C. V. Raman Polytechnic

Lecture note

Mechanics

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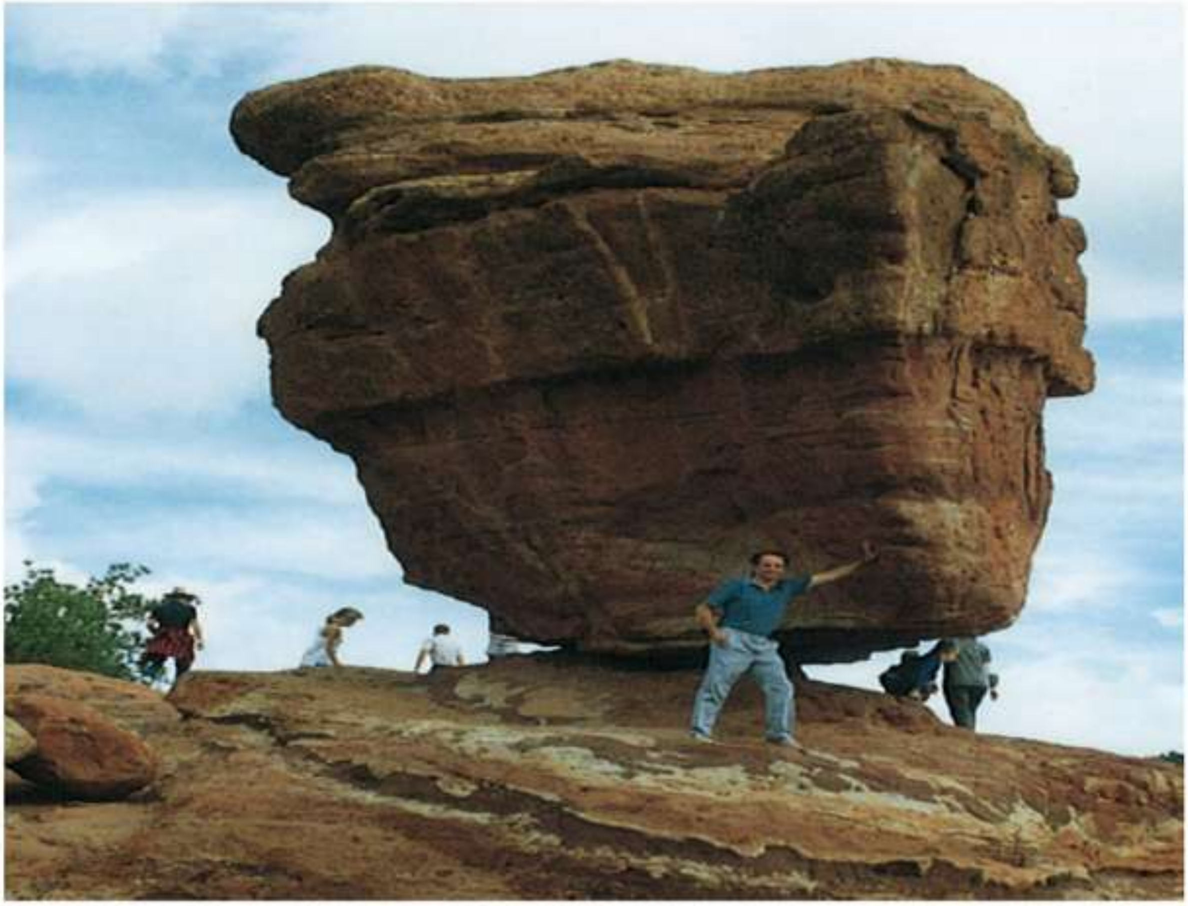
UNIT II

EQUILIBRIUM OF FORCES



PRINCIPLES OF EQUILIBRIUM

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. Two force principle. As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

2. Three force principle. As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.

3. Four force principle. As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

METHODS FOR THE EQUILIBRIUM OF COPLANAR FORCES

Though there are many methods of studying the equilibrium of forces, yet the following are important from the subject point of view

1. Analytical method.

2. Graphical method

ANALYTICAL METHOD FOR THE EQUILIBRIUM OF COPLANAR FORCES

The equilibrium of coplanar forces may be studied, analytically, by Lami's theorem as discussed below :

LAMI'S THEOREM

It states that If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two. Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q, and R are three forces and α, β, γ are the angles as shown in fig. below

Proof

Consider three coplanar forces *P*, *Q*, and *R* acting at a point *O*. Let the opposite angles to three forces be α , β and γ as shown in Fig.

Now let us complete the parallelogram *OACB* with *OA* and *OB* as adjacent sides as shown in the figure. We know that the resultant of two forces *P* and *Q* will be given by the diagonal *OC* both in magnitude and direction of the parallelogram *OACB*.

Since these forces are in equilibrium, therefore the resultant of the forces *P* and *Q* must be in line with *OD* and equal to *R*, but in opposite direction.

From the geometry of the figure, we find that

$BC = P$ and $AC = Q$

$\therefore \angle AOC = (180° - \beta)$

and $\angle ACO = \angle BOC = (180° - \alpha)$

$\angle CAO = 180° - (\angle AOC + \angle ACO)$

$= 180° - [(180° - \beta) + (180° - \alpha)]$

= 180° – 180° + β – 180° + α

= α + β – 180

But α + β + γ = 360°

Subtracting 180° from both sides of the above equation,

(α + β – 180°) + γ = 360° – 180° = 180°

or ∠ CAO = 180° – γ

We know that in triangle AOC

$$\frac{OA}{\sin\angle ACO} = \frac{\mathrm{AC}}{\sin\angle AOC} = \frac{\mathrm{OC}}{\sin\angle CAO}$$

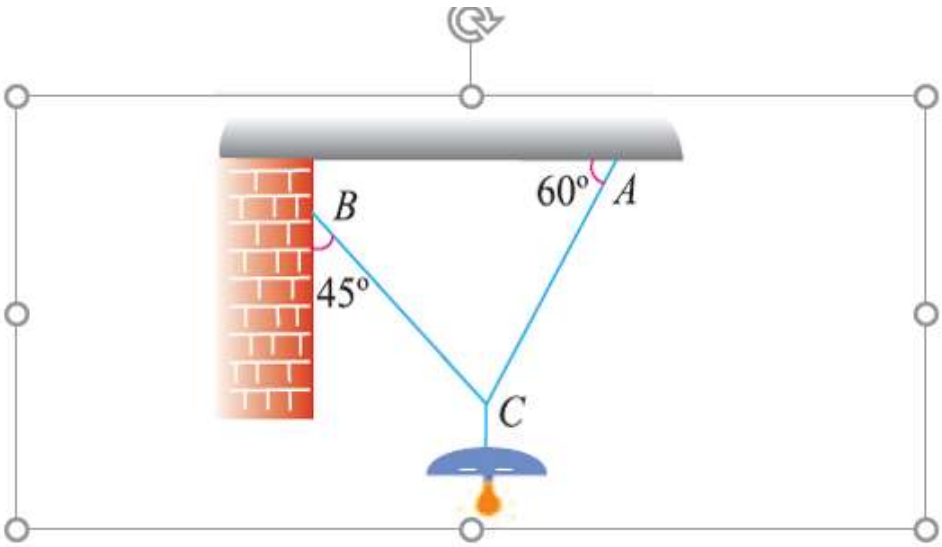
$$\frac{OA}{\sin\angle 180-\alpha} = \frac{\mathrm{AC}}{\sin\angle 180\text{-}\beta} = \frac{\mathrm{OC}}{\sin\angle 180-\gamma}$$

or $\frac{\mathrm{P}}{\sin\alpha} = \frac{\mathrm{Q}}{\sin\beta} = \frac{\mathrm{R}}{\sin\gamma}$ … … … … . .[sin (180° – θ) = sin θ]

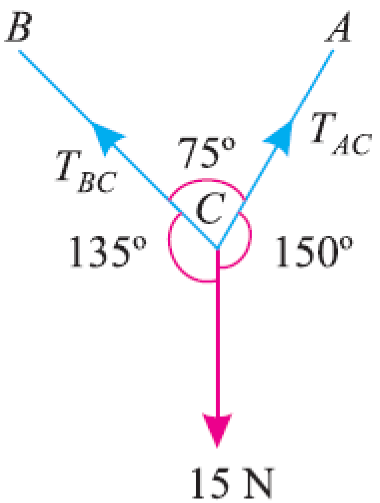
(Proved)

PROBLEM.

An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. Using Lami’s theorem, or otherwise, determine the forces in the strings AC and BC.



Given : Weight at *C* = 15 N
Let *TAC* = Force in the string *AC*, and
TBC = Force in the string *BC*.
The system of forces is shown in Fig. 5.4. From the geometry of the
figure, we find that angle between *TAC* and 15 N is 150° and angle between
TBC and 15 N is 135°.
∠*ACB* = 180° – (45° + 60°) = 75°



Applying Lami's equation at *C*,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

$T_{AC} = \frac{15 \sin 135°}{\sin 75°}$= 10.98 N …………….Ans

STATES OF THE BODY

Consider a body acted upon by a number of coplaner non-concurrent forces. A little consideration will show, that as a result of these forces, the body may have any one of the following states:

1. The body may move in any one direction.

2. The body may rotate about itself without moving.

3. The body may move in any one direction and at the same time it may also rotate about itself.

4. The body may be completely at rest.

CONDITION OF EQUILIBRIUM.

Consider a body acted upon by a number of coplaner non-concurrent forces. A little consideration will show, that as a result of these forces, the body may have any one of the following states:

1. If the body moves in any direction, it means that there is a resultant force acting on it. Alittle consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of allthe forces (ΣH) and vertical component of all the forces (ΣV) must be zero.

Mathematically,

$\Sigma H = 0$ and $\Sigma V = 0$

2. If the body rotates about itself, without moving, it means that there is a single resultantcouple acting on it with no resultant force. A little consideration will show, that if the bodyis to be at rest or in equilibrium, the moment of the couple causing rotation must be zero.

Or in other words, the resultant moment of all the forces (ΣM) must be zero. Mathematically,

$\Sigma M = 0$

3. If the body moves in any direction and at the same time it rotates about itself, if means that

there is a resultant force and also a resultant couple acting on it. A little consideration willshow, that if the body is to be at rest or in equilibrium, the resultant force causing movements and the reusltant moment of the couple causing rotation must be zero. Or in otherwords, horizontal component of all the forces (ΣH), vertical component of all the forces

(ΣV) and resultant moment of all the forces (ΣM) must be zero. Mathematically,

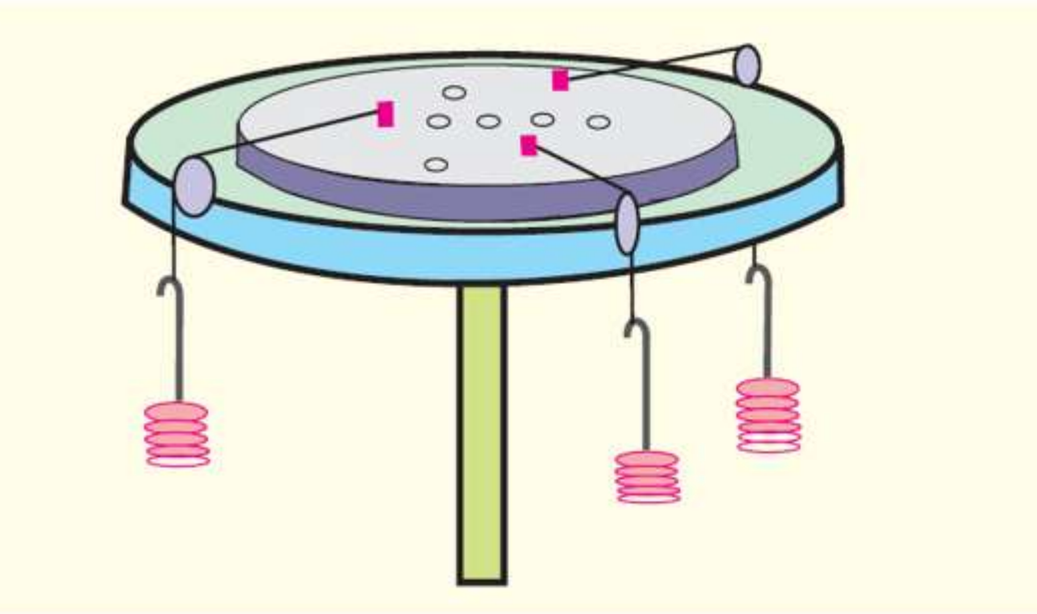
$\Sigma H = 0$,$\Sigma V = 0$ and $\Sigma M = 0$

4. If the body is completely at rest, it necessarily means that there is neither a resultant forcenor a couple acting on it. A little consideration will show, that in this case the following conditions are already satisfied :

$\Sigma H = 0$,$\Sigma V = 0$ and $\Sigma M = 0$

GRAPHICAL METHOD FOR THE EQUILIBRIUM OF COPLANAR FORCES

The table shown below is in equilibrium under the action of three strings.



Sometimes, the analytical method is too tedious and complicated. The equilibrium of such forces may also be studied graphically by drawing the vector diagram. This may also be done by studying the

1. Converse of the Law of Triangle of Forces.

2. Converse of the Law of Polygon of Forces.

CONVERSE OF THE LAW OF TRIANGLE OF FORCES

If three forces acting at a point be represented in magnitude and direction by the three sides of a triangle, taken in order, the forces shall be in equilibrium.

CONVERSE OF THE LAW OF POLYGON OF FORCES

If any number of forces acting at a point be represented in magnitude and direction by the sides of a closed polygon, taken in order, the forces shall be in equilibrium