

C. V. Raman Polytechnic

Lecture note

Fluid Mechanics

Prepared by

Soumya Dash

Assistant Professor
Mechanical Engineering Department

Fluid mechanics is a branch of engineering science which deals with the behavior of fluids (liquid or gases) at rest as well as in motion.

Definition of Fluid.

Fluid is a substance which has the ability to flow and undergo shear deformation when subjected to shear stresses.

Properties of Fluids

1. Density or Mass Density

Density or mass density of fluid is defined as the ratio of the mass of the fluid to its volume.

$\rho = \frac{\text{Mass of the fluid}}{\text{Volume of the fluid}}$

Unit: Kg/m^3

2. Specific Weight or Weight Density.

Specific weight or weight density of a fluid is defined as the ratio of weight of a fluid to its volume.

$W = \frac{\text{weight of a fluid}}{\text{Volume of the fluid}}$

$W = \rho \times g$

Unit: N/m^3

3. Specific Volume

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass of fluid.

$$V = 1/\rho$$

Unit: m^3/Kg

4. Specific Gravity or Relative Density

Specific gravity is defined as the ratio of the density (or weight density) of a fluid to the density (or weight density) of a standard fluid.

Specific Gravity S = $\frac{\text{Density of liquid}}{\text{Density of water}}$

For liquids, standard fluid is water and for gases, standard fluid is air. Specific gravity is also called relative density.

It is dimensionless quantity and is denoted by symbol S.

Specific gravity of mercury is 13.6.

Note: The value of density of water is $1000 \text{ kg}/m^3$

(At 40°C at mean sea level.)

The value of density of air is $1.225 \text{ kg}/m^3$

(At 15°C at mean sea level.)

Problem:

Calculate specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

Solution: 5

Volume of 1 litre of liquid = $10^{-3}m^3$

Specific Weight $W = \text{weight of a fluid} \cdot \text{Volume of the fluid} = 7N10^{-3}m^3$
 $= 7000 N/ m^3$Ans

Density $\rho = \frac{w}{g} = \frac{7000}{9.81} = 713.5 \text{ Kg}/m^3$
.....Ans

Specific Gravity $S = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} = 0.7135$Ans

Problem:

Calculate specific weight, density and weight of 1 litre of petrol of specific gravity = 0.7

Solution:

Specific Gravity $S = \frac{\text{Density of liquid}}{\text{Density of water}}$

$0.7 = \frac{\text{Density of liquid}}{1000 \text{ Kg}/m^3}$

Density of petrol $\rho = 0.7 \times 1000 = 700 \text{ Kg}/m^3$

Specific Weight $W = \rho \times g = 700 \text{ Kg}/m^3 \times 9.81 = 6867 \text{ N}/m^3$

Specific Weight $W = \text{weight of petrol} \cdot \text{Volume of petrol} \cdot$

Weight of petrol = $w \times \text{Vol. of petrol} = 6867 \text{ N}/m^3 \times 10^{-3}m^3$
 $= 6.867 \text{ N}$Ans

5.Viscosity ₆

Viscosity is defined as the property of fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid.

Let, $(u + du)$ = Velocity of top layer.

u = Velocity of adjacent bottom layer.

dy = Distance between these two layers as shown in fig.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to y .

Shear stress is denoted by symbol τ (Tau). 7

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \times \frac{du}{dy}$$

$$\mu = \frac{\tau}{\frac{du}{dy}}$$

Unit of viscosity

N.sm² = Pa.s In SI system,

Kgf.secm² = In MKS system,

dyne.secm² = poise. In CGS system,

$$1 \text{ N.sm}^2 = 10 \text{ poise.}$$

$$1 \text{ poise} = 10 \text{ N.sm}^2$$

$$1 \text{ Centipoise (cp)} = 100 \text{ poise}$$

Newton's Law of Viscosity

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

Mathematically,

$$\tau = \mu \times \frac{du}{dy}$$

Where μ = Constant of proportionality is called as co-efficient of dynamic viscosity or viscosity. 8

Variation of Viscosity with Temperature

The viscosity of fluid depends on two contributing factors, namely,

- Cohesion between the fluid molecules
- Transfer of momentum between the molecules

For Liquid,

$$\mu = \mu_0 (1 + \alpha t + \beta t^2) \dots\dots\dots 1$$

where μ = viscosity of the liquid at $t^\circ\text{C}$ in poise

- μ_0 = viscosity of the liquid at 0°C in poise
- α and β are constant characteristics of the liquid.

Equation 1 shows that with the increase in temperature the viscosity of a liquid decreases.

For water,

$$\mu_0 = 0.00179 \text{ poise}, \alpha = 0.03368, \beta = 0.000221$$

For gas

$$\mu = \mu_0 + \alpha t - \beta t^2 \dots\dots\dots 2$$

where μ = viscosity of the gas at $t^\circ\text{C}$ in poise

μ_0 = viscosity of the gas at 0°C in poise

α and β are constant characteristics of the gas.

For air, $\mu_0 = 0.000017$ poise, $\alpha = 0.56 \times 10^{-7}$, $\beta = 9$

$\beta = 0.1189 \times 10^{-9}$. Equation 2 shows that increase in temperature increases the viscosity of a gas.

Kinematic viscosity (ν)

It is defined as the ratio between dynamic viscosity and density of fluid.

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

Unit: m^2/sin SI

cm^2/s in CGS

$$1 \text{ cm}^2/\text{s} = 1 \text{ stoke}$$

$$100 \text{ cm}^2/\text{s} = 1 \text{ Centistoke.}$$

Example 1: A plate, 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2N per unit area i.e., 2N/m² to maintain this speed. Calculate the Fluid viscosity between the plates.

Solution:

Given data:

$$dy = 0.025 \text{ mm} = 0.025 \times 10^{-3} \text{ m}, \tau = 2 \text{ N} / \text{m}^2$$

$$v = 60 \text{ cm} / \text{s} = 0.6 \text{ m} / \text{s}$$

To find: $\mu = ?$

Change in velocity $du = \text{velocity of top plate} - \text{velocity of bottom} = 0.6 \text{ m / s} - 0 = 0.6 \text{ m / s} ,$

(Since bottom plate is fixed its velocity is 0)

$$\mu = \tau \frac{dy}{du} = 2 \times 0.6 \times 0.025 \times 10^{-3} = 2 \times 0.025 \times 10^{-3} \times 0.6 = 8.33 \times 10^{-5} \text{ N.sm}^2$$

$$= 8.33 \times 10^{-5} \times 10 \text{ poise} = 8.33 \times 10^{-4} \text{ poise} \dots\dots\dots \text{Ans.}$$

6.Surface tension

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquid such that the contact surface behaves like a membrane under tension.

It is denoted by Greek letter σ (sigma).

SI unit of surface tension is N/m, and MKS unit is kgf/m. 11

Consider three molecules A, B & C of a liquid in a mass of liquid. The molecule A is attracted in all directions equally by the surrounding molecules of the liquid. Thus, resultant force acting on molecule A is zero. But molecule B, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus, a net resultant force on molecule B is acting in the downward direction. The molecule C, situated on the free surface of liquid, does experience resultant downward force. All the molecules on the free surface of the liquid act like a very thin film under tension of the surface of the liquid act as though it is an elastic membrane under tension.

Surface Tension on Liquid Droplet

Consider a small spherical droplet of a liquid of radius 'r'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let, σ = Surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure)

d = Dia. of droplet. ¹²

(a) Droplet (b) Surface Tension (c) Pressure Force

Let droplet is cut into two halves. Tensile force due to surface tension acting around the circumference of the cut portion as shown in fig. and is equal to

$$= \sigma \times \text{Circumference}$$

$$= \sigma \times \pi d \dots\dots\dots i$$

$$\text{Pressure force on the area} = p \times \pi \frac{d^2}{4} \dots\dots\dots ii$$

These two forces will be equal and opposite under equilibrium condition i.e.

$$p \times \pi \frac{d^2}{4} = \sigma \times \pi d$$

$$P = \frac{4\sigma}{d} \dots\dots\dots iii$$

Equation iii shows that with increase of diameter of the droplet, pressure intensity inside the droplet decreases.

Surface Tension on a Hollow Bubble

A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension.

$$\text{Hence, } p \times \pi \frac{d^2}{4} = 2(\sigma \times \pi d)$$

$$P = \frac{8\sigma}{d}$$

Surface Tension on a Liquid Jet

Consider a liquid jet of diameter 'd' and length 'L' as shown in fig.

Let σ = Surface tension of the liquid

p = Pressure intensity inside the liquid jet above the outside pressure.

Consider the equilibrium of the semi jet, we have

Force due to pressure = $p \times$ area of semi jet = $p \times L \times d$ i

Force due to surface tension = $\sigma \times 2L$ ii

Equating both equations,

$$p \times L \times d = \sigma \times 2L$$

$$\therefore P = 2\sigma d$$

Example:

The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (at atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Solution:

Given data:

Diameter of droplet $d = 0.04 \text{ mm} = 0.04 \times 10^{-3} \text{ m}$

Pressure outside the droplet = 10.32 N / cm²
= $10.32 \times 10^4 \text{ N / m}^2$

Surface tension $\sigma = 0.0725 \text{ N / m}$

Find:

pressure within the droplet =?

The pressure inside the droplet, in excess of outside pressure

is given by equation $P = 4\sigma d = 4 \times 0.0725 \times 0.04 \times 10^{-3} = 7250 \text{ N / m}^2$

Pressure inside the droplet = $7250 \text{ N / m}^2 + 10.32 \times 10^4 \text{ N / m}^2$
= 110450 N / m^2
= 11.045 N / cm^2 Ans.

Example:

The pressure inside the droplet of water is to be 0.02 N/cm² greater than the outside pressure. The surface tension of water in contact with the air at 20°C is 0.0725 N/m. Calculate the diameter of water droplet. 15

Solution:

Given data:

Pressure inside the droplet = $0.02 \text{ N / cm}^2 = 0.02 \times 10^4 \text{ N / m}^2$

Surface tension $\sigma = 0.0725 \text{ N / m}$

To find:

Diameter of water =?

$$P = 4\sigma d$$

$$d = \frac{4\sigma p}{4} = \frac{4 \times 0.0725 \times 0.02 \times 10^4}{4} = 0.00145 \text{ m} =$$

1.45mm.....Ans.

7.Capillarity

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

The rise of the liquid surface is known as capillary rise while the fall of liquid surface is known as capillary depression or fall. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Expression for Capillary Rise

Consider a glass tube of small diameter 'd' opened at the both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of liquid. ¹⁶

Let h = height of liquid in the tube.

Under a state of equilibrium, weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to the surface tension.

Let σ = Surface tension of liquid

d = Diameter of the tube

θ = Angle of contact between liquid and glass tube.

ρ = Density of the liquid.

Weight of liquid of height h in the tube

$$= (\text{Area of tube} \times h) \times \rho \times g = \pi d^2 \times h \times \rho \times g \dots\dots\dots i$$

Vertical component of the surface tension force

$$= \sigma \times \pi d \times \cos \theta \dots\dots\dots ii$$

Under equilibrium condition, equation i = equation ii

$$\pi d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos \theta$$

$$h = \frac{4\sigma \times \cos \theta}{\rho \times g \times d} \quad 17$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity. Then rise of water is given by $h = \frac{4\sigma}{\rho \times g \times d}$

Expression for Capillary Fall

If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than general level of the outside liquid as shown in fig.

Mercury

Let h = height of depression in tube

Two forces are acting on the mercury inside the tube.

First one is due to surface tension acting in the downward direction and is equal to $= \sigma \times \pi d \times \cos \theta$ 1

Second force is due to hydrostatic force acting upward and is equal to intensity of the pressure at the depth 'h' X Area
 $= h \times \rho \times g \times \pi d^2$ 2

Under equilibrium condition, equation 1 = equation 2 ¹⁸

$$h \times \rho \times g \times \frac{\pi d^2}{4} = \sigma \times \pi d \times \cos \theta$$

$h = \frac{4\sigma \times \cos \theta}{\rho \times g} \times \frac{1}{d}$, Value of θ for mercury and glass tube is 128° .

PROBLEM:

Calculate the capillary rise in a glass tube of 2.5mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tension $\sigma = 0.0725\text{N/m}$ for water and $\sigma = 0.52\text{N/m}$ for mercury in contact with air. Specific gravity of mercury is 13.6 and angle of contact is 130° .

Solution:

Given.

Diameter of tube $d = 2.5 \text{ mm} = 2.5 \times 10^{-3}\text{m}$

surface tension $\sigma = 0.0725 \text{ N/m}$ for water

surface tension $\sigma = 0.52 \text{ N/m}$ for mercury

Specific gravity of mercury is 13.6

Rise of water is given by $h = \frac{4\sigma \times \cos \theta}{\rho \times g} \times \frac{1}{d} =$

$$\frac{4 \times 0.0725 \times \cos 0^\circ}{1000 \times 9.81} \times \frac{1}{2.5 \times 10^{-3}}$$

$= 0.118 \text{ m}$

Fall of mercury is given by $h = \frac{4\sigma \times \cos \theta}{\rho \times g} \times \frac{1}{d}$

$$= \frac{4 \times 0.52 \times \cos 130^\circ}{13.6 \times 1000 \times 9.81} \times \frac{1}{2.5 \times 10^{-3}} = -0.004\text{m}$$

-ve sign indicates the capillary