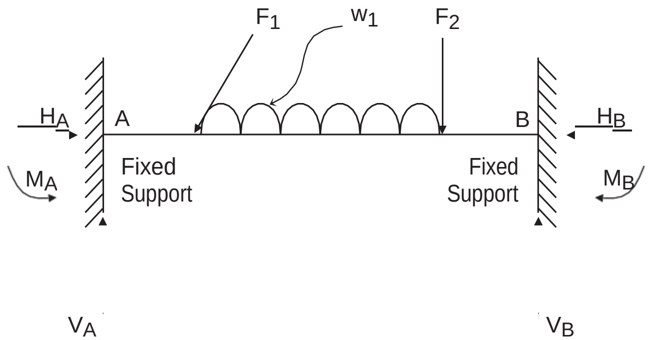
Propped cantilever

(ii) Continuous beam : In this beam nos. of support are more than two as shown in below fig.



Continuous beam

(iii) Fixed beam : In this beam both ends are with fixed support as shown in below fig.



Fixed beam

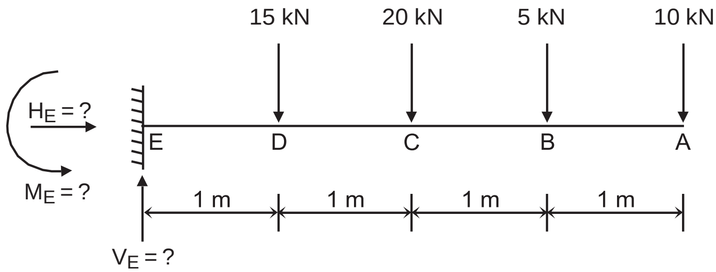
BEAM REACTIONS

Beam supports reaction can be finding by two methods. (I) Analytical method (II) Graphical Method. In analytical method, we have to use conditions of equilibrium to solve unknown support reaction as discuss in this topic.. The beam we have to consider are:
(A) Cantilever beam (B) Simply supported beam & (C) Overhang beam.

Beam reaction for cantilever beam

As we know the cantilever beam have one end fixed support and other end as free i.e., no support. We can find beam support reactions by using conditions of equilibrium by taking some examples.

Example 1 *A cantilever beam of 4m length having fixed support on left hand is carrying point loadsof 10 kN, 5 kN, 20 kN and 15 kN at an interval of 1m from free end respectively find the support reactionfor the beam.*



Solution:

First draw the beam and loading on the beam as per given data as shown in fig.

(a) Using equilibrium condition $\sum V = 0$ with +ve sign for ↑ upward force. Assume vertical reaction at E V_E as upward load.

$\therefore\ V_E - 15 - 20 - 5 - 10 = 0$

$\therefore\ V_E = 50$ kN ↑ **(Answer)**

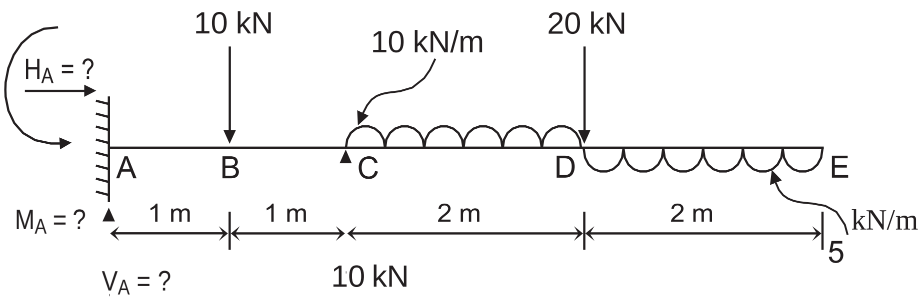
(b) Using equilibrium condition $\Sigma H = 0$ with +ve sign for → eastward force. As no horizontal load is acting on beam, $H_E = 0$ kN → **(Answer)**

(c) Using equilibrium condition $\Sigma M = 0$ with +ve sign for clockwise. ↻
Consider moment at fixed support E, we get

$\Sigma M_E = (15 \times 1) + (20 \times 2) + (5 \times 3) + (10 \times 4) - M_E = 0$

$\therefore\ M_E = 15 + 40 + 15 + 40 = 110$ kN·m ↺ anticlockwise

Example .2 *Determine the support reaction of cantilever beam as shown in fig.*



Solution:

Applying three equilibrium conditions one by one to get reaction at support.

(a) $\Sigma H = 0$. As no Horizontal force acting on beam, $H_A = 0$. **(Answer)**

(b) $\Sigma V = 0$ with + ve sign as ↑ upward and assuming V_A as upward.

$\therefore\ V_A - 10 + 10 - (10 \times 2) - 20 + (5 \times 2) = 0$

$\therefore\ V_A = 10 - 10 + 20 + 20 - 10 = 30$ kN ↑ **(Answer)**

(c) $\Sigma M_A = 0$ with + ve sign as ↻ clockwise and assuming M_A as anticlockwise.

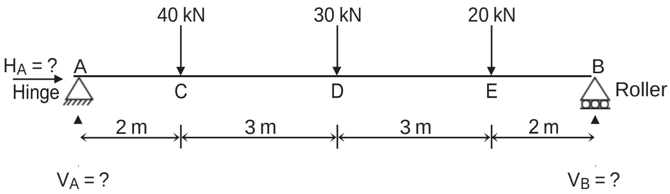
$\therefore\ (10 \times 1) - (20 \times 2) + (\{10 \times 2\} \times 3) + (20 \times 4) - (\{5 \times 2\} \times 5) - M_A = 0$

$\therefore\ M_A = 10 - 40 + 60 + 80 - 50 = 60$ kN·m anticlockwise **(Answer)**

Beam reaction for simply supported beam

Example- 3

A simply supported beam of span 10m carries three points loads of 40 kN, 30 kN and 20 kNfrom left hinge support at the distance 2 m, 5 m and 8 m respectively in downward direction. The right-handsupport is roller. Find support reaction for the beam.



Solution:

First draw space diagram of the beam from given data as shown in fig. Now applying three equilibrium condition for the beam.

(a) $\Sigma H = 0$ since there is no horizontal load on the beam. $\therefore H_A = 0$ kN **(Answer)**

(b) $\Sigma V = 0$ with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore V_A - 40 - 30 - 20 + V_B = 0.$

$\therefore V_A + V_B = 90$ kN. We have to use this equation as check point of our calculation.

(c) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A.

$\Sigma M_A = (40 \times 2) + (30 \times 5) + (20 \times 8) - (V_B \times 10) + (V_A \times 0) = 0$

$\therefore 10\,V_B = 80 + 150 + 160 = 390$

$\therefore \quad V_B = \frac{390}{10} = 39$ kN ↑ **(Answer)**

(ii) Consider moment at other support point B.

$\Sigma M_B = (V_A \times 10) - (40 \times 8) - (30 \times 5) - (20 \times 2) + (V_B \times 0) = 0$

$\therefore 10V_A = 320 + 150 + 40 = 510$

$\therefore V_A = \frac{510}{10} = 51$ kN ↑ **(Answer)**

(d) Now we can check our calculation for perfectness in equation obtain in (b).

If it fulfills, our calculation has no error.

$\therefore V_A + V_B = 90$ put values of V_A and V_B, we get

LHS = 51 + 39 = 90 = RHS. **OK.** Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until it's satisfied the check point equation.

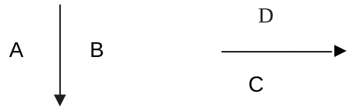
BEAM REACTION BY GRAPHICAL METHOD

In this method, wehave to study only for simply supporting beam carrying only point load .

Funicular Polygon graphical method

It is necessary to understand, certain technical terminologies for graphical method. This graphical method is also known as Funicular Polygon Method.

Bow's notation : Forces are identified by two different identical capital letters, placed on both sides.

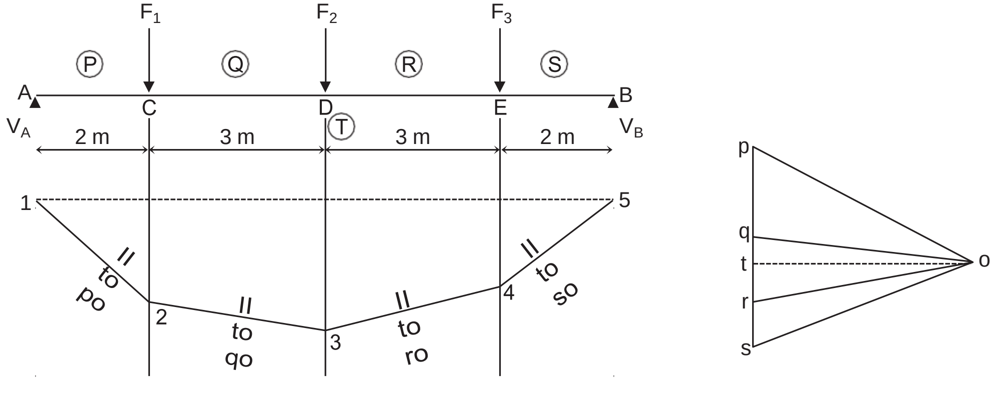


Bow's notation

(i) **Space diagram :** A diagram showing all the forces in position along with their magnitude and direction acting on a body is known as space diagram. Span & position of load shown as per suitable linear scale i.e., 1 cm =10m.

(ii) **Vector diagram :** All the forces on the body is represented one by one in vectorial form by magnitude and direction. The direction is represented by its original direction, while magnitude is represented by some suitable force scale i.e., 1 cm = N

Now, we can understand the steps of drawing funicular polygon. The graphical method to determinesupport reaction is given in following.



(a) Space diagram with Length scale (b) Vector diagram with Force scale

Funicular polygon graphical method

Step-1: Draw the space diagram which shows position, direction & magnitude of all the forces acting on the body (beam) as shown if fig.(a). The distance between forces may be drawn with some length scale i.e. 1 cm = m.

Step-2: Give bow's notations to all forces by placing alphabets on both sides of arrow. Reaction is considered as force acting on the body.

Step-3: Draw vector diagram for the given forces with some suitable force scale i.e. 1 cm =10Nor kN which represents the magnitude of each force. All the forces were drawn; one by one taken in order; in a vector diagram as shown in fig.(b).

Step-4: Take some convenient point O in front of vectorial form of forces & join all the points of vector diagram with this point O.

Step-5: Now select a point 1 on the line of action of first force R_A & through it draw a line parallel to "op" which cross at point 2 on force F_1. Now through point 2 draw line 2-3 parallel to "oq". Similarly draw lines 3-4 & 4-5 parallel to vector diagram lines "or" and "os" respectively on space diagram.

Step-6: Now join first start point 1 & last end point 5 obtained on line of force R_B as dotted line 1-5 in space diagram as shown in fig. (a). Draw a parallel line to 1-5 on vector diagram passing through point O as "ot" as dotted line as shown in fig. (b).

Step-7: Now measure "pt" & "ts" length on vector diagram & convert it by force scale as reaction R_A and R_B respectively.

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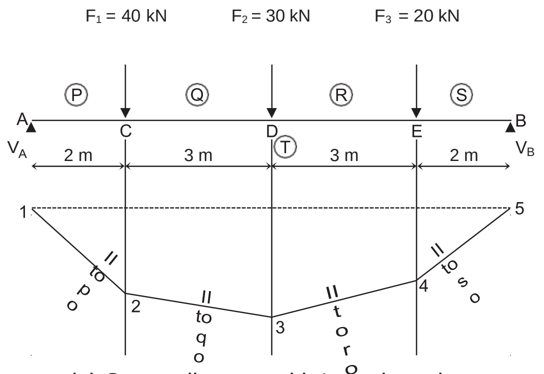
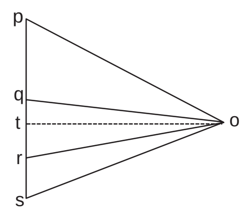
Example1 *Solve example 7 as shown in below fig. by graphical method.*



(a) Space diagram with Length scale



(b) Vector diagram with Force scale

Solution:

Step-1: Draw space diagram from given data with length scale as 1 cm = 1 m as shown in fig.(a).

Step-2: Gives bow's notations to all forces including reactions V_A & V_B by placing alphabets on either side space on line of direction. Here for force F_1 = 40 kN, we have placed "P" & "Q" on either sides of space on line of direction of force F_1 means force F_1 is now F_{pq} as per bow's notation. Similarly placed "R", "S" & "T" as bow's notation as shown in fig.(a).

Step-3: Draw vector diagram for the forces on the beam with force scale as 1 cm = 20 kN as shown in fig.(b). Select start point "p" & draw parallel line as line of action of force F_{pq} (in this case vertically downward) and get point "q" on it by converting magnitude of force F_{pq} as 40 kN as per scale as 2 cm from point "p". Thus line "pq" represent vectorial form of force F_{pq}. Similarly draw all the forces taken in order in vector form in vector diagram as "qr" & "rs" for force F_{qr} (F_2) & F_{rs} (F_3) respectively.

Step-4: Take some convenient point "o" in front of vectorial form of forces. Join all points of vector diagram p, q, r & s with o and obtained line po, qo, ro & so as shown in fig.(b).

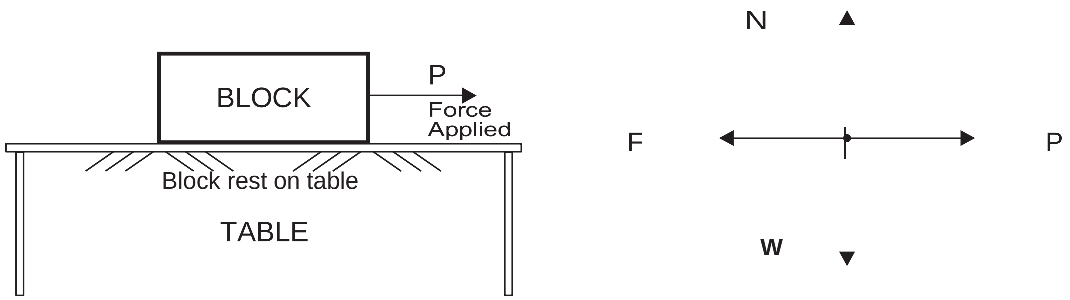
Step-5: Now on space diagram extend all line of action for all the forces. Select start point "1" on line of reaction R_A. Through this point "1", draw line parallel to line "po" of vector diagram

.**Step-6:** Now join first start point 1 & last end point 5 obtained on line of force R_B as dotted line 1-5 in space diagram as shown in fig. (a). Draw a parallel line to 1-5 on vector diagram passing through point O as "ot" as dotted line as shown in fig. (b).

Step-7: Now measure "pt" & "ts" length on vector diagram & convert it by force scale as reaction R_A and R_B respectively.

CH- 3 FRICTION

Consider a block of weight W resting on a table. An external horizontal variable force P is apply on the block as shown in fig. (a). The block is in equilibrium and therefore, $\Sigma V = 0$ and the weight W of the block is resisted by normal reaction N offered by the top of table. $\therefore$ N = W.



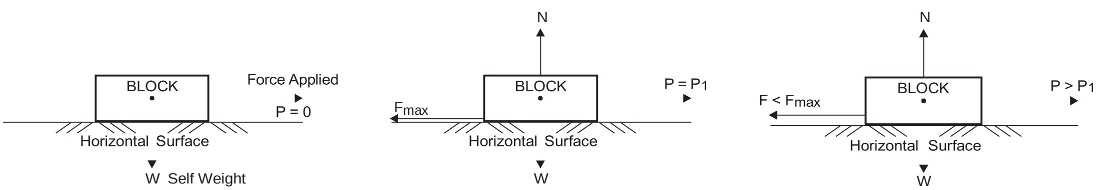
Friction

Now, try to move this block on the top of the table maintain contact surface of the block and top of the table. The rough surface of the table top will offer internal resistance to the motion. This resistance to the motion, which always opposes motion, is force of friction or simply **friction,** denoted by F.

Limiting friction

Consider again the block of weight W place on the rough horizontal surface At this stage, we have not applied any external force on the block i.e., P = 0. Therefore, Internal resistive frictional force F will not develop i.e., F = 0 and the block will remain at rest position.

Let us apply; gradually more & more external force P on the block. As we increase P, resistive frictional force F should also increase. A stage will reach, when the block will just start to move or we can call it about to move, (impending motion) $P = P_1$ as shown in fig. (b). Note that in this case, motionhas not occurred, but when negligible force is applied i.e., if tapping with finger or pen is made, the blockwill start moving. In this case, we say that motion is impending or just going to begin, frictional force F has reached its maximum value or limiting frictional force F_{max}



(a) Block at rest (b) Block in pending (just) to move (c) Block in motion

Fig. Limiting friction

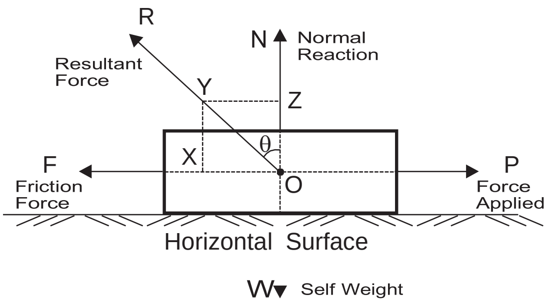
Coefficient of friction (λ)

Coefficient of friction is define as ratio of the maximum frictional force $F_{max.}$, which resist the motion of two surfaces in contact, to the normal reaction force N, which pressing the two surfaces together. It is usually symbolize by the Greek letter mu (μ). Mathematically, $\mu = F_{max}$, where $F_{max.}$ is the maximum frictional force and N is the normal force .

Angle of Friction (□)

Consider a block is resting on a horizontal surface and subjected to horizontal pull P as shown in the fig 3.4. Let R is resultant force of two forces, frictional force F and normal reaction force N, which acts at angle θ to normal reaction, then angle θ is call the angle of friction.

$$\tan\theta = \frac{ZY}{OZ} = \frac{\text{Friction force}}{\text{Normal reaction}} = \frac{F}{N}$$



As P increases, F increases and θ also increases. It can reach maximum value of σ, when F reachlimiting frictional force $F_{max.}$, at this stage, angle θ known as angle of friction σ. Mathematically,

$$\tan \sigma = \frac{F_{max}}{N} = \mu \text{ (Coefficient of friction)}$$

$$\sigma = \tan^{-1}(\mu)$$

Angle of repose (ε)

Angle of repose can be define as the minimum angle of the inclined plane such that an object placed on it just begins to slide. Consider the block of weight W resting on inclined plane, which makes an angle θ with the horizontal as shown in fig When θ be less the block will rest on the plane. Now, θ is increase gradually; a stage will reach, at which the block starts sliding down the plane. The angle θ, at which motion is impending call the angle of repose. Thus, the maximum inclination of the plane at which the body can repose, without any external force, is call Angle of Repose. It is usually symbolize by the Greek letter phai (ϕ).

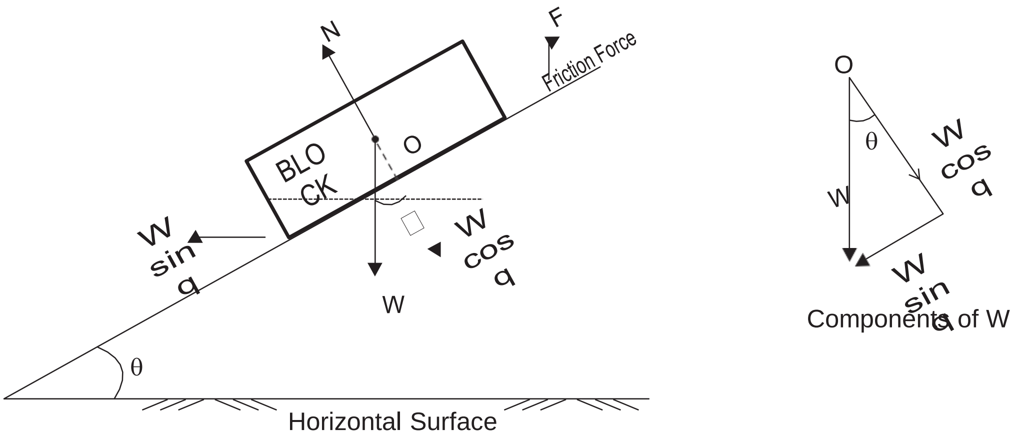


Fig. Angle of repose

Since, block is at rest and hence in equilibrium, conditions of equilibrium can be apply.

(i) Resolving weight of block W, normal to the inclined plane, we get N = W cos θ …(i)

(ii) Resolving weight of block W, along to the inclined plane, we get F = W sin θ …(ii)

At just start of block sliding, θ reach at ϕ, F becomes F_{max}

But, we know $F_{max} = \mu N$

Put the values from equation (i) & (ii), we get,

$$W \sin \phi = \mu W \cos \phi$$

$$\tan \phi = \mu = \tan \sigma$$

So, Angle of repose ϕ = Angle of friction σ

Types of Friction:- There two types of friction (1) static friction (2) dynamic friction

(a) Static Friction

Static friction can act between two objects, when objects are stationary. The maximum frictional force present in the body, when it is in rest position (i.e., limiting friction is a kind of static friction). When body just tends to move on surface of another body is call static friction. It is denote as F_s. Magnitude of static friction is $F_s \leq \mu_s N$.

Here μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

(b) Kinetic friction / Dynamic friction

If two surfaces are in contact and moving relative to one another, then the friction between them called kinetic friction. This happens, when the value of applied force exceeds the limiting friction and body is moving. Kinetic friction is less then limiting friction. It is denote by F_k. Once the applied external force exceeds P_1 the body will move, the magnitude of kinetic friction F_k is given by $F_k = \mu_k N$. Where μ_k is the coefficient of kinetic friction and N is the magnitude of the normal force.

Dynamic friction can be sub divided in to two types. (i) Sliding friction & (ii) Rolling friction.

(i) Sliding friction : It is the frictional force, which comes into play, when one body sides over the other under action of external force.

(ii) Rolling friction : It is the frictional force, which comes into play, when one body rolls over the other under the action of external force.

Laws of friction

1. Force of friction depends upon the material of contact surfaces and the roughness of contact surfaces.
2. Force of friction is independent of the area of contact surfaces.
3. Force of friction is independent of the relative velocity of contact surfaces.
4. Ratio of friction force and normal reaction is known as the coefficient of friction and its value for the given two surfaces will always constant.
5. Coefficient of static friction is greater than coefficient kinetic friction.

EQUILIBRIUM OF A BODY ON A HORIZONTAL PLANE SURFACE

Now, it's clear that, a body lying on a rough horizontal plane surface is always in equilibrium. But body will start moving in the direction of the force, when an external force P is applied on it. This external force P can be apply in the two different ways. (i) Parallel to plane (i.e., horizontal) & (ii) Inclined to horizontal plane.

Let us discuss these cases one by one.

Equilibrium of a body on a horizontal plane with horizontal externalforce

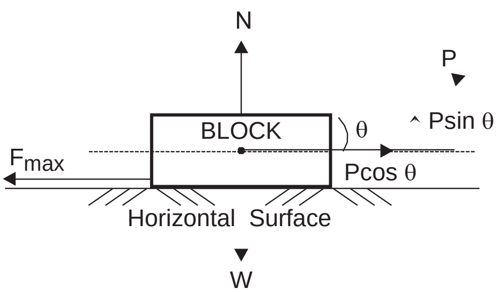
For such case, equilibrium equations were applied on the body as horizontally (parallel to plane) & vertically (normal to plane).

(i) $\Sigma H = 0 \therefore F_{max.} = P$, where $F_{max.}$ = Frictional force & P = External force applied.
(ii) $\Sigma V = 0 \therefore N = W$, where N = normal reaction & W = Self-weight of the body.

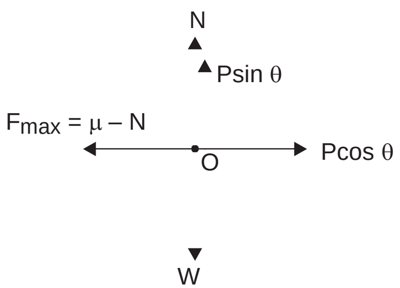
Now the value of the frictional force F_{max} is obtained from the relation:
(iii) $F_{max.} = \mu N$, where μ = Coefficient of friction & N = Normal force of reaction.
Using above three equations, we can solve the problems.

Equilibrium of a body on a horizontal plane with inclined externaforce

For such case, inclined force was resolved (parallel to the plane and perpendicular to the plane) as explained in unit 1 of this book. Now equilibrium equations were applied on the body as horizontally (parallel to plane) & vertically (normal to plane).



(a) Body



(b) Free body diagram

Equilibrium of body on horizontal plane

(iv) $\Sigma H = 0 \therefore F_{max.} = P \cos \theta$, where F = Frictional force & P = External force applied.

(v) $\Sigma V = 0 \therefore W = N + P \sin \theta$, where N = normal reaction & W = Self-weight of the body.

Now the value of the force of friction F_{max} is obtained from the relation:

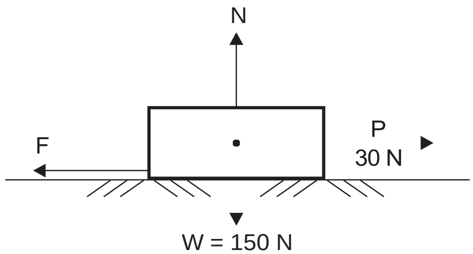
(vi) $F_{max} = \mu N$, where μ = Coefficient of friction & N = Normal force of reaction.

Using above three equations, we can solve the problems. We calculate some examples to understand all above points.

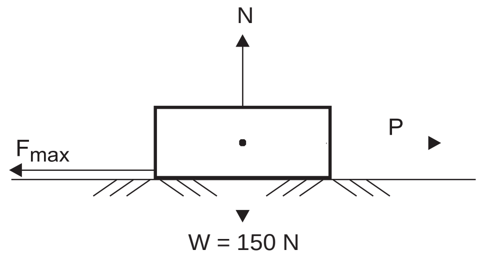
Example 1. *A block of weight 150 N is resting on the horizontal surface. The coefficient of friction betweenthe block and horizontal surface is 0.25.*

(a) Explain what happens to the block, if horizontal external force of P = 30 N is apply, as shown in fig.

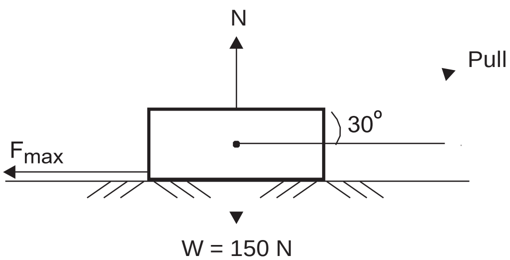
(b) Now determine the external force required to just start the motion of the block, when P is a pull horizontal force, (c) P is a pull force inclined at 30° with horizontal and (d) P is push force inclined at 30° with horizontal.



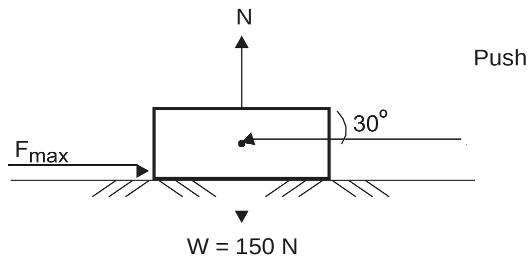
(a) Ext. force P = 30 N



(b) Horizontal Ext. force



(c) Pull force at 30° with horizone



(d) Push force at 30° with horizone

Solution:

(a) W = 150 N, μ = 0.25 & P = 30 N. [fig. 3.7(a)]

Apply equilibrium condition equations,

(i) $\Sigma V = 0, \therefore N = W = 150$ N

(ii) $\Sigma H = 0, \therefore F = P = 30$ N

(iii) $F_{max.} = \mu N = 0.25 \times 150 = 37.5$ N

Since, $F < F_{max}$, the block will be in equilibrium. It will not move as applied force is less than F_{max}. **(Answer)**

(b) W = 150 N, μ = 0.25 [Fig. 3.7(b)]

(i) $\Sigma V = 0, \therefore N = W = 150$ N

(ii) We know that $F_{max.} = \mu N = 0.25 \times 150 = 37.5$ N

(iii) From $\Sigma H = 0$, $P = F_{max} = 37.5$ N **(Answer)**

(c) W = 150 N, μ = 0.25 [fig. 3.7(c)]

(i) From $\Sigma V = 0$, $N = 150 - P \sin 30$

$$N = 150 - 0.5\,P$$

(ii) As we know that $F_{max.} = \mu N$

$$F_{max} = 0.25\,(150 - 0.5P)$$

$$F_{max} = 37.5 - 0.125\,P$$

(iii) From $\Sigma H = 0$, $P \cos 30 = F_{max}$

Put F_{max} from (ii),

$$P \cos 30 = 37.5 - 0.125\,P$$

$$\therefore 0.866\,P + 0.125\,P = 37.5$$

$\therefore P = 37.84$ N **(Answer)**

(d) W = 150 N, μ = 0.25 [Fig. 3.7(d)]

Here it should be clear that Push type of externally applied force (P) is apply on the body. Hence force of friction (Fmax) will be in opposite direction to that of probable motion or applied external force.

(i) From $\Sigma V = 0$, $N = 150 + P \sin 30$

$$N = 150 + 0.5\,P$$

(ii) As we know that, $F_{max} = \mu N$

$$F_{max} = 0.25\,(150 + 0.5P)$$

$$F_{max} = 37.5 + 0.125\,P$$

(iii) From $\Sigma H = 0$, $P \cos 30 = F_{max}$

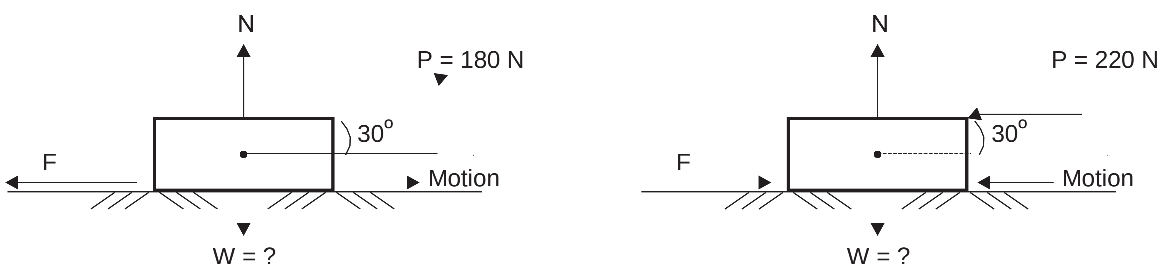
Now put value of F_{max} from (ii), we get

$$0.866\,P = 37.5 + 0.125\,P$$

$$\therefore 0.741\,P = 37.5$$

$\therefore P = 50.61$ N **(Answer)**

Example 2. *A body is resting on a rough horizontal plane. It requires an external force of 180 N (pull type), inclined at 30° to the horizontal plane, just to start the motion. Furthermore, it is also observed that an external force of 220 N (push type) inclined at 30° to the horizontal plane, can also just start the motion. Find the weight the body and coefficient of friction.*



(a) Pull of 180 N at 30°　　(b) Push of 220 N at 30°

Solution:

The forces acting on the body shown in fig. 3.9 (a) & (b) for the external force pull & push respectively.

Here, $\theta = 30^o$ & pull P = 180 N and push = P = 220 N

Resolving all forces horizontally & vertically for both cases, we get;

(A) Case I as external force is apply as pull of 180 N at inclination of 30° with horizontal.

(i) $\Sigma H = 0 \quad \therefore F = P \cos\theta$

$\therefore F = 180 \times \cos 30$

$\therefore F = 155.9$ N

(ii) $\Sigma V = 0 \quad \therefore W = N + P \sin\theta$

$\therefore W = N + 180 \times \sin 30$

$\therefore W = N + 90$

$\therefore N = W - 90$

(iii) We know that frictional force = $F = \mu N = \mu (W - 90)$

$\therefore 155.9 = \mu (W - 90) \qquad \ldots (i)$

(B) Case II as external force is apply as push of 220 N at inclination of 30° with horizontal.

(i) $\Sigma H = 0 \quad \therefore F = P \cos\theta$

$\therefore F = 220 \times \cos 30$

$\therefore F = 190.5$ N

(ii) $\Sigma V = 0 \quad \therefore N = W + P \sin\theta$

$\therefore N = W + 220 \cdot \sin 30$

$\therefore N = W + 110$

(iii) We know that frictional force = $F = \mu N = \mu (W + 110)$

$\therefore 190.5 = \mu (W + 110) \qquad \ldots (ii)$

(C) Dividing equation (i) by (ii), we get;

$$\frac{155.9}{190.5} = \frac{\mu(W - 90)}{\mu(W + 110)} = \frac{(W - 90)}{(W + 110)}$$

$\therefore 155.9 (W + 110) = 190.5 (W - 90)$

$\therefore 155.9 W + 17149 = 190.5 W - 17145$

$\therefore 34.6 W = 34294$

$\therefore W = 991.16$N **(Answer)**

(D) Substituting the value of W in equation (i), we get;

$\therefore 155.9 = \mu (W - 90) = \mu (991.16 - 90) = \mu (901.16)$

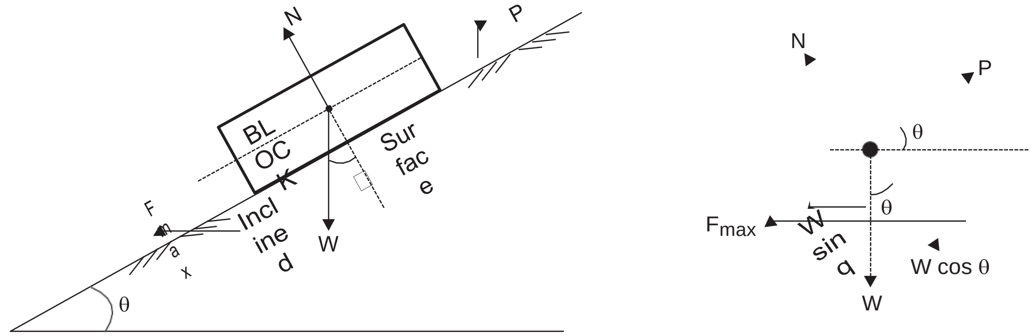
$$\therefore \mu = \frac{155.9}{901.16}$$

$\therefore \mu = 0.173$ **(Answer)**

Equilibrium of a body on an inclined plane with parallel externalforce to plane

In such case, equilibrium of the body is study by resolving all the forces acting on the body as parallel (along) to plane surface & normal (perpendicular) to plane surface with external force to be apply as (i) Pull for upward motion & (ii) Push for downward motion of the body.

(a) External force is applied as pull for upward motion of the body on inclined plane surface:



(a) Body on inclined plane (b) Free body diagram

Equilibrium of body on inclined plane surface

(i) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = F_{max} + W \sin \theta$

$P = \mu N + W \sin \theta$ [as $F_{max} = \mu N$] …(i)

(ii) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos \theta$ …(ii)

Substituting equation (ii) in equation (i), we get, $P = \mu W \cos \theta + W \sin \theta$ …(iii)

Now μ = Coefficient of friction = $\tan \sigma = \frac{\sin \alpha}{\cos \alpha}$, Substituting in equation (iii), we get,

$$P = \frac{\sin \alpha}{\cos \alpha} W \cos \theta + W \sin \theta$$

$\therefore P \cos \sigma = W \sin \sigma \cos \theta + W \cos \sigma \sin \theta$

$\therefore P \cos \sigma = W \sin (\sigma + \theta)$

$$\therefore P = \frac{W \sin(\alpha + \theta)}{\cos \alpha} \quad \text{…(iv)}$$

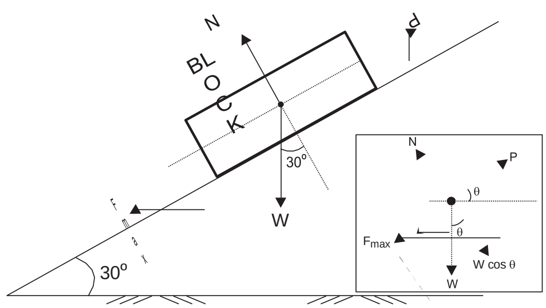
Thus, an externally applied force P can be obtained by using equation (iv) for upward motion of body.

(b) External force is applied as push of force for downward motion of the body on inclined plane surface:

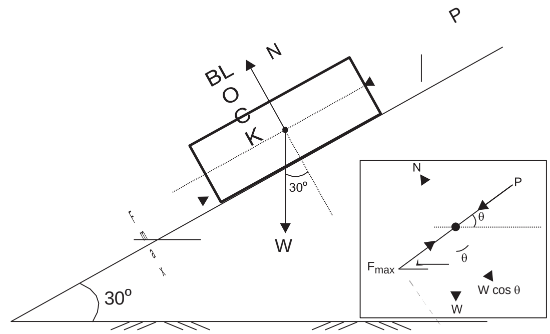
In the similar way as obtained in (a), we get,

$$P = \frac{W \sin(\alpha - \theta)}{\cos \alpha} \quad \text{…(v)}$$

Thus, the external force to be apply P for downward motion of body can be find by using equation (v). We can solve some examples to understand above points.



(a) Upward motion of clock on inclined plane



(b) Downward motion of clock on inclined plane

Solution:

Given data : $\theta = 30^o$, $\mu = 0.25$ & mass = m = 10 kg., $W = mg = 10 \times 9.8$ N = 98 N.

(A) Block motion (move) upward on inclined plane surface : [fig. 3.11(a)]

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos\theta$

$\therefore N = 98 \times \cos 30^o$

$\therefore N = 84.87$ N …(i)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get, $P = F_{max} + W \sin\theta$

$\therefore P = \mu N + W \sin\theta$ …(ii)

Putting values of μ, θ, N & W, we get;

$P = (0.25 \times 84.87) + (98 \times \sin 30)$

$\therefore P = 70.22$ N **(Answer)**

(B) Block motion (move) downward on inclined plane surface : [fig. 3.11(b)]

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos\theta$

$\therefore N = 98 \times \cos 30^o$

$\therefore N = 84.87$ N …(i)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get, $P = F_{max} - W \sin\theta$

$\therefore P = \mu N - W \sin\theta$ …(ii)

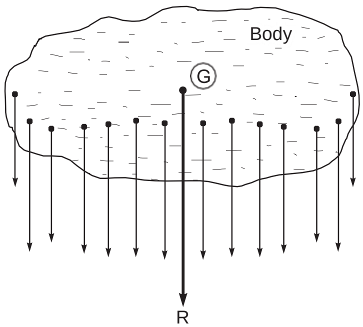
Putting values of μ, θ, N & W, we get;

$P = (0.25 \times 84.87) - (98 \times \sin 30)$

$\therefore P = -27.78$ N (Push) **(Answer)**

CHAPTER – 4 Center of Gravity (CG)

Every particle of a body is attract by the earth towards its centre.The force of attraction, which is proportional to the mass of the particles of the body, acts vertically downwards and known a weight of the body. As the distance between the different particles of a body & the centre of the earth is taken to be same (because of very small size of the body as compared to theearth), these forces may be taken to act along parallel lines as shown in fig.below. The resultant R of all such parallel forces acts on one point G. This point through which the whole weight of the body acts, is known as center of gravity (CG), irrespective of the position of the body. It may be note that everybody has onlyone & one CG. If you balance the body on this point of CG, it will balance.



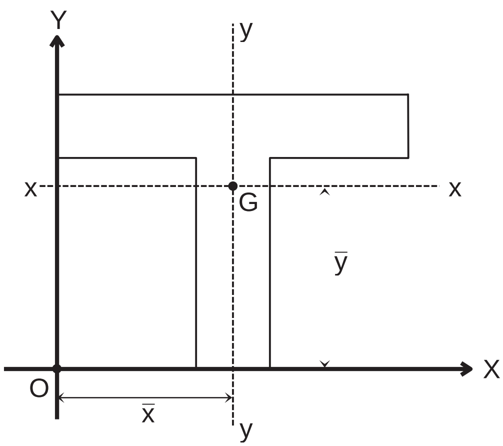
Center of gravity (CG)

Centroid

The plane figures (like triangle, circle etc.) have only areas, but no mass. The center of area of two-dimension figures known as centroid. Centroid is the point in a plane section such that for any axis through that point moment of area is zero.

Axis of Reference

The C.G. or centroid of a body is always calculate with reference to the assumed axis. This assumed axis known as axis of reference. The axis of reference, of plane figure generally taken as the left most line (OY) of the figure for calculating $\bar{x}$ and the lowest line (OX) of the figure for calculating $\bar{y}$, as shown in fig.4.2.



Axis of reference

Axis of Symmetry

The axis (x-x axis or y-y axis) which divide the figure into two identical parts is call axis of symmetry.

If figure is symmetrical about y-y axis, $\bar{x}$ is directly available & $\bar{y}$ needs to be calculated.

If figure is symmetrical about x-x axis, $\bar{y}$ is directly available & $\bar{x}$ needs to be calculated.

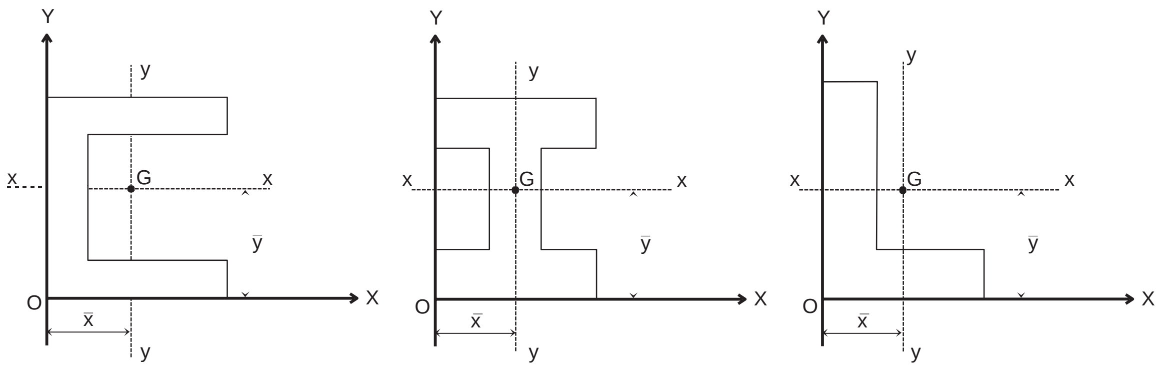
Take some examples to observe different type of symmetry.

(a) **T-Section** Section is symmetrical about y-y axis. So $\bar{x}$ is directly available and $\bar{y}$ is to be calculated.

(b) **C- Section (Channel**Section is symmetrical about x-x axis. So $\bar{y}$ is directlyavailable and $\bar{x}$ is to be calculated.

(c) **I-Section** Section is symmetrical about both x-x and y-y axis. So $\bar{x}$ and $\bar{y}$ both aredirectly available.

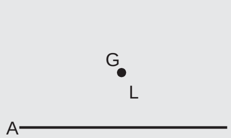
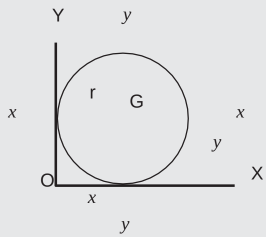
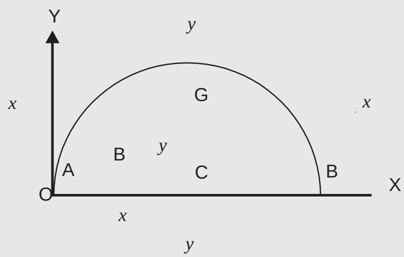
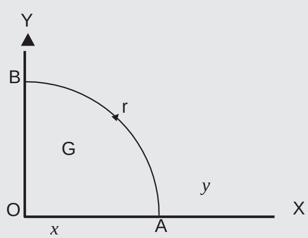
(d) **L-Section (Angle Section** Section is not symmetrical about any axis. So $\bar{x}$ and $\bar{y}$ both are required to be calculated.



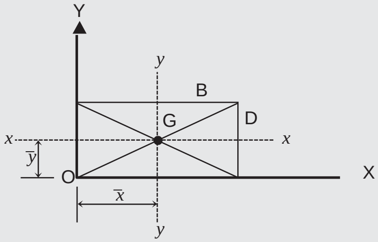
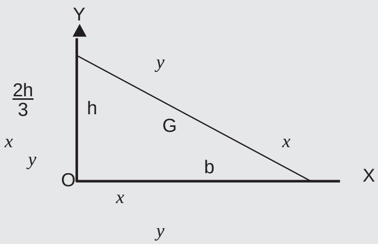
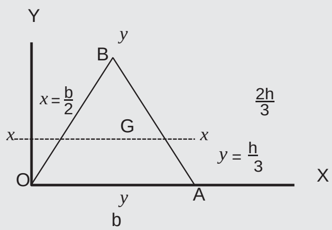
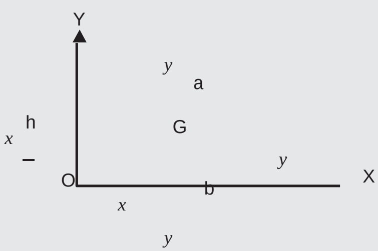
(a) C – Section (b) I – Section (c) L – Section

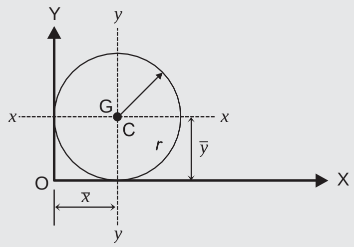
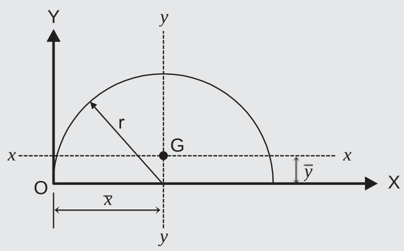
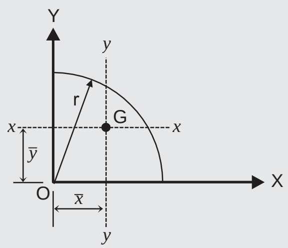
Axis of symmetry

Centroid of standard shapes [1D & 2D elements]
One Dimensional shape (Wires)

<table>
<tr><th>Sr. No.</th><th>Geometrical Shape</th><th>Length</th><th>$\bar{x}$</th><th>$\bar{y}$</th></tr>
<tr><td>1.</td><td>
Straight wire AB</td><td>L</td><td colspan="2">Centre of Length
$\left(\frac{L}{2}\right)$</td></tr>
<tr><td>2.</td><td>
Wire ring</td><td>2κr</td><td colspan="2">Centre of Circle (r)
$x = r$ $y = r$</td></tr>
<tr><td>3.</td><td>
Semicircular wire AB</td><td>κr</td><td>r</td><td>$\frac{2r}{\pi}$</td></tr>
<tr><td>4.</td><td>
Quarter-circular wire AB</td><td>$\frac{\pi r}{2}$</td><td>$\frac{2r}{\pi}$</td><td>$\frac{2r}{\pi}$</td></tr>
</table>

(A) Two-Dimensional shape (plane figures)

Sr. No.	Geometrical Shape	Area	$\bar{x}$	$\bar{y}$
1.	 Rectangular or square	$A = B \cdot D$	$\frac{B}{2}$	$\frac{D}{2}$
2.	 Right angle triangle	$A = \frac{1}{2} b \cdot h$	$\frac{1}{3} b$	$\frac{1}{3} h$
3.	 Symmetrical triangle	$A = \frac{1}{2} \cdot b \cdot h$	$\frac{b}{2}$	$\frac{h}{3}$
4.	 Trapezium	$A = (a+b)\frac{h}{2}$	$\frac{b}{2}$	$\frac{h}{3}\left(\frac{b+2a}{b+a}\right)$

5.	 Circle	$A = \kappa r^2$ or $A = \frac{\pi}{4} d^2$	r	r
6.	 Semicircle	$A = \frac{\pi r^2}{2}$	r	$\frac{4r}{3\pi}$
7.	 Quarter circle	$A = \frac{\pi r^2}{2}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$

CENTRIOD OF COMPOSITE FIGURES

Steps for finding centriod of Composite figures

To find CG of composite figures, we have to follow following steps.

Step-1: Divide the given composite (compound) shape into various standard figures. These standard figures include square, rectangles, circles, semicircles, triangles and many more. In dividing the composite figure, include parts with holes (cut out) are to treat as components with negative values. There is also possibility of rotation (90^o, 180^o, 270^o & 360^o) of standard figure to adjust in composite section. Make sure that you break down every part of the compound shape in to various components with designate name (Component-1, Component-2 & so on) before proceeding to the next step.

Step-2: Calculate the area of each component as per standard shape from table 4.1 (B). Make the area negative for designated areas that act as holes (cut out).

Step-3: The given figure should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite figure, while the Y-axis as the vertical line passing through left most point of the given composite figure.

Step-4: Get the distance of the centroid of each component, as divided into standard figure in step-1, from the X-axis and Y-axis as reference lines.

Step-5: Make a calculation in tableas shown below.

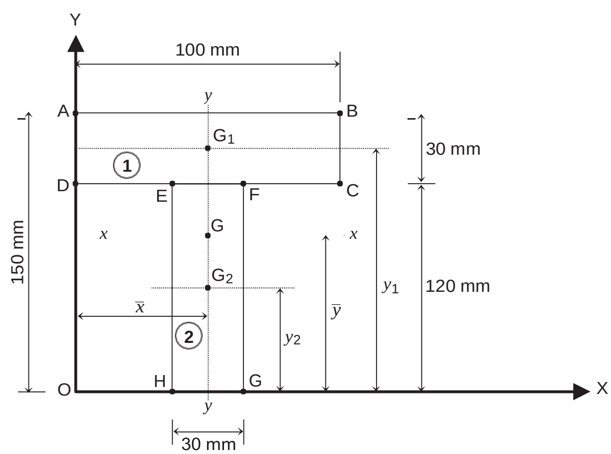
Sr. No.	Component Name	Area of Component A in mm²	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Component 1	A_1	x_1	y_1	A_1x_1	A_1y_1
2	Component 2	A_2	x_2	y_2	A_2x_2	A_2y_2
n	Component n	A_n	x_n	y_n	A_nx_n	A_ny_n
	Summation	ΣA =	---	---	ΣA·x =	ΣA·y =

Step-6: Use the equations to find the coordinates ($\bar{x}, \bar{y}$) of centroid (CG) from reference lines.

(a) $\bar{x} = \dfrac{\Sigma A \cdot x}{\Sigma A}$ and (b) $\bar{y} = \dfrac{\Sigma A \cdot y}{\Sigma A}$

Let to explain above point, take some examples of composite figure (section), as per syllabus composite figure must be composed of not more than three geometrical figures.

Example 1. *Find the centroid (CG) of a 100 mm × 150 mm × 30 mm T-section as shown in the figure.*



Solution:

.

Sr. No.	Component Name	Area of Component A in mm^2	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Top Rectangle- 1 ABCD (100 x 30 mm)	100 x 30 = 3000	$\frac{100}{2}$ = 50	120 + $\frac{30}{2}$ = 135	150000	405000
2	Vertical Rectangle-2 EFGH (30 x 120 mm)	30 x 120 = 3600	50 from symmetry	$\frac{120}{2}$ = 60	180000	216000
	Summation	ΣA = 6600	---	---	ΣA.x = 330000	ΣA·y = 6210000

.

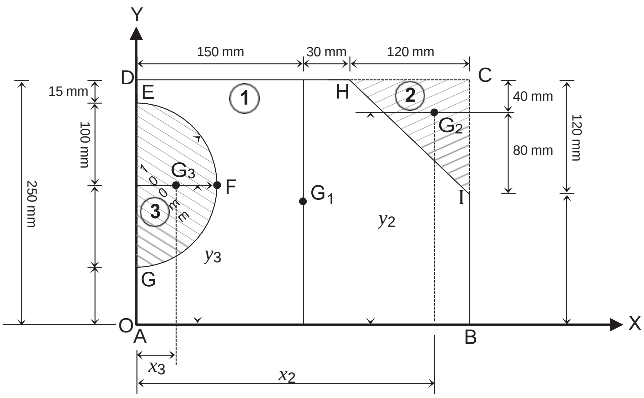
(a) $\bar{x} = \dfrac{\Sigma A \cdot x}{\Sigma A} = \dfrac{330000}{6600}$ = 50.00 mm **(Answer)**

[As per symmetry of composite figure about yy axis (vertical), we can directly,

Find $\bar{x} = \dfrac{\text{Total width}}{2} = \dfrac{100}{2}$ = 50.00 mm; As we obtained by calculations.]

(b) $\bar{y} = \dfrac{\Sigma A \cdot y}{\Sigma A} = \dfrac{621000}{6600}$ = 94.09 **mm (Answer)**

Example 2. *Find the centroid of the given composite figure shown in figure.*

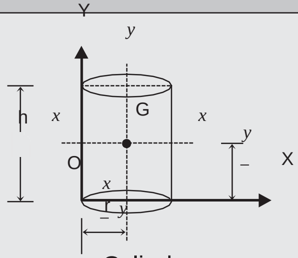
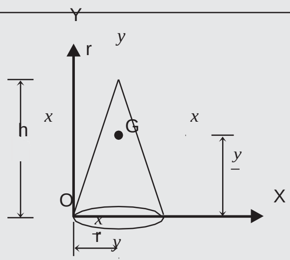
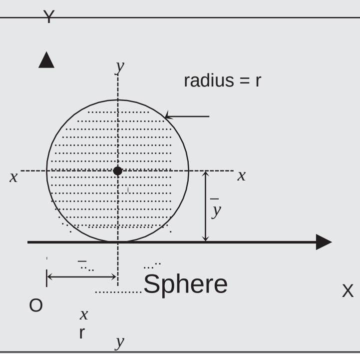
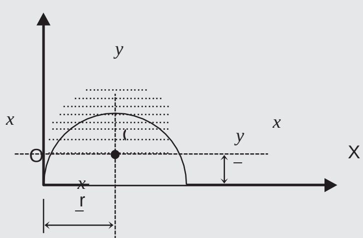


Sr. No.	Component Name	Area of Component A in mm²	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Rectangle- 1 [ABCD] (300 x 250 mm)	300 x 250 = 75000	$\frac{300}{2} = 150$	$\frac{250}{2} = 250$	11250000	9375000
2	CUT of Triangle- 2 [CHI] (120 mm Base & Height both)	$120 \times \frac{120}{2} = 7200$	300 – 40 = 260	250 – 40 = 210	–1872000	–1512000
3	CUT of Semiciecle-3 [EFG] (Radius = 100 mm)	$\frac{\pi \times 100^2}{2} = -15707.96$	$\frac{4 \times 100}{(3\pi)} = 42.44$	35 + 100 = 135	–666645.82	–2120574.60
	Summation	ΣA = 52092.04	---	---	ΣA·x = 8711354.18	ΣA·y = 5742425.40

(a) $\bar{x} = \frac{\Sigma A \cdot x}{\Sigma A} = \frac{8711354.18}{52092.04} = 167.23$ mm **(Answer)**

(b) $\bar{y} = \frac{\Sigma A \cdot y}{\Sigma A} = \frac{5742425.40}{52092.04} = 110.23$ mm **(Answer)**

Center of Gravity (CG) of Three Dimensional Standard Solid

	Geometrical Shape	Volume	$\bar{x}$	$\bar{y}$
1.	 Cylinder	$V = \kappa r^2 h$	r	$\frac{h}{2}$
2.	 Cone	$V = \frac{\pi}{3} r^2 h$	r	$\frac{h}{4}$
3.	 Sphere	$V = \frac{4}{3} \kappa r^3$	r	r
4.		$V = \frac{2}{3} \kappa r^3$	r	$\frac{3r}{8}$

y

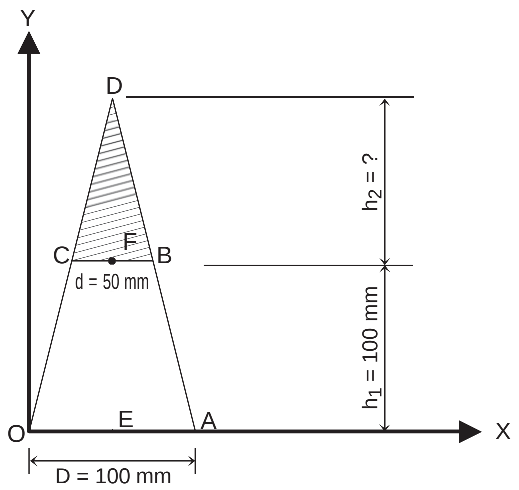
CENTRE OF GRAVITY (CG) OF COMPOSITE SOLIDS

.

Sr. No.	Component Name	Volume of Component V in mm^3	Distance of CG of component from Reference lines		V·x	V·y
			x	y		
1	Component 1	V_1	x_1	y_1	V_1x_1	V_1y_1
2	Component 2	V_2	x_2	y_2	V_2x_2	V_2y_2
n	Component n	V_n	x_n	y_n	V_nx_n	V_ny_n
	Summation	$\Sigma V =$	---	---	$\Sigma V \cdot x =$	$\Sigma V \cdot y =$

(a) $\bar{x} = \dfrac{\Sigma V \cdot x}{\Sigma V}$ and (b) $\bar{y} = \dfrac{\Sigma A_x}{\Sigma A}$

Example .*A frustum of cone is having base diameter 100 mm and top diameter 50 mm with height as100 mm. Find the CG of this frustum.*



Solution:

Here to get frustum, we have to substrate upper cone BCD from full cone OAD as shown in fig. For full cone OAD, compare triangles DOE & DCF, we get

$$\frac{DE}{OE} = \frac{DF}{CF}$$

$$\therefore \frac{DE}{50} = \frac{(DE - 100)}{25}$$

$\therefore$ 25 DE = 50 DE – 5000

$\therefore$ 25 DE = 5000

$\therefore$ DE = 200 mm

$\therefore$ h_2 = DF

= DE – EF

= 200 – 100

$\therefore$ h_2 = 100 mm

.

Sr. No.	Component Name	Volume of Component V in mm^3	Distance of CG of component from Reference lines		V·x	V·y
			x	y		
1	Full cone OAD (D = 100 mm & H = 200 mm)	$\kappa R^2 H$ $= \kappa \times 50^2 \times 200$ $= 1570796.33$	$\frac{D}{2}$ $= \frac{100}{2}$ $= 50$	$\frac{H}{4}$ $= \frac{200}{4}$ $= 50$	78539816.50	78539816.50
2	Cut of Upper Cone BCD (d = 50 mm & h_2 = 100 mm)	$-\kappa r^2 h$ $= \kappa \times 25^2 \times 100$ $= -196349.54$	50 From Symmetry	$h_1 + \frac{h_2}{4}$ $= 160 + \frac{100}{4}$ $= 200$	–9817477.04	–24543692.50
	Summation	ΣV = 1374446.79	---	---	ΣV.x = 68722339.46	ΣV·y = 53996124.00

(a) $\bar{x} = \frac{\Sigma V \cdot x}{\Sigma V} = \frac{343145693.60}{4289321.17} = 50.00$ mm **(Answer)**

[As per symmetry of composite figure about YY axis (vertical), we can directly,

Find $\bar{x} = \frac{\text{Total width}}{2} = \frac{100}{2} = 50.00$ mm; As we obtained by calculations.]

(b) $\bar{y} = \frac{\Sigma V \cdot y}{\Sigma V} = \frac{53996124.00}{4289321.17} = 39.29$ mm **(Answ**

CHAPTER 5 SIMPLE MACHINE

(i) Simple machine : Simple machine is a device in which effort is applied at one place and work is done at some other place. Simple machines are run manually, not by electric power.

e.g. pulley, bicycle, sewing machine & simple screw jack, etc.

(ii) Compound machine : If a machine, consists of many simple machines, it is called compound machine. Such machines are run by electric or mechanical power. Such machines work at higher speed. Using compound machines more work is done at less effort.

e.g. scooter, lathe, crane & grinding machine, etc.

(iii) Lifting machine : Lifting machine is a device in which heavy load can be lifted by less effort.

e.g. lift, crane, etc.

(iv) Simple lifting machine : Simple lifting machine is a device in which heavy load can be lifted by small effort manually.

e.g. simple pulley, simple crew jack, etc

TECHNICAL TERMS RELATED TO SIMPLE LIFTING MACHINES

(i) Load (W) : The weight of lifted elements is called load (W).

(ii) Effort (P) : The force apply to lift the load (W) is called effort (P).

(iii) Mechanical advantage (MA) : The ratio of load (W) lifted and effort (P) required to lift the load is called Mechanical advantage. It is always express as pure number mathematically.

$$MA = \frac{\text{Load lifted}}{\text{Effort required}}$$

$$\therefore \ MA = \frac{W}{P}$$ Where, W = Load in N OR kN & P = Effort in N

TECHNICAL TERMS RELATED TO SIMPLE LIFTING MACHINES

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$$MA = \frac{\text{Load lifted}}{\text{Effort required}}$$

$$\therefore\ MA = \frac{W}{P}$$ Where, W = Load in N OR kN & P = Effort in N

(vii) Velocity ratio (VR) : The ratio of distance moved by effort (y) and the distance moved by load (x) is called velocity ratio. It is unit less quantity and hence expressed as pure number. Mathematically –

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$\therefore\ VR = \frac{y}{x}$$

It is also noted here that Velocity ratio is constant for a particular machine. It will not change over period of time.

(viii) Efficiency (η) : The ratio of work done by the machine (output) and work done on the machine (input) is called efficiency of the machine. The output and input mathematically express as

(a) Input = Effort × Distance moved by effort

$\therefore$ Input = $P \cdot y$ and

(b) Output = Load × Distance moved by load

$\therefore$ Output = $W \cdot x$

It is express as percentage. Mathematically, Efficiency = $\frac{\text{output}}{\text{input}} \times 100\%$

We know that output = $W \cdot x$ and input = $P \cdot y$

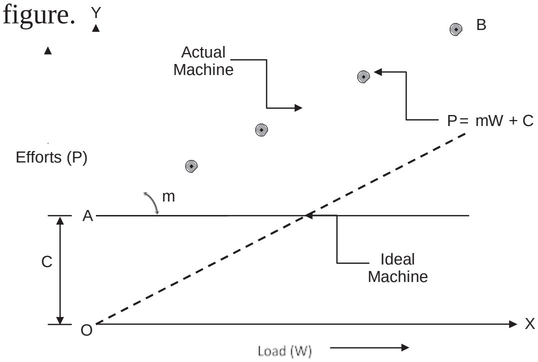
$$\therefore\ \eta = \frac{\text{output}}{\text{input}} \times 100$$

$$= \frac{W \cdot x}{P \cdot y} \times 100 = \frac{W/P}{y/x} \times 100$$

$$\therefore\ \eta = \frac{MA}{VR} \times 100\%$$

$$\therefore\ \eta = \frac{\text{output}}{\text{input}} \times 100\% = \frac{MA}{VR} \times 100\%$$

(ix) Law of machine : For a particular machine, if we record various values of effort (P) required to lift the corresponding loads (W) and plot a graph between effort and load, we shall get a straight line AB as shown in figure.



Law of Machine

Mathematically, the law of machine is given by relation :

$P = mW + C$

Where, P = Effort applied, W = Load lifted, m = constant (coefficient of friction) = slope of line AB and C = constant = machine friction

Following observations are made from the graph :

(a) On a machine, if W = 0, effort C is required to run the machine. Hence, effort C is requiredto overcome machine friction.

(b) If line AB passes through origin, no effort is required to balance friction. Such a graph is for Ideal machine.

(c) If line AB crosses *x*-*x* axis, without effort (p), some load can be lifted, which is impossible. Hence, line AB never crosses *x*-*x* axis.

(x) Maximum mechanical advantage (MA_{max}) : We know that $MA = \frac{W}{P}$.

To get maximum MA, put P from law of machine as, $P = mW + C$

$$\therefore MA_{max} = \frac{W}{mW + C}$$

$$= \frac{1}{m + \frac{C}{W}} \qquad \text{neglecting } \frac{C}{W}\text{, we get}$$

$$\therefore MA_{max} = \frac{1}{m}$$

(xi) Maximum efficiency (η_{max}) : We know that, Velocity Ratio (VR) is constant for a given machine and MA varies.

Now $\eta = \frac{MA}{VR}$ $\qquad \because$ Substitute MA as $MA_{max} = \frac{1}{m}$ to get η_{max},

$$\therefore \eta_{max} = \frac{\frac{1}{m}}{VR}$$

$$\therefore \eta_{max} = \frac{1}{m \times VR}$$

(xii) Ideal machine : A machine having 100% efficiency is called an ideal machine. In an ideal machine friction is zero.

For ideal machine, Output = Input or MA = VR

(xiii) Effort lost in friction (P_f) : In a simple machine, effort required to overcome the friction between various parts of a machine is called effort lost in friction.

Let P = Effort, P_o = Effort for ideal machine, P_f = Effort lost in friction

$\therefore$ Effort lost in friction, $P_f = P - P_o$

For Ideal machine MA = VR

$$\therefore \frac{W}{P_o} = VR$$

$$\therefore P_o = \frac{W}{VR} = \text{Ideal effort}$$

Due to friction, Actual P > Ideal Effort P_o

$$\therefore P_f = P - P_o$$

$$\therefore P_f = P - \frac{W}{VR}$$

(xiv) Friction load (W_f) : Total friction force produced, when machine is in motion, is called friction load.

Let W = Load (Actual), W_o = Load for Ideal machine and P = Effort

For ideal machine, MA = VR

$W_o = P \times VR$ = Ideal load

Now, friction load $W_f = W_o - W$

$\therefore W_f = (P \times VR) - W$

(xv) Reversible machine : If a machine is capable of doing some work in the reverse direction, after the effort is removed, is called reversible machine.

For reversible machine, $\eta \geq 50\%$

(xvi) Non-reversible machine or self-locking machine : If a machine is not capable of doing some work in the reverse direction, after the effort is removed, is called non-reversible machine or self-locking machine. Generally all lifting machines are self-locking machines.

For non-reversible machine, $\eta < 50\%$

(xvii) Condition for reversibility of machine :

Let W = Load lifted, P = Effort required, x = Distance moved by load and y = Distance moved by effort and $P \cdot y$ = input & $W \cdot x$ = output

We know that, Machine friction = Input – Output = $P \cdot y - W \cdot x$

For a machine to reverse,

Output $\geq$ Machine friction

$$\therefore W \cdot x \geq P \cdot y - W \cdot x$$

$$\therefore 2\,W \cdot x \geq P \cdot y$$

$$\therefore \frac{W \cdot x}{P \cdot y} \geq \frac{1}{2}$$

$$\therefore \frac{\text{output}}{\text{input}} \geq 0.5$$

$$\therefore \eta \geq 50\%$$

For a machine to reverse, $\eta \geq 50\%$

Example 1. *In a lifting machine, an effort of 30 N just lift a load of 720 N. What is the mechanical advantage, if efficiency of machine is 30% at the load ? Calculate velocity ratio of machine.*

Solution :

W = 720 N, P = 30 N and η = 30% = 0.3

(a) $MA = \frac{W}{P} = \frac{720}{30}$

$\therefore$ MA = 24 **(Answer)**

(b) $\eta = \frac{MA}{VR}$

$\therefore 0.30 = \frac{24}{VR}$

$\therefore$ VR = 80 **(Answer)**

Example 2. *The velocity ratio of a machine is 20 and efficiency is 80%. Find how much load will be lifted by an apply effort of 200 N.*

Solution :

VR = 10, η = 80% = 0.80, P = 200 N

(a) $\eta = \frac{MA}{VR}$

$0.80 = \frac{MA}{20}$

$\therefore$ MA = 16

(b) $MA = \frac{W}{P}$

$16 = \frac{W}{200}$

$\therefore$ W = 3200 N **(Answer)**

Example 3. *A single purchase crab which has the following details : It is observed that an effort of 60 N lifts a load of 1800 N and an effort of 120 N lifts a load of 3960 N. The Velocity Ratio VR of machine is 42. (a) Establish the law of machine. (b) Find the efficiency in any one case of above.*

Solution :

(i) When P_1 = 60 N, W_1 = 1800 N (ii) When P_2 = 120 N, W_2 = 3960 N (iii) VR = 42

(A) Law of machine

(a) Put the value of P and W of two observations in Law of machine.

P = mW + C

$\therefore \quad 60 = m \times 1800 + C$ ... (i)

$\underline{-120 = -m \times 3960 \pm C}$ ...(ii)

$\therefore \quad -60 = -2160\ m$ ...(i) – (ii)

$\therefore\ m = \frac{60}{2160}$

$\therefore\ m = 0.0277$

(b) Substitute the value of m in equation (i),

$\therefore\ 60 = 0.0277 \times 1800 + C$

$\therefore\ 60 = 49.86 + C$

$\therefore\ C = 10.14$

(c) Law of machine for the given machine is

$P = 0.0277\ W + 10.14$ **(Answer)**

(B) Efficiency for case-1

(a) $MA = \frac{W}{P} = \frac{1800}{60} = 30$

(b) $VR = 42$

(c) $\eta = \frac{MA}{VR} = \frac{30}{42} = 0.7142$

$\therefore\ \eta = 71.42\%$ **(Answer)**

Example 4. *Fill in the blanks given below for a simple lifting machine having velocity ratio VR = 30. Find maximum efficiency the machine can reach stating whether the machine is reversible or not.*

Sr. No.	Load (W) in kN	Effort (P) in kN	Efficiency in %
1	100	9.82	______
2	600	49.82	______
3	1000	______	______

Solution :

(A) For first observation :

W = 100 kN, P = 9.82 kN and VR = 30

(i) $MA = \frac{W}{P} = \frac{100}{9.82} = 10.18$

(ii) $\eta = \frac{MA}{VR} \times 100$

$= \frac{10.18}{30} \times 100$

$\therefore\ \eta = 33.93\%$ **(Answer)**

(B) For second observation :

W = 600 kN, P = 49.82 kN and VR = 30

(i) $MA = \frac{W}{P} = \frac{600}{49.82} = 12.04$

(ii) $\eta = \frac{MA}{VR} \times 100$

$= \frac{12.04}{30} \times 100$

$\therefore \eta = 40.13\%$ **(Answer)**

(C) For third observation :

(I) We know the law of machine as P = mW + C

Put the values of two observations

(i) P = 9.82 kN and W = 100 kN

(ii) P = 49.82 kN and W = 600 kN

We get,

$\therefore \quad 9.82 = m \times 100 + C$ ...(i)

$49.82 = m \times 600 \pm C$ ...(ii)

$-40 = -500\,m$ ...(i) – (ii)

$\therefore \quad m = 0.08$

(II) Substitute m = 0.08 in equation (i)

$9.82 = 0.08 \times 100 + C$

$\therefore 9.82 = 8 + C$

$\therefore C = 1.82$

(III) Law of machine is

$P = 0.08\,W + 1.82$

(IV) Now, when W = 1000 kN,

$P = 0.08 \times 1000 + 1.82$

$\therefore P = 81.82$ kN **(Answer)**

(V) (i) $MA = \frac{W}{P} = \frac{1000}{81.82} = 12.22$

(ii) $\eta = \frac{MA}{VR} = \frac{12.22}{30} \times 100 = 40.73\%$ **(Answer)**

(D) $\eta_{max} = \frac{1}{m \times VR} \times 100$

$= \frac{100}{0.08 \times 30}$

$= 41.67\%$

$\therefore \eta_{max} = 41.67\%$ **(Answer)**

(E) In all the above observations, η is less than 50%. Hence, the machine is non-reversible (self-locking machine). **(Answer)**

Example 5. *In a lifting machine an effort of 30 N can lift a load of 350 N and an effort of 40 N can lift a load of 500 N. If velocity of machine is 20, prove that maximum efficiency is 75%.*

Solution :

VR = 20, η_{max} = 75%, P_1 = 30 N and W_1 = 350 N, P_2 = 40 N and W_2 = 500 N

(a) Put the values of two observations in Law of machine,

P = mW + C

$$\therefore \quad 30 = m \times 350 + C \quad \text{...(i)}$$

$$-40 = -m \times 500 - C \quad \text{...(ii)}$$

$$-10 = -150\, m \quad \text{...(i) – (ii)}$$

$\therefore$ m = 0.067

(b) Substitute, m = 0.067 in equation (i), we get

30 = 0.067 × 350 + C

$\therefore$ C = 6.55

(c) $$\eta_{max} = \frac{1}{m \times VR} = \frac{1}{0.067 \times 20}$$

$\therefore$ η_{max} = 0.746 = 74.6% ≅ 75% **(Answer)**

Example 6. *In a machine whose velocity ratio is 6 and which lifts the load of 100 N with an effort of 20 N. Find (i) Efficiency of machine, (ii) Effort lost in friction, (iii) Frictional load, (iv) Ideal effort and (v) ideal load.*

Solution :

Here VR = 6, W = 100 N & P = 20 N

(i) Mechanical Advantage, $MA = \frac{W}{P} = \frac{100}{20} = 5$

(ii) Efficiency of machine, $\eta = \frac{MA}{VR} = \frac{5}{6} = 0.8333$

$\therefore$ Efficiency of machine, η = 83.33% **(Answer)**

(iii) Effort lost in friction, $P_F = P - \frac{W}{VR} = 20 - \frac{100}{6}$

$\therefore$ P_F = 3.33 N **(Answer)**

(iv) Frictional load, $W_F = (P \times VR) - W$

$\therefore$ $W_F = (20 \times 6) - 100$

$\therefore$ Frictional load, W_F = 20 N **(Answer)**

(v) Ideal effort (P_o), $P_o = \frac{W}{VR} = \frac{100}{6}$

$\therefore$ P_o = 16.67 N **(Answer)**

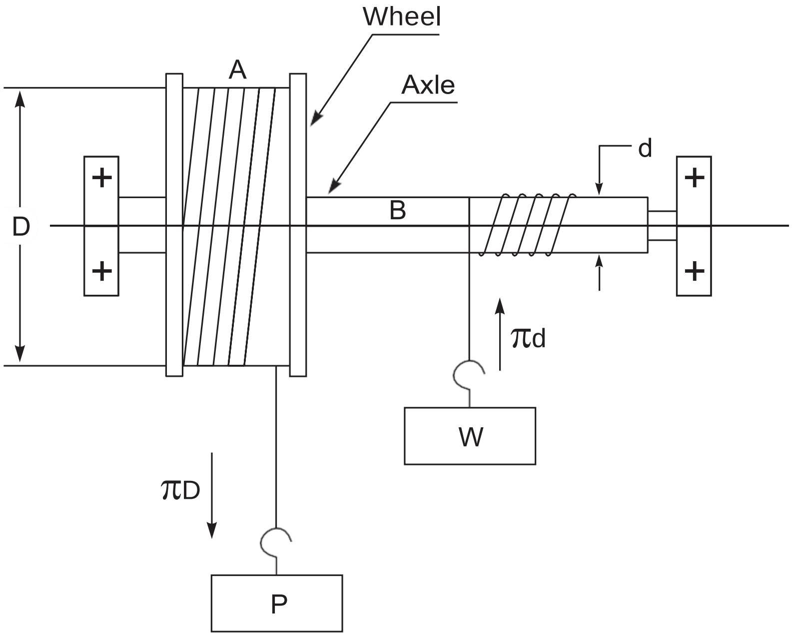
(vi) Ideal Load (W_o), $W_o = P \times VR$

$\therefore W_o = 20 \times 6 = 120$ N **(Answer)**

VELOCITY RATIO FOR DIFFERENT SIMPLE LIFTING MACHINES

(a) Simple axle & wheel

In fig. 5.2 is shown a simple axle and wheel in which the wheel A and axle B are keyed to the same shaft.



Simple axle & wheel

The string is wound around the axle B, which carries the load W to be lifted. A second string is wound around the wheel A in opposite direction to that of string on axle B, so that downward motion of effort P will lift the load W.

Let D = Diameter of wheel and d = Diameter of axle, then

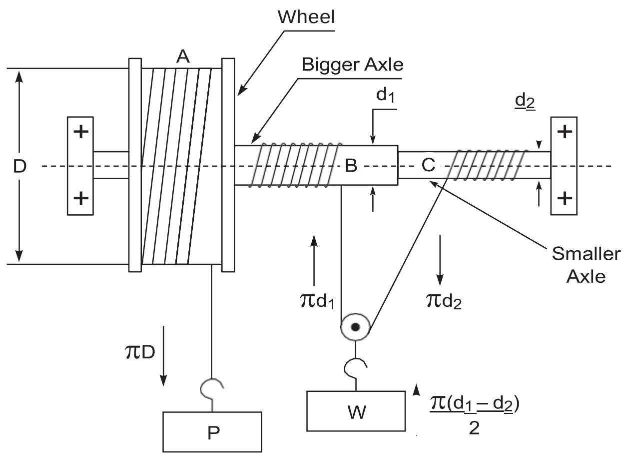
$$VR = \frac{\text{Distance moved by Effort}}{\text{Distance moved by Load}}$$

$$= \frac{y}{x} = \frac{\pi D}{\pi d}$$

$$\therefore VR = \frac{D}{d}$$

(b) Differential axle and wheel

In fig. 5.3 is shown a differential axle and wheel. In this case, the load axle BC is made of two parts of different diameters & effort wheel A are key to same shaft.



Differential axle and wheel

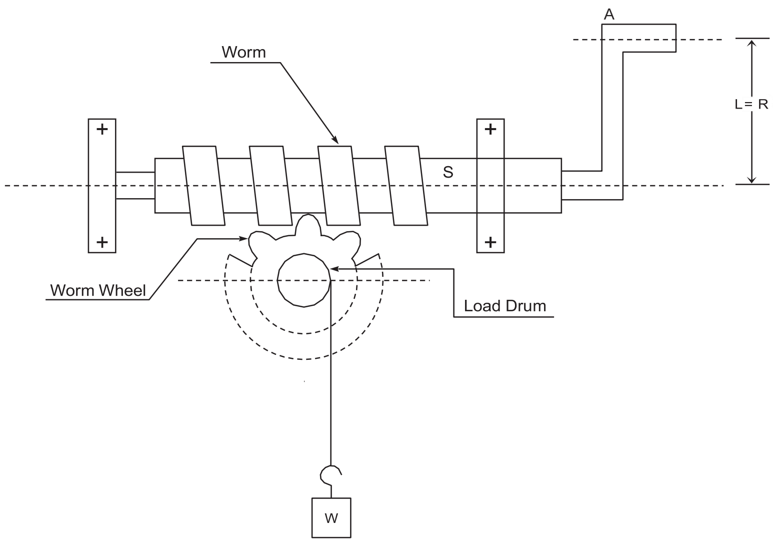
The effort string is wound round the wheel A and another string is wound round the axle B which after passing round the pulley (to which the weight to be lift is attached) is wound round the axle C in opposite direction to that of axle B. So unwinds string from wheel A, other string also unwinds from axle C. But it winds on axle B to lift the load W.

Let D = Diameter of wheel, d_1 = Diameter of bigger axle & d_2 = Diameter of smaller axle, then

$$VR = \frac{2D}{d_1 - d_2}$$

(c) Worm and worm wheel

It consists of a square threaded screw 'S' known as worm and a toothed wheel known as worm wheel geared with each other as shown in fig. 5.4. A wheel or handle A is attach to the worm to apply effort P. A load drum is securely mount on the worm wheel.



Worm and worm wheel

Let R = Radius of effort wheel = Length of handle, r = Radius of load drum, T = no. of teeth on worm wheel and n = no. of worm thread (single, double etc.), then

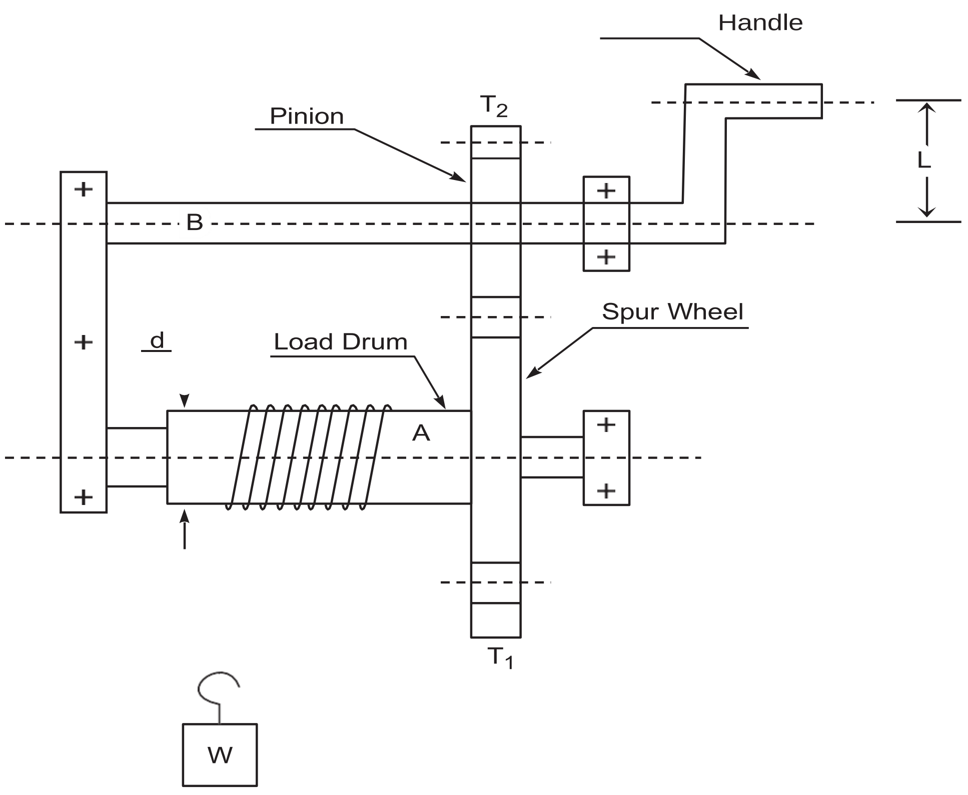
$$VR = \frac{RT}{r} \text{ or } VR = \frac{RT}{nr}$$

(d) Single purchase crab winch

In a single purchase crab winch, a rope is fix to the load drum A and is wound a few turns round it. The free end of the rope lift up the load W. A toothed spur wheel (T_1) is rigidly mount on the load drum A. Another toothed pinion wheel (T_2) is gear with spur wheel as shown in fig. 5.5.

Let l = Length of handle, r = Radius of load drum, T_1 = No. of teeth on main gear (spur wheel) and T_2 = No. of teeth on pinion, then

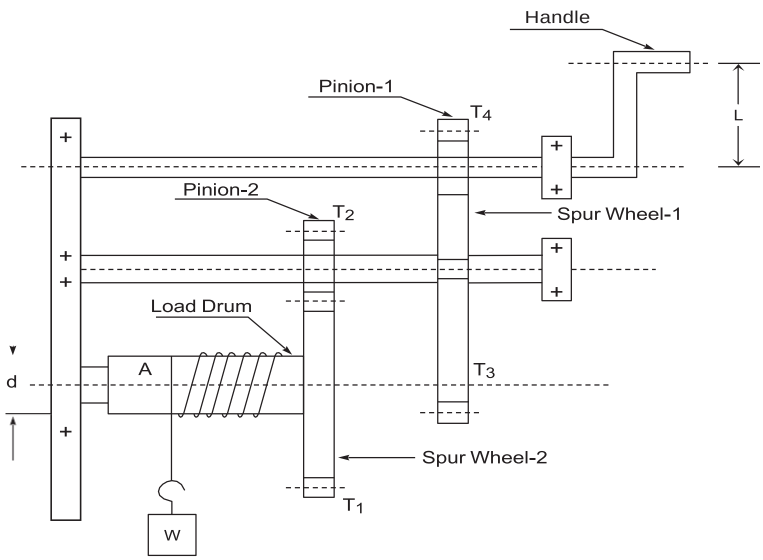
$$VR = \frac{l}{r} \times \frac{T_1}{T_2}$$



Single purchase crab winch

(e) Double purchase crab winch

A double purchase crab winch is as intensified design of a single purchase crab winch, to obtain higher value of VR. In this, there are two spur wheel of teeth T_1 and T_3 as well as two pinion teeth T_2 and T_4.



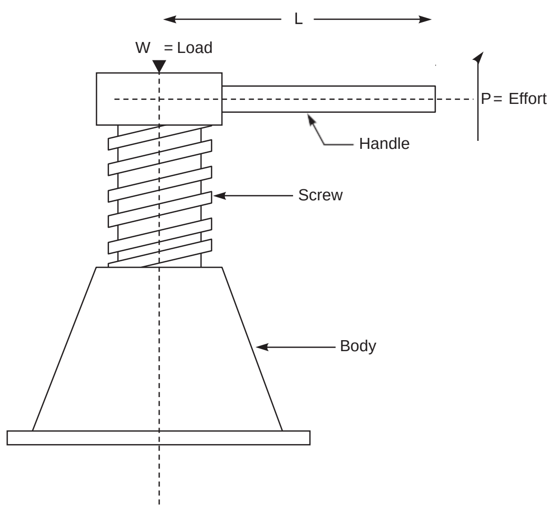
Double purchase crab winch

Let l = Length of handle, r = Radius of load drum, T_1 & T_3 = No. of teeth on main gears (spur wheel), T_2 & T_4 = No. of teeth on pinions, then

$$VR = \frac{l}{r} \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

(f) Simple screw jack

It consists of a screw, fitted in nut, which forms the body of the jack. In which screw is rotate by the application of an effort P, at the end of the lever handle, for lift the load W considering a single threaded simple screw jack.



Simple screw jack

Let l = Length of handle & p = Pitch of screw, then

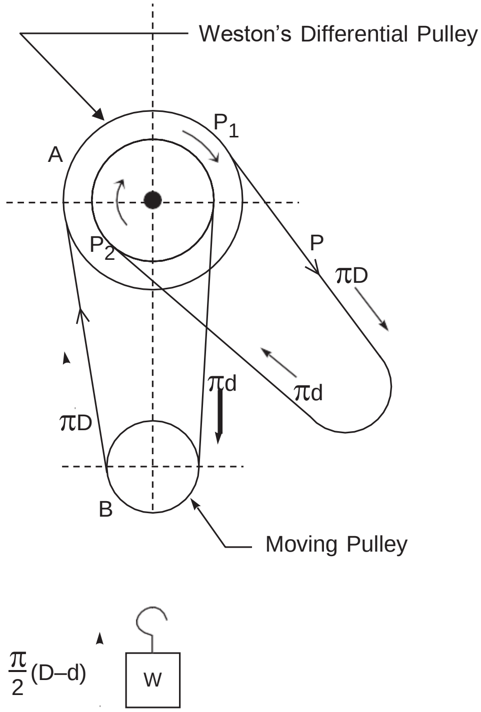
$$VR = \frac{2\pi l}{p}$$

(g) Weston's differential pulley block

It consists of two pulley blocks A and B. The upper block A has two pulleys (P_1 & P_2), one having its diameter a little larger than that of the other. i.e. both of pulley behaves as one pulley with two grooves. The lower block B also carries a pulley, to which the load W is attach to lift up. A continuous chain passes around the pulley P_1 then around the lower block pulley and then finally round the pulley P_2. The effort P is apply to the chain passing over the pulley P_1, so that load W can be lift up as shown in fig.5.8.

Let D = Diameter of bigger pulley and d = Diameter of smaller pulley, then

$$VR = \frac{2D}{D-d}$$



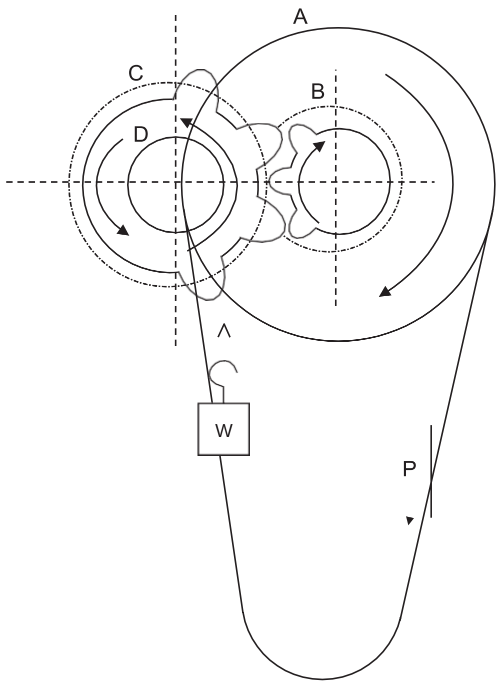
Weston's differential pulley block

(h) Geared pulley block

In consists of a cog wheel A, around which is passed an endless chain. A small gear wheel B known as pinion is key to the same shaft as that of A. The wheel axle B is gear with another bigger wheel C called the spur wheel. A cogwheel D is key to the same shaft as that of spur wheel C.

The load W is attach to a chain that passes over the cogwheel D and the effort P is applied to the endless chain, which passes over the wheel A as shown in fig. 5.9.

Geared pulley block



Let T_1 = No. of cogs on effort wheel A, T_2 = No. of teeth on pinion wheel B, T_3 = No. of teeth on spur wheel C, T_4 = No. of cogs on load wheel D, then

$$VR = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

Example 7. *In a double purchase crab winch the pinion has 10 and 20 teeth and spur wheels have 40 and50 teeth. The handle is 30 cm long and load axle drum is 20 cm diameter. Find the effort required to lift aload of 1500 N when efficiency is 40%.*

Solution :

$$l = 30 \text{ cm}, r = \frac{20}{2} = 10 \text{ cm}, T_1 = 40, T_2 = 10, T_3 = 50 \ \& \ T_4 = 20, \eta = 0.40$$

(a) $$VR = \frac{l}{r} \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

$$\therefore VR = \frac{30}{10} \times \frac{40 \times 50}{10 \times 20} = 30$$

(b) $\eta = \frac{MA}{VR}$

$0.40 = \frac{MA}{30}$

$\therefore$ MA = 12

(c) $MA = \frac{W}{P}$

$12 = \frac{1500}{P}$

$\therefore$ P = 125 N **(Answer)**

Example 8. *In a worm and worm wheel, worm wheel has 120 teeth. Length of handle is 30 cm and diameter of load drum is 10 cm. To lift a load of 1800 N effort of 350 N is required. If maximum efficiency is 40%, find the law of machine. Worm is single threaded.*

Solution :

T = 120, R = 30 cm, $r = \frac{10}{2}$ = 5 cm, n = 1 (screw is single threaded) and η_{max} = 40%

(a) $VR = \frac{RT}{nr} = \frac{30 \times 120}{1 \times 5} = 720$

(b) $\eta_{max} = \frac{1}{m \times VR}$

$\therefore 0.40 = \frac{1}{m \times 720}$

$\therefore$ m = 0.00347

(c) P = mW + C

350 = 0.00347 × 1800 + C

$\therefore$ C = 343.75

(d) Law of machine, P = 0.00347 W + 343.75 **(Answer)**

Example 9. *A single purchase crab which has the following details : (1) Length of lever = 80 cm, (2) Number of teeth on pinion = 20, (3) Number of teeth on spur wheel = 120, (4) Diameter of load drum (axle) = 30 cm. It is observe that an effort of 80 N lifts a load of 2000 N and an effort of 160 N lifts a load of 4200 N (1) Establish the law of machine, (2) Find the efficiency in any one case.*

Solution :

l = 80 cm, $r = \frac{30}{2}$ = 15 cm, T_1 = 120 & T_2 = 20

(a) $VR = \frac{l}{r} \times \frac{T_1}{T_2} = \frac{80}{15} \times \frac{120}{20}$

$\therefore$ VR = 32

(b) $MA = \frac{W}{P} = \frac{2000}{80} = 25$

(c) Efficiency for reading No. 1

$$\eta = \frac{MA}{VR} \times 100 = \frac{25}{32} \times 100$$

$\eta = 78.125\%$ **(Answer)**

(d) Law of machine

(i) Put value of two observation in law of machine P = mW + C, we get

$$80 = m \times 2000 + C \quad ...(i)$$

$$160 = m \times 4200 + C \quad ...(ii)$$

$-80 = -2200$ m [Subtracting (ii) from (i)]

$\therefore m = 0.036$

(ii) Put value of m in equation (i), we get

$80 = m \times 2000 + C$

$\therefore 80 = 0.036 \times 2000 + C$

$\therefore C = 8$

(ii) Thus Law of machine P = mW + C by putting value of m & C, we get

P = 0.036W + 8 **(Answer)**

Example 10. *In a differential axle and wheel, the diameter of the effort wheel is 400 m. The radii of the axles are 150 mm and 100 mm respectively. The diameter of the rope is 1 cm. Find the load which can be lifted by an effort of 200 N assuming efficiency of machine to be 75%.*

Solution :

D = 400 mm, $d_1 = 2 \times 150 = 300$ mm, $d_2 = 2 \times 100 = 200$ mm,

t_1 = diameter of the rope = 1 cm = 10 mm & $\eta = 75\%$

(a) For differential axle and wheel,

$$VR = \frac{2(D + t_1)}{(d_1 + t_1) - (d_2 + t_1)} = \frac{2(D + t_1)}{(d_1 - d_2)}$$

$$\therefore VR = \frac{2 \times (400 + 10)}{300 - 200}$$

$\therefore VR = 8.2$

(b) $\eta = \frac{MA}{VR} \times 100$

$75 = \frac{MA}{8.2} \times 100$

$\therefore$ MA = 6.15

$$\therefore\ 6.15 = \frac{W}{200}$$

$\therefore$ W = 1230 N **(Answer)**

Example 11. *In a double purchase crab winch number to teeth on pinion are 120 and 150 and that of spur are 300 and 400. Diameter of axle is 20 cm. Find V.R. Also find friction in terms of effort and load when effort of 105 N is required to lift a load of 1.834 kN. Take length of handle as 80 cm.*

Solution :

$$l = 80 \text{ cm},\ r = \frac{20}{2} = 10 \text{ cm},\ T_1 = 300,\ T_2 = 120\ \&\ T_3 = 400\ \&\ T_4 = 150$$

(a) $$VR = \frac{l}{r} \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

$$\therefore\ VR = \frac{80}{} \times \frac{300 \times 400}{} = 53.33$$ **(Answer)**

10 120 × 150

(b) Effort lost in friction : $P_f = P - \frac{W}{VR}$

Put P = 105 N and W = 1.834 kN = 1834 N

$\therefore P_f = 105 - \frac{1834}{53.33}$

$\therefore P_f = 70.61$ N **(Answer)**

(c) Friction load : $W_f = 105 \times 53.33 - 1834$

$\therefore W_f = 3765.65$ N **(Answer)**