C. V. Raman Polytechnic

DEPARTMENT OF MECHANICALENGINEERING

LECTURE NOTES ON
STRENGTH OF MATERIALS (SOM)
SUBJECT CODE: TH – 2

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COURSE OBJECTIVES:

At the end of the course the students will be able to:

1. Determine the stress, strain under uniaxial loading (due to static or impact load and temperature) in simple and single core composite bars.

2. Determine the stress, strain and change in geometrical parameters of cylindrical and spherical shells due to pressure.

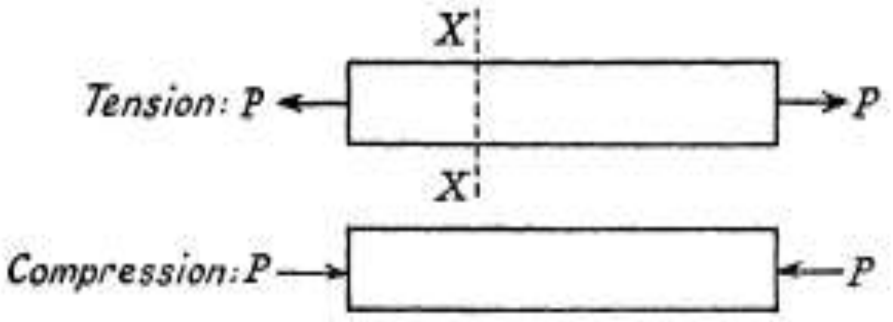
3. Realise about shear stress besides normal stress and computation of resultant stress in two dimensional objects.

4. Draw bending moment and shear force diagram and locating points in a beam where the effect is maximum or minimum.

5. Determine the bending stress and torsional shear stress in simple cases.

6. Underst about critical load in slender columns thus realizing combined effect of axial and bending load.

Load.
The simplest type of load *(P)* is a direct pull or push, known as *tension* or *compression,*

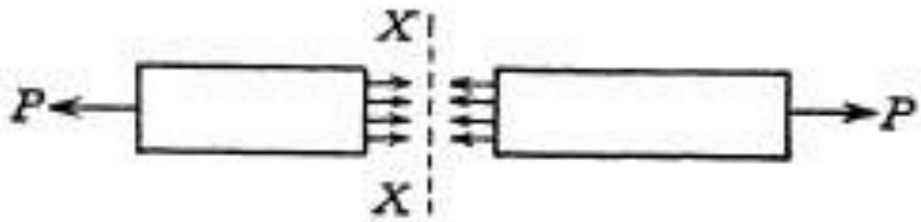


Stress (σ)

$\sigma = \frac{P}{A}$

Where P = Load acting on the member.
A = Cross sectional area.

Let the member is imagined cut through the section XX , each portion is in equilibrium under the action of the external load P and the stresses at XX. Stresses which are normal to the plane on which they act are called direct stresses, and are either tensile or compressive.



Strain.(ε)

Strain is a measure of the deformation produced in the member by the load. Direct stresses produce a change in length in the direction of the stress. If a rod of length *l* is in tension and the elongation produced is $\boldsymbol{\delta l}$, then the direct strain is defined as the ratio of elongation to original length

$\varepsilon = \frac{\delta l}{l}$

Where δl = Elongation of the member.
l = Original length.

Normally, tensile strains will be considered positive and compressive strains is negative. Strain is a *ratio* and hence it is dimensionless.

HOOKE'S LAW.

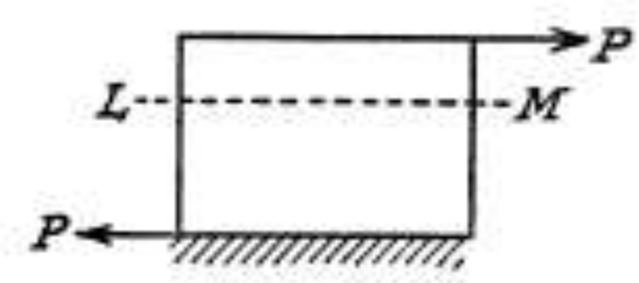
It states that strain is proportianal to stress within elastic limit.

Modulus of Elasticity (Young's Modulus) (E).

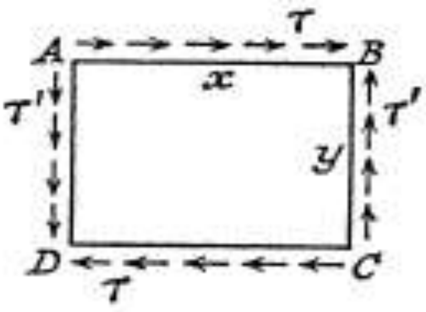
Within the limits for which Hooke's law is obeyed, the ratio of the direct stress to the strain produced is called Young's Modulus or the Modulus of elasticity (E).

$E = \frac{\sigma}{\varepsilon}$

Shear Stress.
If the applied load *P* consists of two equal and opposite parallel forces not in the same line, then there is a tendency for one part of the body to slide over or shear from the other part across any section LM.



Complementary Shear Stress. Let ABCD , be a
small rectangular element of sides *x, Y,*
and *:l* perpendicular to the figure. Let there be a shear stress *T* acting on planes AB and CD



Compound Stress and Strain

Oblique Stress.

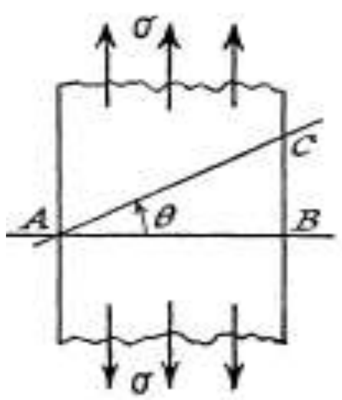
Previous chapters have dealt with either a pure normal, or direct, stress or a pure shear stress. In many instances, however, both direct and shear stresses are
brought into play, and the resultant stress across any section will be neither normal nor tangential to the plane. If , is the resultant stress, making an angle ∅ with the normal to the plane on which it acts.

it is usually more convenient to calculate the normal and tangential components $\boldsymbol{\sigma}$ and τ.

$\emptyset = \tan^{-1}(\tau / \boldsymbol{\sigma})$

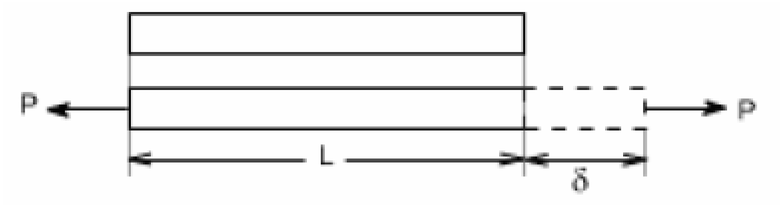
Simple Tension.

If a bar is under the action of a tensile stress along its length then any transverse section will have a pure normal stress acting on it. The problem is to find the stress acting on any plane
AC at an angle θ to AB. This stress will not be normal to the plane, and may be resolved into two components.



Elongation

For a prismatic bar loaded in tension by an axial force P. The elongation of the bar can be determined as ,$\delta l =$ PL/AE



Elongation of composite body

Elongation of a bar of varying cross section A1,A2,-----etc,An of lengths l1, l2,---etc respectively.

Principle of Superposition

The principle of superposition states that when there are numbers of loads are acting together on an elastic material, the resultant strain will be the sum of individual strains caused by each load acting separately.

$$\delta l = \frac{Pl}{AE} = \frac{1}{AE}\,(P_1 l_1 + P_2 l_2 + P_3 l_3 + \ldots)$$

P_1 = Force acting on section 1,

l_1 = Length of section 1,

P_2, l_2 = Corresponding values of section 2, and so on.

MODULUS OF ELASTICITY (OR YOUNG'S MODULUS)

The ratio of tensile or compressive stress to the corresponding strain is a constant. This ratio is known as Young's Modulus or Modulus of Elasticity and is denoted by

$$E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{\text{Compressive Stress}}{\text{Compressive Strain}}$$

$$E = \frac{\sigma}{\varepsilon}$$

MODULUS OF RIGIDITY OR SHEAR MODULUS.

The ratio of shear stress to the corresponding shear strain within the elastic limit, is known as Modulus or Rigidity or Shear Modulus. This is denoted by G

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\emptyset}$$

BULK MODULES (K)

When a body is subjected to three mutually perpendicular stresses of equal intensity the ratio of direct stress to the corresponding volumetric strain is known as bulk modules.

TWO DIMENSIONAL STRESS SYSTEM.

Longitudinal Strain.

The ratio of axial deformation to the original length of the body is known as longitudinal (or linear).

Let l = Length of the body,

P = Tensile force acting on the body.

δl = Increase in the length of the body in the direction of P.

Longitudinal strain = $\frac{\delta l}{l}$

Lateral strain.

Let b = Breadth of the body,

δb = Decrease in breadth.

δd = Decrease in depth.

1 If longitudinal strain is tensile, the lateral strains will be compressive.

2. If longitudinal strain is compressive then lateral strains will be tensile.

3.Thus every longitudinal strain in the direction of load is accompanied by lateral strains of the opposite kind in all directions perpendicular to the load.

Lateral strain.= $\frac{\delta b}{b}$

POISSON'S RATIO.

Ratio of lateral strain to the longitudinal strain is called Poisson's ratio, denoted by μ.

Lateral strain = μ x Longitudinal strain.

PRINCIPLE OF SUPERPOSITION.

If a body is subjected to a number of Loads at various sections,the total deformation of the body will be equal to the algebraic sum of deformation of the individual sections

$\delta l = \delta l_1 + \delta l_2 + \delta l_3 + \delta l_4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

δl = Total deformation of the body

$\delta l_1 =$ Deformation of the section 1

$\delta l_2 =$ Deformation of the section 2

$\delta l_3 =$ Deformation of the section 3

$\delta l_4 =$ Deformation of the section 4

THERMAL STRESS IN COMPOSITE BAR

When a composite material is heated to someTemperature remains the same for all the materials but strain rate is different due to thermal expansion of materials. The below figure shows the thermal expansion on composite bar.

ELASTIC CONSTANTS

If a body is stressed within elastic limit then there is changes in length along x-direction, y-direction and z - direction.

Various types of elastic constants.

1.Elasticity Modulus (E)0r Young's Modulus

2. Poisson's Ratio (μ)

3. Shear Modulus (G)

4. Bulk Modulus (K)

Relation between K and E

$E = 3K (1-2\mu)$

Relation between G and E

$E= 2G (1+\mu)$

Relation between K and E & G

$E = \frac{9GK}{G+3K}$

THERMAL STRESSES IN SIMPLE BAR

Let L = original length of the body

Δ t = Increase in temperature

α = Coefficient of liner expansion.

We know that the increase in length due to increase of temperature

$\delta L = L \times \alpha \times \Delta t$

$$\varepsilon = \frac{\delta L}{L} = \frac{L \times \alpha \times \Delta t}{L} = \alpha \times \Delta t$$

TEMPERATURE STRESSES IN COMPOSITE BAR

If a compound bar made up of several materials is subjected to a change in temperature there will be tendency for the components parts to expand different amounts due to the unequal coefficient of thermal expansion. If the parts are constrained to remain together then the actual change in length must be the same for each. This change is the resultant of the effects due to temperature and stresses condition.

let σ_1 = Stress in brass

ε_1 = Strain in brass

α_1= Coefficient of liner expansion for brass

A_1= Cross sectional area of brass bar

$\sigma_2, \varepsilon_2, \alpha_2, A_2$ = Corresponding values for steel.

ε = Actual strain of the composite bar per unit length. As compressive load on the brass in equal to the tensile load on the steel,

therefore σ 1. A1 =σ 2. A2 strain in brass

$\varepsilon_1 = \alpha_1 \Delta t - \varepsilon$

$\varepsilon_2 = \varepsilon - \alpha_2 \Delta t$

$\varepsilon_1 + \varepsilon_2 = \alpha_1 \Delta t - \varepsilon + \varepsilon - \alpha_2 \Delta t$

$= \alpha_1 \Delta t - \alpha_2 \Delta t$

$\varepsilon_1 + \varepsilon_2 = \Delta t(\alpha_1 - \alpha_2)$

PROBLEM

An aluminium alloy bar fixed at its both ends is heated through 20K find the stress developed in the bar. Take modules of elasticity and coefficient of linear expansion for the bar material as 80 GPa and 24 $\times 10^{-6}$/K respectively.

Data Given

Δ t = 20K

E = 80GPa = 80 $\times 10^3$ N/mm^2

α = 24 $\times 10^{-6}$/K

Solution:

$\sigma t = \alpha \times \Delta t \times E$

$= 24 \times 10^{-6} \times 20 \times 80 \times 10^3$

= 38.4 N/mm^2

= 38.4GPa……………………..(Ans)

PROBLEM

A flat steel bar 200mm X 20mm X 8mm is placed between two aluminium bars 200mm X 20mm X 6mm. So as to form a composite bar. All the three bars are fastened together at room temperature. Find the stresses in each bar where the temperature of the whole assembly in raised through 500 c, Assume Es =200GPa, Ea =80GPa, $\alpha_s = 12 \times 10^{-6}/^0c$,

$\alpha_a = 24 \times 10^{-6}/^0c$

Data Given

Δ t = 50^{0}c

E_s =200 $\times 10^3$ N/mm^2

E_a =80GPa = 80 $\times 10^3$ N/mm^2

$\alpha_s = 12 \times 10^{-6}/^0c$

$\alpha_a = 24 \times 10^{-6}/^0c$

Solution:

As = 20 x 8 = 160 mm2

Aa = 2 x 20 x 6 = 240 mm2

$\sigma s = \frac{\mathrm{Aa}}{\mathrm{As}} \times \sigma a$

$= \frac{240}{160} \times \sigma a$

$= 1.5 \times \sigma a$

$\varepsilon_a + \varepsilon_s = \Delta t(\alpha_a - \alpha_s)$

$= 50(24\times10^{-6} - 12\times10^{-6})$

$\frac{\sigma a}{\mathrm{Ea}} + \frac{\sigma s}{\mathrm{Es}} = 50(12\times10^{-6})$

$\frac{\sigma a}{\mathrm{Ea}} + \frac{1.5\sigma a}{\mathrm{Es}} = 50(12\times10^{-6})$

$\frac{\sigma a}{80 \times 103} + \frac{1.5\sigma a}{200 \times 103} = 50(12\times10^{-6})$

$\Rightarrow \sigma a = 30 \text{ N/mm}^2$

$= 30$ MPa

$\sigma s = 1.5 \times \sigma a$

$= 1.5 \times 30 \text{ N/mm}^2$

= 45 MPa……………………(Ans)