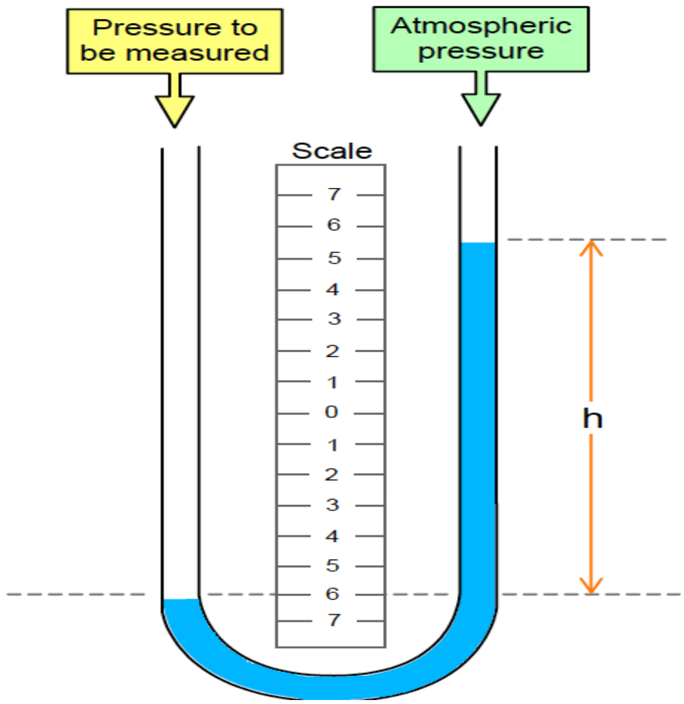
FLUID MECHANICS (Th-3)



Prepared by:
Soumya Dash
Assistant Professor
Mechanical Engineering Department
C V Raman Polytechnic
Bhubaneswar-751019

PROPERTIES OF FLUIDS AND FLUID STATICS

Introduction to Fluid Mechanics

Definition of a fluid

A fluid is defined as a substance that deforms continuously under the action of a shear stress, however small magnitude present. It means that a fluid deforms under very small shear stress, but a solid may not deform under that magnitude of the shear stress.

By contrast a solid deforms when a constant shear stress is applied, but its deformation does not continue with increasing time. In Fig.1.1, deformation pattern of a solid and a fluid under the action of constant shear force is illustrated. We explain in detail here deformation behavior of a solid and a fluid under the action of a shear force.

In Fig.1, a shear force ***F*** is applied to the upper plate to which the solid has been bonded, a shear stress resulted by the force equals to, where *A* is the contact area of the upper plate. We know that in the case of the solid block the deformation is proportional to the shear stress *t* provided the elastic limit of the solid material is not exceeded. When a fluid is placed between the plates, the deformation of the fluid element is illustrated in Fig.1.3. We can observe the fact that the deformation of the fluid element continues to increase as long as the force is applied. The fluid particles in direct contact with the plates move with the same speed of the plates. This can be interpreted that there is no slip at the boundary. This fluid behavior has been verified in numerous experiments with various kinds of fluid and boundary material. **In short, *a fluid continues in motion under the application of a shear stress and can not sustain any shear stress when at rest.***

Fig. 1 Deformation of solid under a constant shear force

Properties of fluid

Some of the basic properties of fluids are discussed below-

Density : As we stated earlier the density of a substance is its mass per unit volume. In fluid mechanic it is expressed in three different ways-

Mass density r is the mass of the fluid per unit volume (given by Eq.1.1)

Unit- kg/m^3

Dimension- ML^{-3}

Typical values: water- *1000* *kg/* m^3

Air- $1.23\ kg/m^3$ at standard pressure and temperature (STP)

Specific weight, w: - As we express a mass *M* has a weight ***W=Mg*** . The specific weight of the fluid can be defined similarly as its weight per unit volume.

$$w = \rho g \qquad \text{L-2.1}$$

Unit: N/m^3

Dimension: $ML^{-2}T^{-2}$

Typical values; water- $9.810 N/m^3$

Air- $12.07 N/m^3$ (STP)

Relative density (Specific gravity), ***S*** :-

Specific gravity is the ratio of fluid density (specific weight) to the fluid density (specific weight) of a standard reference fluid. For liquids water at $4^0 C$ is considered as standard fluid.

$$S_{liquid} = \frac{\rho_{liquid}}{\rho_{water\ at\ 4^0C}} \qquad \text{L-2.2}$$

Similarly for gases air at specific temperature and pressure is considered as a standard reference fluid.

$$S_{gas} = \frac{\rho_{gas}}{\rho_{gas\ at\ STP}} \qquad \text{L-2.3}$$

Units: pure number having no units.

Dimension:- $M^oL^oT^o$

Typical vales : - Mercury- 13.6

Water-1

Specific volume v_s : - Specific volume of a fluid is mean volume per unit mass *i.e.* the reciprocal of mass density.

$$v_s = \frac{1}{\rho} \qquad \text{L-2.4}$$

Units:- m^3/kg

Dimension: $M^{-1}L^3$

Typical values: - Water - $10^{-3} m^3/kg$

Air- $1.23 \times 10^{-3} m^3/kg$

Viscosity

In section L1 definition of a fluid says that under the action of a shear stress a fluid continuously deforms, and the shear strain results with time due to the deformation. Viscosity is a fluid property, which determines the relationship between the fluid strain rate and the applied shear stress. It can be noted that in fluid flows, shear strain rate is considered, not shear strain as commonly used in solid mechanics. Viscosity can be inferred as a quantative measure of a fluid'sresistance to the flow. For example moving an object through air requires very less force compared to water. This means that air has low viscosity than water.

Let us consider a fluid element placed between two infinite plates as shown in fig (Fig-2.1). The upper plate moves at a constant velocity δu under the action of constant shear force δF. The shear stress, *t* is expressed as

$$\tau = \lim_{\delta A \to 0} \frac{\delta F}{\delta A} = \frac{dF}{dA}$$

where, δA is the area of contact of the fluid element with the top plate. Under the action of shear force the fluid element is deformed from position *ABCD* at time *t* to position AB'C'D' at time $t + \delta t$ (fig-L2.1). The shear strain rate is given by

Shear strain rate $= \lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$ L2.6

Where α is the angular deformation

From the geometry of the figure, we can define

For small $\delta \alpha$, $\tan \delta \alpha = \frac{\delta u \ \delta t}{\delta y}$

Therefore,

$$\frac{\delta \alpha}{\delta t} = \frac{\delta u}{\delta y}$$

The limit of both side of the equality gives $\frac{d\alpha}{dt} = \frac{du}{dy}$ L-2.5

The above expression relates shear strain rate to velocity gradient along the *y* -axis.

Newton's Viscosity Law

Sir Isaac Newton conducted many experimental studies on various fluids to determine relationship between shear stress and the shear strain rate. The experimental finding showed that a linear relation between them is applicable for common fluids such as water, oil, and air. The relation is

$$\tau \ \alpha \ \frac{d\alpha}{dt}$$

Substituting the relation gives in equation(L-2.5)

$$\tau \, \alpha \, \frac{du}{dy} \qquad \text{L-2.6}$$

Introducing the constant of proportionality

$$\tau = \mu \frac{du}{dy}$$

where μ is called absolute or dynamic viscosity. Dimensions and units for μ are $ML^{-1}T^{-1}$ and $N-s/m^2$, respectively. [In the absolute metric system basic unit of co-efficient of viscosity is called poise. 1 poise = $N-s/m^2$]

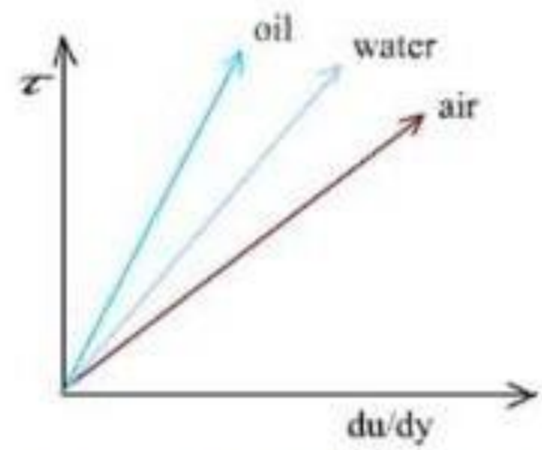


Fig.L-2.2: Relationship between shear stress and velocity gradient of Newtonian fluids

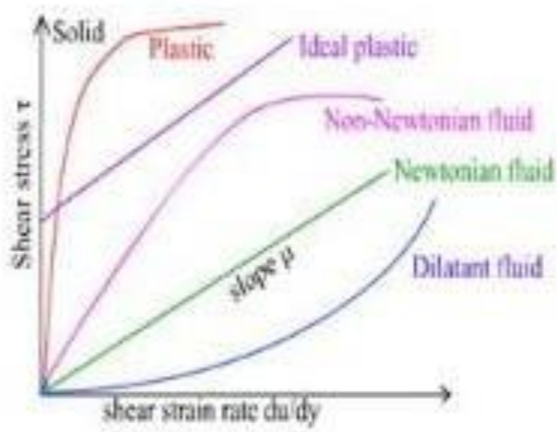


Fig.L-2.3: Relationship between shear stress and shear strain rate of diferent fluids

Typical relationships for common fluids are illustrated in Fig-L2.3.

The fluids that follow the linear relationship given in equation (L-2.7) are called Newtonian fluids.

Kinematic viscosity v

Kinematic viscosity is defined as the ratio of dynamic viscosity to mass density

$$\nu = \frac{\mu}{\rho} \qquad \text{L-2.8}$$

Units: m^2/s

Dimension: L^2T^{-1}

Typical values: water $1.14\times10^{-6} m^2 s^{-1}$ air $1.46\times10^{-5} m^2/s$

Non - Newtonian fluids

Fluids in which shear stress is not linearly related to the rate of shear strain are non– Newtonian fluids. Examples are paints, blot, polymeric solution, etc. Instead of the dynamic viscosity apparent viscosity, μ_{ap} which is the slope of shear stress versus shear strain rate curve, is used for these types of fluid.

Based on the behavior of μ_{ap}, non-Newtonian fluids are broadly classified into the following groups –

a. *Pseudo plastics* (shear thinning fluids): μ_{ap} decreases with increasing shear strain rate. For example polymer solutions, colloidal suspensions, latex paints, pseudo plastic.
b. *Dilatants* (shear thickening fluids) μ_{ap} increases with increasing shear strain rate.

   Examples: Suspension of starch and quick sand (mixture of water and sand).

c. *Plastics* : Fluids that can sustain finite shear stress without any deformation, but once shear stress exceeds the finite stress τ_y, they flow like a fluid. The relation between the shear stress and the resulting shear strain is given by

$$\tau = \tau_y + \mu_{ap}\left(\frac{du}{dy}\right)^n \qquad \text{L-2.9}$$

Fluids with n = 1 are called Bingham plastic. some examples are clay suspensions, tooth paste and fly ash.

d. *Thixotropic* fluid(Fig. L-2.4): μ_{ap} decreases with time under a constant applied shear stress.

Example: Ink, crude oils.

e. *Rheopectic fluid* : μ_{ap} increases with increasing time.

Example: some typical liquid-solid suspensions.

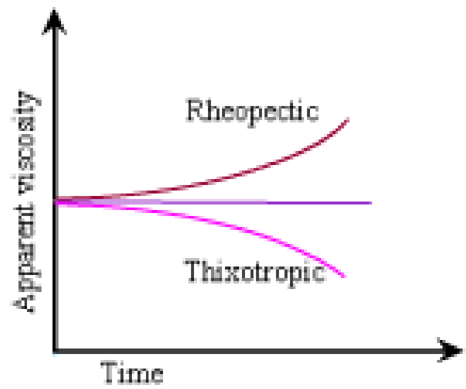
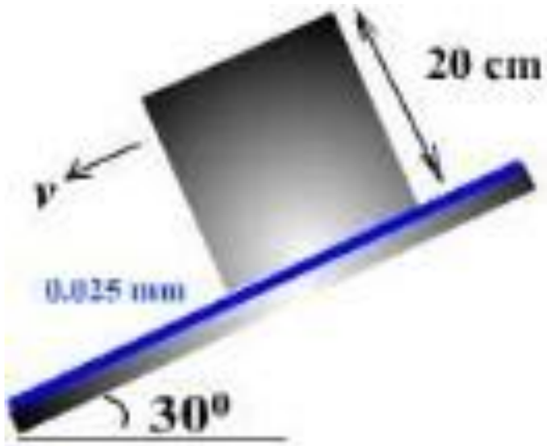


Fig. L-2.4: Thixotropic and Rheopectic fluids

Example

As shown in the figure a cubical block of 20 cm side and of 20 kg weight is allowed to slide down along a plane inclined at 30^0 to the horizontal on which there is a film of oil having viscosity $2.16x10^{-3}$ N-s/m^2 .What will be the terminal velocity of the block if the film thickness is 0.025mm?



Given data : Weight = 20 kg

Block dimension = 20x20x20 cm^3

Driving force along the plane $F = W \sin 30^0 = 98.1N$

Shear force $\tau = F / A = 2452.5N / m^2$

Contact area, $A = 0.2 \times 0.2 m^2$

Also, $\tau = \mu \frac{dv}{dy}$

Answer: 28.38m/s.

Example

If the equation of a velocity profile over a plate is *v = 5y 2 + y* (where *v* is the velocity in m/s) determine the shear stress at *y =0* and at *y =7.5cm* . Given the viscosity of the liquid is 8.35 poise.

Solution

Given Data: Velocity profile $v = 5y^2 + y$

$\mu = 8.35 poise$

$$\text{Velocity gradient, } \frac{dv}{dy} = 10y + 1$$

$$\tau = \mu.\frac{dv}{dy} = \mu.(10y+1)$$

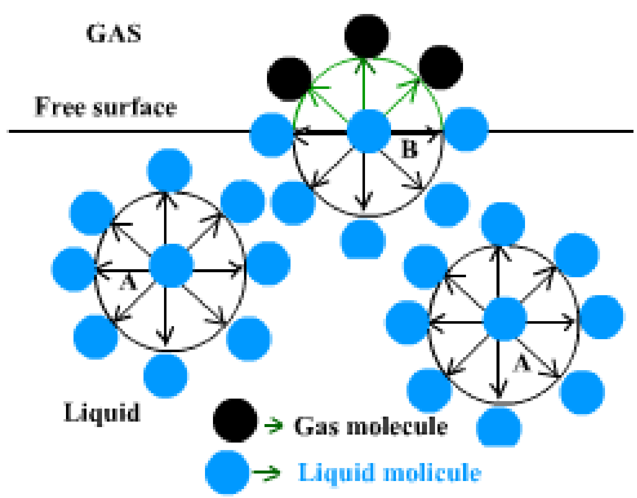
Substituting $y = 0$ *and* $y = 0.075$ on the above equation, we get shear stress at respective depths.

Answer: 0.835 ; $1.46\ N/m^2$

Surface tension and Capillarity

Surface tension

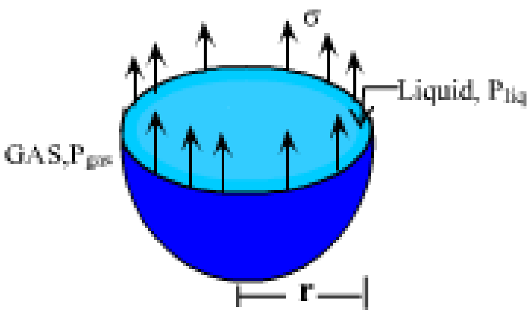
In this section we will discuss about a fluid property which occurs at the interfaces of a liquid and gas or at the interface of two immiscible liquids. As shown in Fig (L - 3.1) the liquid molecules- 'A' is under the action of molecular attraction between like molecules (cohesion). However the molecule ‘B' close to the interface is subject to molecular attractions between both like and unlike molecules (adhesion). As a result the cohesive forces cancel for liquid molecule 'A'. But at the interface of molecule 'B' the cohesive forces exceed the adhesive force of the gas. The corresponding net force acts on the interface; the interface is at a state of tension similar to a stretched elastic membrane. As explained, the corresponding net force is referred to as surface tension, σ. In short it is apparent tensile stresses which acts at the interface of two immiscible fluids.



Dimension: MT^{-2}
Unit: N/m
Typical values: Water $0.074\,N/m$ at 20°C with air.

Note that surface tension decreases with the liquid temperature because intermolecular cohesive forces decreases. At the critical temperature of a fluid surface tension becomes zero; i.e. the boundary between the fluids vanishes.

Pressure difference at the interface



Surface tension on a droplet

In order to study the effect of surface tension on the pressure difference across a curved interface, consider a small spherical droplet of a fluid at rest.

Since the droplet is small the hydrostatic pressure variations become negligible. The droplet is divided into two halves as shown in Fig.L-3.2. Since the droplet is at rest, the sum of the forces acting at the interface in any direction will be zero. Note that the only forces acting at the interface are pressure and surface tension. Equilibrium of forces gives

$$\left(P_{liq} - P_{gas}\right)\pi r^2 = \sigma\left(2\pi r\right) \qquad \text{L - 3.1}$$

Solving for the pressure difference and then denoting $\Delta P = P_{liq} - P_{gas}$ we can rewrite equation (L- 3.1) as

$$\Delta P = \frac{2\sigma}{r}$$

Contact angle and welting

As shown in fig. a liquid contacts a solid surface. The line at which liquid gas and solid meet is called the contact line. At the contact line the net surface tension depending upon all three materials - liquid, gas, and solid is evident in the contact angle, θ_c. A force balance on the contact line yields:

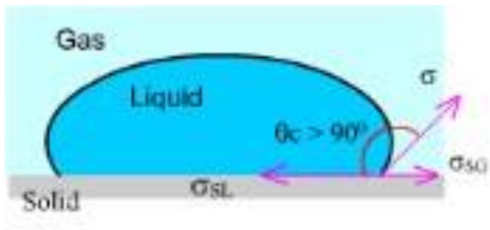


Fig : L-3.3: Contact line for wetting condition

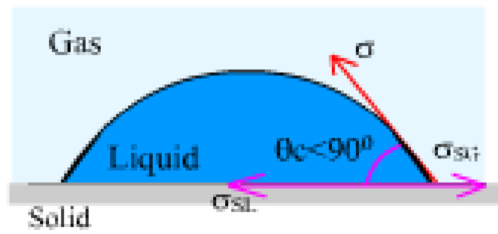


Fig : L-3.4: Contact line for non-wetting condition

$$\sigma_{gas} - \sigma_{solid} = \sigma \cos\theta_c$$

here σ_{gas} is the surface tension of the gas-solid interface, σ_{solid} is the surface tension of solid-liquid interface, and is σ the surface tension of liquid-gas interface.

Typical values:

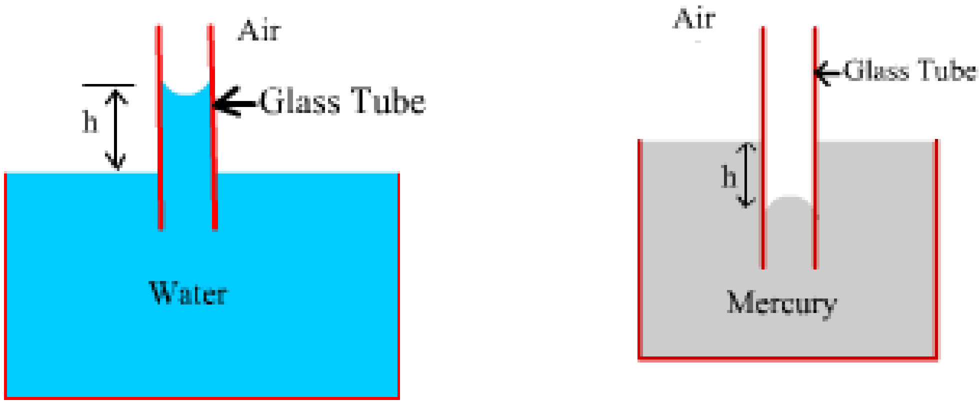
$\theta_C \approx 0°$ for air-water- glass interface

$\theta_C \approx 140°$ for air-mercury–glass interface

If the contact angle $\theta_c < 90°$ the liquid is said to wet the solid. Otherwise, the solid surface is not wetted by the liquid, when $\theta_c > 90°$.

Capillarity

If a thin tube, open at the both ends, is inserted vertically in to a liquid, which wets the tube, the liquid will rise in the tube (fig : L -3.4). If the liquid does not wet the tube it will be depressed below the level of free surface outside. Such a phenomenon of rise or fall of the liquid surface relative to the adjacent level of the fluid is called capillarity. If θ_c is the angle of contact between liquid and solid, *d* is the tube diameter, we can determine the capillary rise or depression, *h* by equating force balance in the z-direction (shown in Fig : L-3.5), taking into account surface tension, gravity and pressure. Since the column of fluid is at rest, the sum of all of forces acting on the fluid column is zero.



The pressure acting on the top curved interface in the tube is atmospheric, the pressure acting on the bottom of the liquid column is at atmospheric pressure because the lines of constant pressure in a liquid at rest are horizontal and the tube is open.

Upward force due to surface tension $= \sigma \cos\theta_c \pi d$

Weight of the liquid column $= \rho g \pi \frac{d^2}{4} h$

Thus equating these two forces we find

$$\sigma \cos\theta_c \pi d = \rho g \pi \frac{d^2}{4} h$$

The expression for *h* becomes

$$h = \frac{4\sigma \cos\theta_c}{\rho g d} \qquad \text{L -3.2}$$

Typical values of capillary rise are

a. Capillary rise is approximately 4.5 mm for water in a glass tube of 5 mm diameter.
b. Capillary depression is approximately - 1.5 mm (depression) for mercury in the same tube.
c. Capillary action causes a serious source of error in reading the levels of the liquid in small pressure measuring tubes. Therefore the diameter of the measuring tubes should be large enough so that errors due to the capillary rise should be very less. Besides this,

capillary action causes the movement of liquids to penetrate cracks even when there is no significant pressure difference acting to move the fluids in to the cracks.

d. In figure (Fig : L - 3.6), a two-dimensional model for the capillary rise of a liquid in a crack width, *b,* is illustrated. The height of the capillary rise can also be computed by equating force balance as explained in the previous section.

Capillary rise, $$h = \frac{2\sigma\cos\theta_c}{b\rho g} \qquad \text{L-3.3}$$

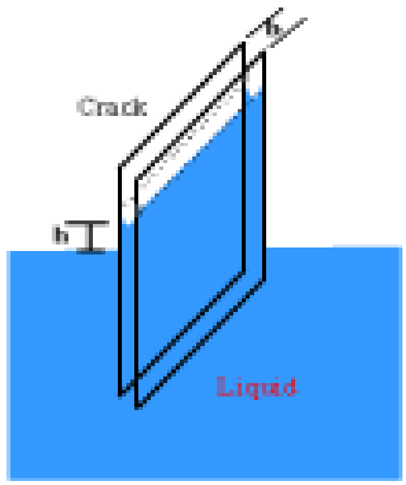


Fig. L-3.6: Capillary rise in a Crack

Vapour Pressure

Since the molecules of a liquid are in constant motion, some of the molecules in the surface layer having sufficient energy will escape from the liquid surface, and then changes from liquid state to gas state. If the space above the liquid is confined and the number of the molecules of the liquid striking the liquid surface and condensing is equal to the number of liquid molecules atany time interval becomes equal, an equilibrium exists. These molecules exerts of partial pressure on the liquid surface known as vapour pressure of the liquid, because degree of molecular activity increases with increasing temperature. The vapour pressure increases with temperature. Boiling occurs when the pressure above a liquid becomes equal to or less then the vapour pressure of the liquid. It means that boiling of water may occur at room temperature if thepressure is reduced sufficiently.

For example water will boil at 60 ° C temperature if the pressure is reduced to 0.2 atm.

Cavitation

In many fluid problems, areas of low pressure can occur locally. If the pressure in such areas is equal to or less then the vapour pressure, the liquid evaporates and forms a cloud of vapour bubbles. This phenomenon is called cavitation. This cloud of vapour bubbles is swept in to an area of high pressure zone by the flowing liquid. Under the high pressure the bubbles collapses.If this phenomenon occurs in contact with a solid surface, the high pressure developed by collapsing bubbles can erode the material from the solid surface and small cavities may be formed on the surface.

The cavitation affects the performance of hydraulic machines such as pumps, turbines and propellers.

Compressibility and the bulk modulus of elasticity

When a fluid is subjected to a pressure increase the volume of the fluid decreases. The relationship between the change of pressure and volume is linear for many fluids. This relationship may be defined by a proportionality constant called bulk modulus.

Consider a fluid occupying a volume *V* in the piston and cylinder arrangement shown in figure. If the pressure on the fluid increase from *p* to $p + \delta p$ due to the piston movement as a result the volume is decreased by ΔV. We can express the bulk modulus of elasticity

$$k = -\frac{\delta p}{\delta v / v} \qquad \text{L - 4.1}$$

The negative sign indicates the volume decreases as pressure increases. As in the limitas $\delta p \rightarrow 0$ then

$$k = -\frac{dp}{dv / v} \qquad \text{L - 4.2}$$

Since $-\frac{dv}{v} = \frac{dp}{p}$ the equation can be rearranged as

$$k = \frac{dp}{d\rho / \rho} \qquad \text{L - 4.3}$$

Dimension :- $ML^{-1}T^{-2}$

Unit :- N / m^2

Typical values:-

Air - 1.03 x 10 5 N/m^2

water $2.05\times10^{9}\ N/m^{2}$ at standard temperature and pressure as compared to that of Mild steel $2.06\times10^{11}\ N/m^{2}$.

The above typical values show that the air is about 20,000 times more compressible than water while water is about 100 times more compressible than mild steel.

Basic Equations

To analysis of any fluid problem, the knowledge of the basic laws governing the fluid flows is required. The basic laws, applicable to any fluid flow, are:

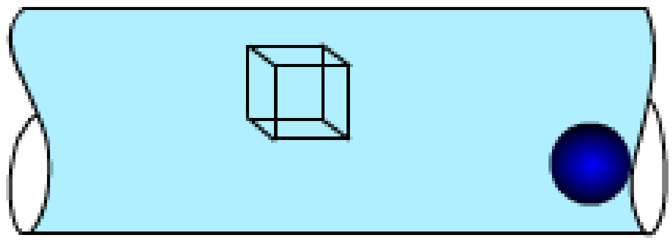
a. Conservation of mass. (Continuity)
b. Linear momentum. (Newton 's second law of motion)
c. Conservation of energy (First law of Thermodynamics)

Besides these governing equations, we need the state relations like $\rho = \rho(P,T)$ and appropriate boundary conditions at solid surface, interfaces, inlets and exits. Note that all basic laws are not always required to any one problem. These basic laws, as similar in solid mechanics and thermodynamics, are to be reformulated in suitable forms so that they can be easily applied to solve wide variety of fluid problems.

System and control volume

A system refers to a fixed, identifiable quantity of mass which is separated from its surrounding by its boundaries. The boundary surface may vary with time however no mass crosses the system boundary. In fluid mechanics an infinitesimal lump of fluid is considered as a system and is referred as a fluid element or a particle. Since a fluid particle has larger dimension than the limiting volume (refer to section fluid as a continuum). The continuum concept for the flow analysis is valid.

control volume is a fixed, identifiable region in space through which fluid flows. The boundary of the control volume is called control surface. The fluid mass in a control volume may vary with time. The shape and size of the control volume may be arbitrary.



System and control volume

When a fluid is at rest, the fluid exerts a force normal to a solid boundary or any imaginary plane drawn through the fluid. Since the force may vary within the region of interest, we conveniently define the force in terms of the pressure, P, of the fluid. The pressure is defined as the *force per unit area.*

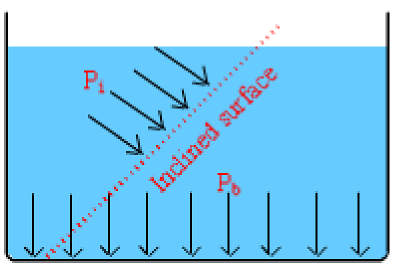


Fig : L - 6.1: Pressure variation at the bottom surface P_b and at the inclined surface P_i

In Fig : L - 6.1 pressure variation of a fluid at different locations is illustrated.

Commonly the pressure changes from point to point. We can define the pressure at a point as

$$P = \lim_{\delta A \to 0} \frac{\delta F}{\delta A} = \frac{dF}{dA} \qquad \text{L - 6.1}$$

where dA is the area on which the force dF acts. It is a scalar field and varies spatially and temporally as given *P = P (x, y, z, t)*

Pascal's Law : Pressure at a point

The Pascal's law states that *the pressure at a point in a fluid at rest is the same in all directions* . Let us prove this law by considering the equilibrium of a small fluid element shown in Fig : L - 6.2

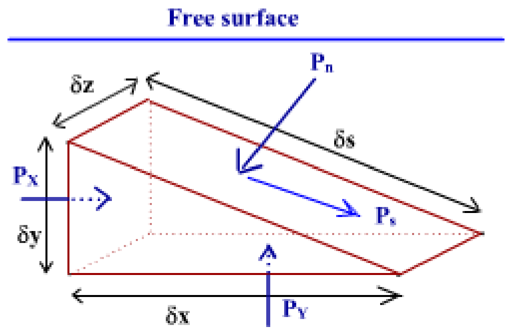


Fig : L -6.2: A fluid element with force components

Since the fluid is at rest, there will be no shearing stress on the faces of the element.

The equilibrium of the fluid element implies that sum of the forces in any direction must be zero. For the x-direction:

Force due to P_x is $P_x \cdot \delta y \cdot \delta z$

Component of force due to P_n

$$= -P_n \cdot \delta n \cdot \delta z \cdot \frac{\delta y}{\delta n}$$
$$= -P_n \cdot \delta y \cdot \delta z$$

Summing the forces we get,

$$P_x \cdot \delta y \cdot \delta z - P_n \cdot \delta y \cdot \delta z = 0$$
$$\text{then } P_x = P_n$$

Similarly in the y-direction, we can equate the forces as given below

Force due to P_y = $P_y \cdot \delta x \cdot \delta z$

Component of force due to P_n

$$= -P_n \cdot \delta n \cdot \delta z \cdot \frac{\delta x}{\delta n}$$
$$= -P_n \cdot \delta x \cdot \delta z$$

$$\text{Weight of the fluid element} = -\text{Specific weight} \times \text{volume of the element}$$
$$= -\rho \cdot g \cdot \frac{1}{2} \cdot \delta x \cdot \delta y \cdot \delta z$$

The negative sign indicates that weight of the fluid element acts in opposite direction of the z-direction.

Summing the forces yields

$$P_y \cdot \delta n \cdot \delta z - P_n \cdot \delta x \cdot \delta z - \frac{1}{2} \cdot \rho \cdot g \cdot \delta x \cdot \delta y \cdot \delta z = 0$$

Since the volume of the fluids $\delta x \cdot \delta y \cdot \delta z$ is very small, the weight of the element is negligible in comparison with other force terms. So the above Equation becomes

$P_y = P_n$

Hence, $P_n = P_x = P_y$

Similar relation can be derived for the z-axis direction.

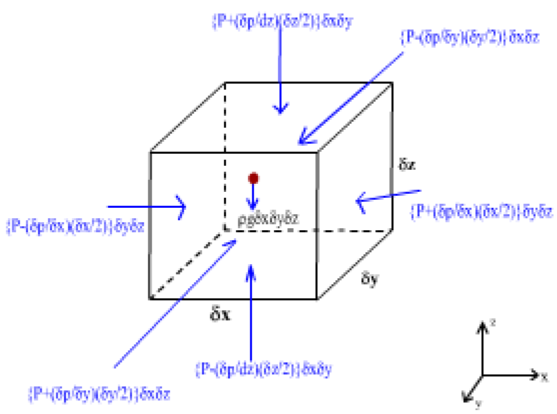
This law is valid for the cases of fluid flow where shear stresses do not exist. The cases are

a. Fluid at rest.
b. No relative motion exists between different fluid layers. For example, fluid at a constant linear acceleration in a container.
c. Ideal fluid flow where viscous force is negligible.

Basic equations of fluid statics

An equation representing pressure field $P = P(x, y, z)$ within fluid at rest is derived in this section. Since the fluid is at rest, we can define the pressure field in terms of space dimensions(x, y and z) only.

Consider a fluid element of rectangular parellopiped shape(Fig : L - 7.1) within a large fluid region which is at rest. The forces acting on the element are body and surface forces.



Body force: The body force due to gravity is

$$d\overline{F}_B = \rho \cdot g \cdot \delta x \cdot \delta y \cdot \delta z \qquad \text{L -7.1}$$

Where $\delta x \cdot \delta y \cdot \delta z$ is the volume of the element.

Surface force: The pressure at the center of the element is assumed to be *P* (x, y, z). Using Taylor series expansion the pressure at point $\left(x, y - \frac{\delta y}{2}, z\right)$ on the surface can be expressed as

$$P\left(x, y - \frac{\delta y}{2}, z\right) = P(x, y, z) + \frac{\delta p}{\delta y}\left(-\frac{\delta y}{2}\right) + \frac{1}{2!}\frac{\partial^2 p}{\partial y^2}\left(-\frac{\delta y}{2}\right)^2 + \dots\dots\dots \qquad \text{L -7.2}$$

When $\delta y \rightarrow 0$, only the first two terms become significant. The above equation becomes

$$P\left(x, y - \frac{\delta y}{2}, z\right) = P(x, y, z) + \frac{\delta p}{\delta y}\left(-\frac{\delta y}{2}\right) \qquad \text{L - 7.3}$$

Similarly, pressures at the center of all the faces can be derived in terms of *P (x, y, z)* and its gradient.

Note that surface areas of the faces are very small. The center pressure of the face represents the average pressure on that face.
The surface force acting on the element in the y-direction is

$$dF_y = \left\{P + \frac{\delta P}{\delta y}\left\{-\frac{\delta y}{2}\right\}\right\}\delta x \cdot \delta y - \left\{P + \frac{\delta P}{\delta y}\left\{\frac{\delta y}{2}\right\}\right\}\delta x \cdot \delta z$$

$$= -\frac{\delta P}{\delta y}\cdot \delta x \cdot \delta y \cdot \delta z \qquad \text{L -7.4}$$

Similarly the surface forces on the other two directions (x and z) will be

$$dF_x = -\frac{\delta P}{\delta x}\cdot \delta x \cdot \delta y \cdot \delta z$$

$$dF_z = -\frac{\delta P}{\delta z}\cdot \delta x \cdot \delta y \cdot \delta z$$

The surface force which is the vectorical sum of the force scalar components

$$dF_s = -\left(\frac{\delta p}{\delta x}\hat{i} + \frac{\delta p}{\delta y}\hat{j} + \frac{\delta p}{\delta z}\hat{k}\right)(\delta x \cdot \delta y \cdot \delta z)$$

$$= -\nabla p \cdot \delta x \cdot \delta y \cdot \delta z \qquad \text{L - 7.5}$$

The total force acting on the fluid is

$$d\overline{F} = d\overline{F_S} + d\overline{F_B}$$

$$= \left(-\nabla p + \rho\vec{g}\right)(\delta x \cdot \delta y \cdot \delta z) \qquad \text{L - 7.6}$$

The total force per unit volume is

$$\frac{dF}{\delta x \cdot \delta y \cdot \delta z} = -\nabla p + \rho\vec{g}$$

For a static fluid, *dF=0* .

Then, $\left(-\nabla p + \rho\vec{g}\right) = 0$ L -7.7

$$\begin{bmatrix}\text{Net pressure force}\\ \text{per unit volume}\\ \text{at a point}\end{bmatrix}+\begin{bmatrix}\text{Body force}\\ \text{per unit volume}\\ \text{at a point}\end{bmatrix}=0$$

If acceleration due to gravity $\vec{g}$ is expressed as $\vec{g}=g_x\hat{i}+g_y\hat{j}+g_z\hat{k}$, the components of Eq(L- 7.8) in the x, y and z directions are

$$-\frac{\delta p}{\delta z}+\rho g_z=0$$

$$-\frac{\delta p}{\delta x}+\rho g_x=0$$

$$-\frac{\delta p}{\delta y}+\rho g_y=0$$

The above equations are the basic equation for a fluid at rest.

Simplifications of the Basic Equations

If the gravity $\vec{g}$ is aligned with one of the co-ordinate axis, for example *z*- axis, then

$$g_x=0$$

$$g_y=0$$

$$g_z=-g$$

The component equations are reduced to

$$\frac{\delta p}{\delta x}=0$$

$$\frac{\delta p}{\delta y}=0 \qquad \text{L} \qquad -7.9$$

$$\frac{\delta p}{\delta z}=-\rho g$$

Under this assumption, the pressure *P* depends on z only. Therefore, total derivative can be used instead of the partial derivative.

$$\frac{dp}{dz} = -\rho g$$

This simplification is valid under the following restrictions

a. Static fluid

b. Gravity is the only body force.

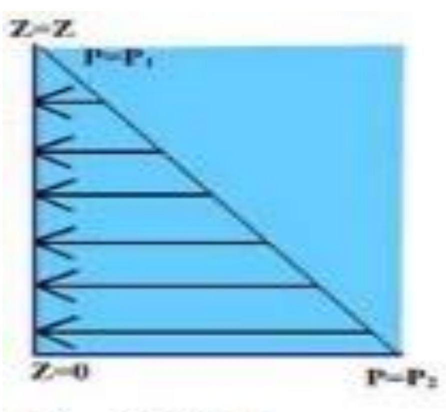
c. The z-axis is vertical and upward.

Pressure variations in an incompressible fluid at rest

In some fluid problems, fluids may be considered homogenous and incompressible *i.e* . density is constant ρ. Integrating the equation (L -7.10) with condition given in figure (Fig : L - 7.2), we have

$$\int_{P_1}^{P_2} dp = \int_{0}^{z} -\rho g \cdot dz$$

$$P_2 - P_1 = -\rho g z$$



Pressure variation in an incompressible fluid

This indicates that the pressure increases linearly from the free surface in an incompressible static fluid as illustrated by the linear distribution in the above figure.

Scales of pressure measurement

Fluid pressures can be measured with reference to any arbitrary datum. The common datum are

1. Absolute zero pressure.
2. Local atmospheric pressure

When absolute zero (complete vacuum) is used as a datum, the pressure difference is called an absolute pressure, P_{abs} .

When the pressure difference is measured either above or below local atmospheric pressure, P_{local} , as a datum, it is called the gauge pressure. Local atmospheric pressure can be measured by mercury barometer.

At sea level, under normal conditions, the atmospheric pressure is approximately 101.043 kPa.

As illustrated in figure(Fig : L -7.2),

When $P_{abs} < P_{local}$

$$P_{gauge} = P_{local} - P_{abs} \qquad \text{L - 7.12}$$

Note that if the absolute pressure is below the local pressure then the pressure difference is known as vacuum suction pressure.

Example 1 :

Convert a pressure head of 10 m of water column to kerosene of specific gravity 0.8 and carbon-tetra-chloride of specific gravity of 1.62.

Solution

Given data:

Height of water column, $h_1 = 10$ m

Specific gravity of water $s_1 = 1.0$

Specific gravity of kerosene $s_2 = 0.8$

Specific gravity of carbon-tetra-chloride, $s_3 = 1.62$

For the equivalent water head

Weight of the water column = Weight of the kerosene column.

□ g □ g

So, $\square$ g $h_1 s_1 =$ $h_2 s_2 =$ $h_3 s_3$

Answer:- 12.5 m and 6.17 m.

Example 2

Determine (a) the gauge pressure and (b) The absolute pressure of water at a depth of 9 m from the surface.

Solution

Given data:

Depth of water = 9 m

the density of water = 998.2 kg/m^3

And acceleration due to gravity = 9.81 m/s^2

Thus the pressure at that depth due to the overlying water is P = r gh = 88.131 kN/m^2

Case a) as already discussed, gauge pressure is the pressure above the normal atmospheric pressure.

Thus, the gauge pressure at that depth = 88.131 kN/m^2

Case b) The standard atmospheric pressure is 101.213 kN/m^2

Thus, the absolute pressure as P_{abs} = 88.131+101.213 = 189.344 kN/m^2

Answer: 88.131 kN/m^2 ; 101.213 kN/m^2

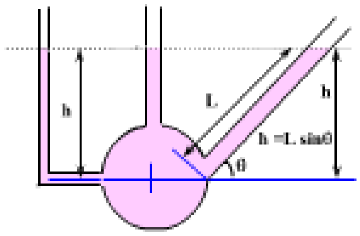
Manometers: Pressure Measuring Devices

Manometers are simple devices that employ liquid columns for measuring pressure difference between two points.

In Figure(L 8.1), some of the commonly used manometers are shown.

In all the cases, a tube is attached to a point where the pressure difference is to be measured and its other end left open to the atmosphere. If the pressure at the point P is higher than the local atmospheric pressure the liquid will rise in the tube. Since the column of the liquid in the tube is at rest, the liquid pressure P must be balanced by the hydrostatic pressure due to the column of liquid and the superimposed atmospheric pressure, P_{atm} .

$$P = \rho g h + P_{atm}$$



Simple Manometer

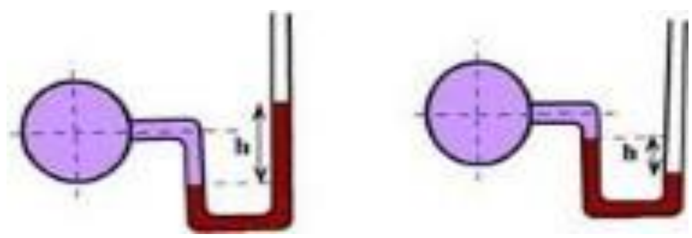
This simplest form of manometer is called a *Piezometer* . It may be inadequate if the pressure difference is either very small or large.

U - Tube Manometer

In (Fig : L -8.2), a manometer with two vertical limbs forms a U-shaped measuring tube. A liquid of different density is used as a manometric fluid. We may recall the Pascal's lawwhich states that the pressure on a horizontal plane in a continuous fluid at rest is the same. Applying this equality of pressure at points B and C on the plane gives

$$P + \rho g h = P_{atm} + \rho_1 g h_1$$

$$P - P_{atm} = \rho_1 g h_1 - \rho g h$$



U-tube Manometer

Inclined Manometer

A manometer with an inclined tube arrangement helps to amplify the pressure reading, especially in low press range. A typical arrangement of the same is shown in Fig. L-8.3.

The pressure at O is

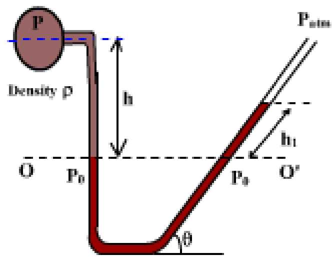
$$P_0 = P + \rho g h$$

The pressure at O is

$$P_{0'} = P_{atm} + \rho_1 g h_1 \sin\theta$$

Equating the pressures, we have

$$P_0 - P_{atm} = \rho_1 g h_1 \sin\theta - \rho g h$$



Inclined Manometer

At the same pressure difference, Equations (1) and (2) indicate that inclined tube manometer amplifies the length of measurement by $\frac{1}{\sin\theta}$, which is the primary advantage of such type of manometer.

Differential Manometers

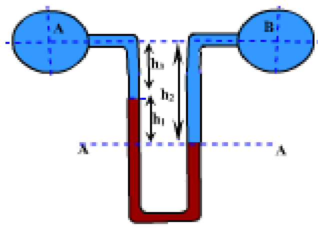
Differential Manometers measure difference of pressure between two points in a fluid system and cannot measure the actual pressures at any point in the system.

Some of the common types of differential manometers are

a. Upright U-Tube manometer
b. Inverted U-Tube manometer
c. Inclined Differential manometer
d. Micro manometer

Upright U-Tube manometer:

As shown in Fig. : L-8.4, an upright U-tube manometer is connected between points A and B. The difference of pressure between the points may be calculated by balancing pressure in a horizontal plane, the lowest interface A-A is used for this case.



Upright U-tube Manometer

$$P_A + \rho_1 g h_1 + \rho_3 g h_3 = P_B + \rho_2 g h_2$$

or

$$P_A - P_B = \rho_2 g h_2 - \rho_1 g h_1 - \rho_3 g h_3$$
$$= (\rho_2 h_2 - \rho_1 h_1 - \rho_3 h_3) g$$

Inverted U-Tube manometer:

The manometer fluid used in this type of manometer is lighter than the working fluids. Thus the height difference in two limbs is enhanced. This is therefore suitable for measurement of small pressure difference in liquids. For the configurations given in Fig. L-8.1.

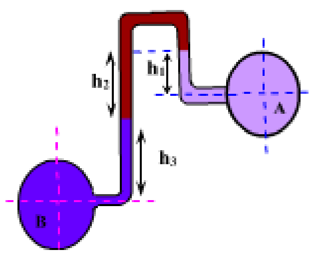


Fig. L-8.5 Inverted Manometer

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_3 g h_3$$ Or $$P_A - P_B = (\rho_1 h_1 - \rho_2 h_2 - \rho_3 h_3) g$$

If the two points A and B are at the same level and the same fluid is used, then $P_1 = P_2 = P$ and $h_2 + h_3 = h_1$.

The above equation becomes $P_A - P_B = (\rho_1 - \rho_3) h_3 g$

Inclined Differential Manometer

In this type of manometer a narrow tube is connected to a reservoir at an inclination. The cross section of the reservoir is larger than that of the tube. Fluctuations in the reservoir may be ignored. As shown in Fig.L-8.6, the initial liquid level in both the reservoir and the tube is at o-o.The application of the differential pressure liquid level of the reservoir drops by , whereas □h is the rising level in the tube. Therefore

$$P_A = P_B + (h + \Delta h)\rho g$$

Since the volume of liquid displaced in the reservoir equals to the volume of liquid in the tube, we can define

$$A \cdot \Delta h = a \cdot L$$

Where 'A' and 'a' are the cross sectional areas of the reservoir and the tube respectively. Then the equation becomes $P_A - P_B = (h + \frac{a}{A} L)\rho g$

In practice, the reservoir area is much larger than that of the tube; the ratio $\frac{a}{A}$ is negligible and the above equation is reduced to $P_A - P_B = \rho g L \sin\theta$; $h = L \sin\theta$

Micro manometer:

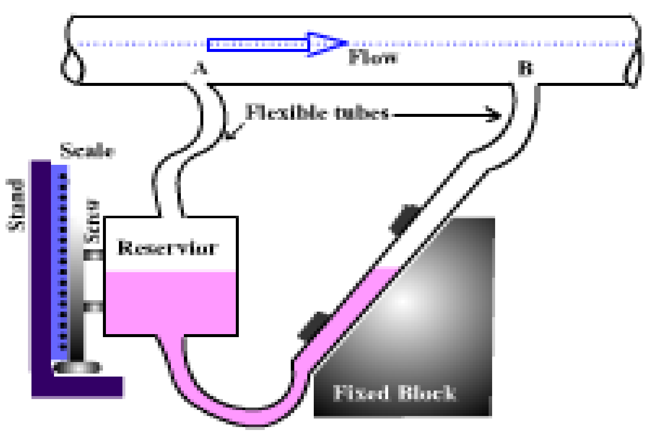


Fig. L-8.6: Micro manometer

A typical micro-manometer tube arrangement as shown in fig has a reservoir which can be moved up and down by means of micrometer screw. A flexible tube is connected between point A and the reservoir. Another flexible tube connecting point B and the other end of the reservoiris placed on an inclined surface. A reference mark 'R' is provided on the inclined portion of the tube. Before application of the pressure, the level of the reservoir is moved so as to coincide this level with the reference mark. When a pressure difference is applied, the liquid levels will be disturbed. The micrometer arrangement is then adjusted to vary the reservoir level so as to coincide with the reference. The extent of movement of the micrometer screw gives the pressure difference between the two points A and B.

Example 1:

Two pipes on the same elevation convey water and oil of specific gravity 0.88 respectively. They are connected by a U-tube manometer with the manometric liquid having a specific gravity of 1.25. If the manometric liquid in the limb connecting the water pipe is 2 m higher than the other find the pressure difference in two pipes.

Solution :

Given data:

Height difference = 2 m

Specific gravity of oil s = 0.88

Specific gravity of manometric liquid s = 1.25

Equating pressure head at section (A-A)

$$P_A + 2\times 1.25\rho_W\ g + (h-2)\rho_W\ g = P_B + h\times 0.88\rho_W\ g$$

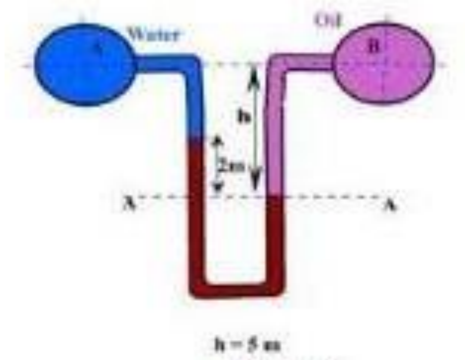


Fig. Ex1

Substituing h = 5 m and density of water 998.2 kg/m^3 we have $P_A - P_B = 10791$

Example 2:

A two liquid double column enlarged-ends manometer is used to measure pressure difference between two points. The basins are partially filled with liquid of specific gravity 0.75 and the lower portion of U-tube is filled with mercury of specific gravity 13.6. The diameter of the basin is 20 times higher than that of the U-tube. Find the pressure difference if the U-tube reading is 25mm and the liquid in the pipe has a specific weight of 0.475 N/m^3.

Solution:

Given data: U-tube reading 25 mm
Specific gravity of liquid in the basin 0.75
Specific gravity of Mercury in the U-tube13.6
As the volume displaced is constant we have,

$$Y = 25\frac{a}{A} = 25 \times \frac{1}{20^2}$$

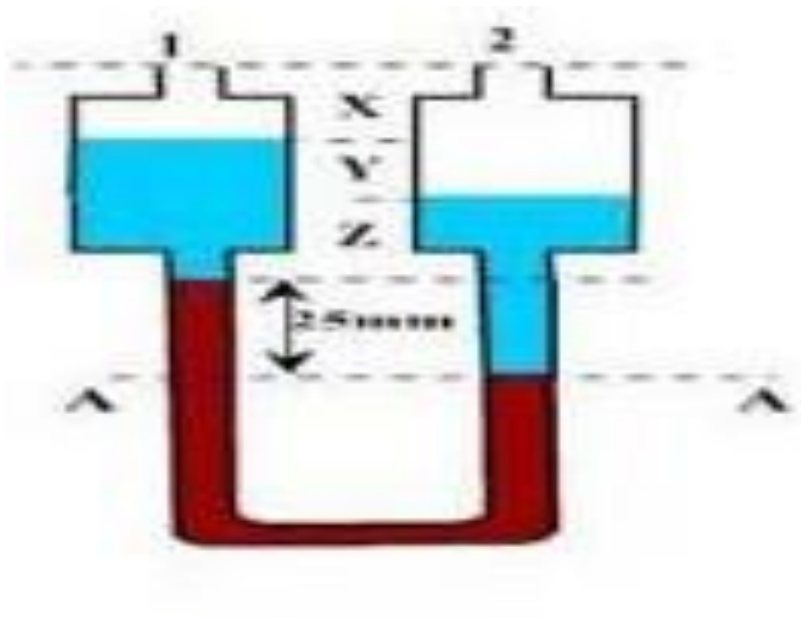


Fig. Ex 2

Equating pressure head at (A--A)

$$P_1 + X\frac{0.475}{1000}\rho_w g + (Z+Y)\rho_w g \times 0.75 + 25\times 13.6\rho_w g$$

$$= P_2 + (X+Y)\frac{0.475}{1000}\rho_w g + (Z+25)\times 0.75\rho_w g$$

Put the value of Y while X and Z cancel out.
Answer: 31.51 kPa

Example 3:

As shown in figure water flows through pipe A and B. The pressure difference of these two points is to be measured by multiple tube manometers. Oil with specific gravity 0.88 is in the upper portion of inverted U-tube and mercury in the bottom of both bends. Determine the pressure difference.

Solution

Given data: Specific gravity of the oil in the inverted tube 0.88
Specific gravity of Mercury in the U-tube13.6

Calculate the Pressure difference between each two point as follow
$P_2 - P_1 = h \rho g = h S \rho_w g$

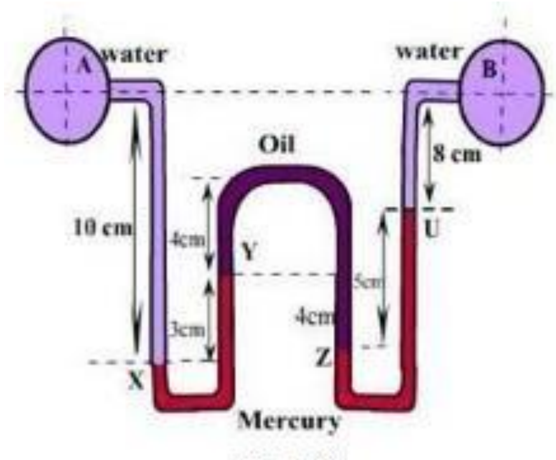


Fig. Ex3

Start from one and i.e. P_A or P_B

$$\text{Now, } P_X = P_A + 10\rho_w g$$
$$\text{Similarly, } P_Y = P_X - 3\times 13.6\rho_w g$$
$$P_Z = P_Y + 4\times 0.88\rho_w g$$
$$P_U = P_Z - 5\times 13.6\rho_w g$$
$$P_B = P_U - 8\rho_w g$$

Rearranging and summing all these equations we have $P_A - P_B = 103.28\ \rho_w\ g$

Example 4:

A manometer connected to a pipe indicates a negative gauge pressure of 70 mm of mercury . What is the pressure in the pipe in N/m^2 ?

Solution :

Given data:

Manometer pressure- 70 mm of mercury (Negative gauge pressure)

A pressure of 70 mm of Mercury, P = r gh = 9.322 kN/m^2

Also we know the gauge pressure is the pressure above the atmosphere.

Thus a negative gauge pressure of 70 mm of mercury indicates the absolute pressure of

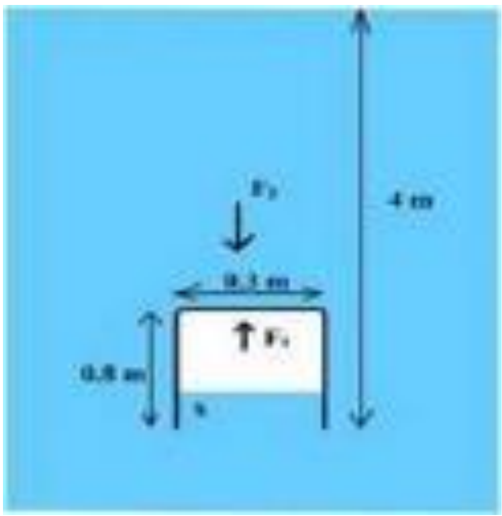
$P_{abs} = 101.213 + (-9.322) = 91.819 \text{ kN/m}^2$

Answer: 91.819 kN/m^2

Example 5:

An empty cylindrical bucket with negligible thickness and weight is forced with its open end first into water until its lower edge is 4m below the water level. If the diameter and length of the bucket are 0.3m and 0.8m respectively and the trapped water remains at constant temperature. What would be the force required to hold the bucket in that position atmospheric pressure being 1.03 N/cm^2

Solution :



Let, the water rises a height *x* in the bucket

By applying the Boyle's Law at constant temperature we have

$$p_1 \times (0.3)^2 \times \frac{\pi}{4} \times (0.8 - x) = p_{atm} \times (0.3)^2 \frac{\pi}{4} \times 0.8$$

$$p_1 = p_{atm} + (4 - x) \times 9810$$

Also, Downward pressure ion the bucket,

Solve for, p_1 and x.

$$p_1 = 6.46 \times 10^4 \, N/m^2$$

$$x = 0.610 m$$

Total upward force exerted by the trapped water $F_1 = p_1 \times \frac{\pi}{4} \times 0.3^2 = 4.57 \times 10^3 N/m^2$

Downward force due to the overlying water and the Atmospheric Pressure

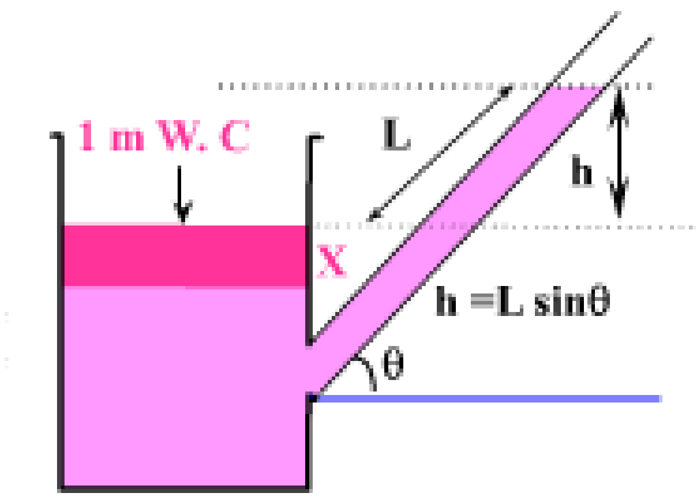
$$F_2 = [1.03\times10^4 + 9810\times(4-0.8)]\times\frac{\pi}{4}\times0.3^2$$

Answer: 1.62KN

Example 6:

A pipe connected with a tank (diameter 3 m) has an inclination of with the horizontal and the diameter of the pipe is 20 cm. Determine the angle ? which will give a deflection of 5 m in the pipe for a gauge pressure of 1 m water in the tank. Liquid in the tank has a specific gravity of 0.88.

Solution :



Given data:

Diameter of tank = 3 m
Diameter of tube = 20 cm
Deflection in the pipe, L = 5 m
From the figure shown
h = L sin
If X m fall of liquid in the tank rises L m in the tube. (Note that the volume displaced is the same in the tank is equal to the volume displaced in the pipe)

$$x\pi\frac{3^2}{4} = L\pi\frac{0.2^2}{4}$$

$$\text{or} \quad x = \frac{0.04\text{L}}{9}.$$

Difference of head = x + h = L sin q + 0.04 L/9

And $$\left\{Lsin\theta + \frac{0.04L}{9}\right\} \times 0.88 = 1$$

Substitute L = 5m in the above equation.

Answer: □ = 12.87 0

Introduction

Designing of any hydraulic structure, which retains a significant amount of liquid, needs to calculate the total force caused by the retaining liquid on the surface of the structure. Other critical components of the force such as the direction and the line of action need to be addressed. In this module the resultant force acting on a submerged surface is derived.

Hydrostatic force on a plane submerged surface

Shown in Fig.L-9.1 is a plane surface of arbitrary shape fully submerged in a uniform liquid. Since there can be no shear force in a static liquid, the hydrostatic force must act normal to the surface.

Consider an element of area $d\overline{A}$ on the upper surface. The pressure force acting on the element is

$$d\overline{F} = -Pd\overline{A}$$

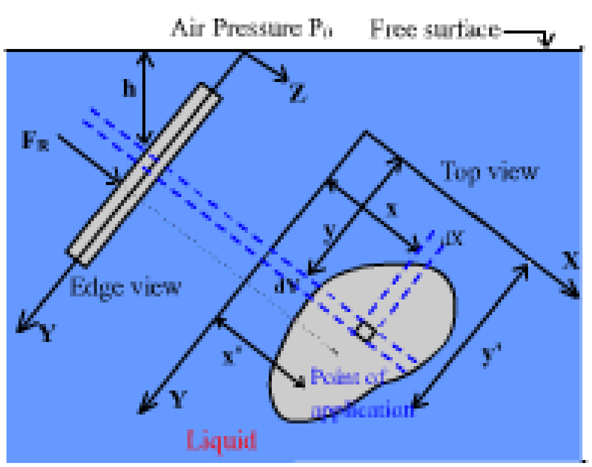


Fig : L - 9.1: Hydrostatic force and center of pressure on an inclined surface

Note that the direction of $d\overline{A}$ is normal to the surface area and the negative sign shows that the pressure force $d\overline{F}$ acts against the surface. The total hydrostatic force on the surface can be computed by integrating the infinitesimal forces over the entire surface area.

$$F = \int_A -P \cdot d\overline{A}$$

If h is the depth of the element, from the horizontal free surface as given in Equation (L2.9) becomes

$$\frac{dP}{dh} = \rho g = w \qquad \text{L-9.1}$$

If the fluid density is ρ constant and P 0 is the atmospheric pressure at the free surface,integration of the above equation can be carried out to determine the pressure at the element as given below

$$P = P_0 + \int_0^h w dh$$
$$= P_0 + wh \qquad \text{L-9.2}$$

Total hydrostatic force acting on the surface is

$$F = \int_A P \cdot d\overline{A}$$
$$= \int_A (P_0 + wh) \cdot d\overline{A}$$
$$= \int_A (P_0 + w \cdot y \sin\theta) \cdot d\overline{A}$$
$$= P_0 A + w \cdot \sin\theta \int_A y \cdot d\overline{A} \qquad \text{L-9.3}$$

The integral $\int_A y \cdot d\overline{A}$ is the first moment of the surface area about the x-axis.

If y_c is the y coordinate of the centroid of the area, we can express

$$\int_A y \cdot d\overline{A} = y_c \cdot A \qquad \text{L-9.4}$$

in which A is the total area of the submerged plane.

Thus

$$F = P_0 \cdot A + w \sin\theta \cdot (y_c A)$$
$$= P_c A \qquad \text{L-9.5}$$

This equation says that the total hydrostatic force on a submerged plane surface equals to the pressure at the centroid of the area times the submerged area of the surface and acts normal to it

Centre of Pressure (CP)

The point of action of total hydrostatic force on the submerged surface is called the Centre of Pressure (CP). To find the co-ordinates of CP, we know that the moment of the resultant force about any axis must be equal to the moment of distributed force about the same axis. Referring to Fig. L-9.2, we can equate the moments about the x-axis.

$$Y_{cp} F = \int_A y \cdot P \cdot dA \qquad \text{L-9.6}$$

Neglecting the atmospheric pressure ($P_0 = 0$) and substituting $F = w \sin\theta \cdot y_c A$, *P=wh* and $h = y \sin\theta$,

We get $Y_{cp} \cdot w \sin\theta \cdot y_c A = w \sin\theta \int_A y^2 \cdot dA$

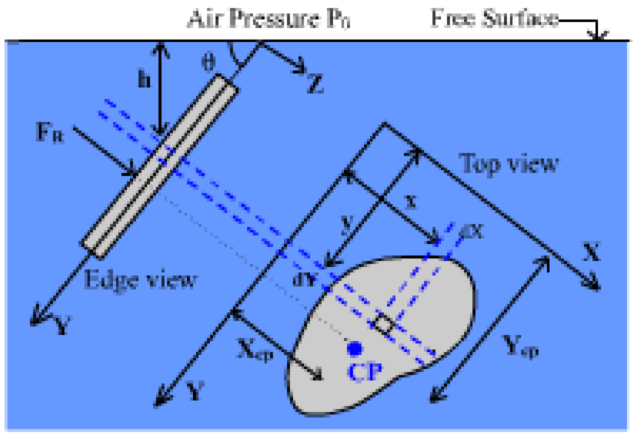


Fig. L-9.2 : Centre of pressure

We get $Y_{cp} \cdot w\sin\theta \cdot y_c A = w\sin\theta \int_A y^2 \cdot dA$

$$Y_{cp} = \frac{\int_A y^2 \cdot dA}{y_c A}$$

$$= \frac{\int_A y^2 \cdot dA}{\int y \cdot dA}$$

$$= \frac{\text{second moment of area about '0'}}{\text{first moment of area about '0'}}$$

From parallel-axis theorem

$$I_{xx} = I_{xc} + A \cdot y_c^{\,2}$$

Where I_{xc} is the second moment of the area about the centroidal axis.

$$Y_{cp} = \frac{I_{xc} + A \cdot y_c^{\,2}}{A \cdot y_c}$$

$$= \frac{I_{xc}}{A \cdot y_c} + y_c \qquad \text{L-9.8}$$

This equation indicates that the centre of the pressure is always below the centroid of the submerged plane. Similarly, the derivation of x_{cp} can be carried out

Hydrostatic force on a Curved Submerged surface

On a curved submerged surface as shown in Fig. L-9.3, the direction of the hydrostatic pressure being normal to the surface varies from point to point. Consider an elementary area $d\overline{A}$ in the curved submerged surface in a fluid at rest. The pressure force acting on the element is

$$d\vec{F} = Pd\vec{A}$$

The total hydrostatic force can be computed as

$$\vec{F} = \int_A -Pd\vec{A}$$

Note that since the direction of the pressure varies along the curved surface, we cannot integrate the above integral as it was carried out in the previous section. The force vector $\overline{F}$ is expressed in terms of its scalar components as

$$\overline{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

in which $F_x, F_y \text{ and } F_z$ represent the scalar components of F in the x , y and z directions respectively.

For computing the component of the force in the x-direction, the dot product of the force and the unit vector ($\hat{i}$ i) gives

$$F_x = \int d\overline{F}\cdot\hat{i}$$

$$= \int_A -PdA\cdot\hat{i}$$

$$= -\int_A PdA_x$$

Where dA_x is the area projection of the curved element on a plane perpendicular to the x-axis. This integral means that each component of the force on a curved surface is equal to the force on the plane area formed by projection of the curved surface into a plane normal to the component. The magnitude of the force component in the vertical direction (z direction)

$$F_z = \int_{A_z} PdA_z$$

Since $P = P_0 + wh$ and neglecting P_0, we can write

$$F_z = \int_{A_z} wh\cdot dA_z$$

$$= \int wd\nu$$

in which is the weight of liquid above the element surface. This integral shows that the z-component of the force (vertical component) equals to the weight of liquid between the submerged surface and the free surface. The line of action of the component passes through the centre of gravity of the volume of liquid between the free surface and the submerged surface

Example 1 :

A vertical gate of 5 m height and 3 m wide closes a tunnel running full with water. The pressure at the bottom of the gate is 195 kN/m ^{2} . Determine the total pressure on the gate and position of the centre of the pressure.

Solution

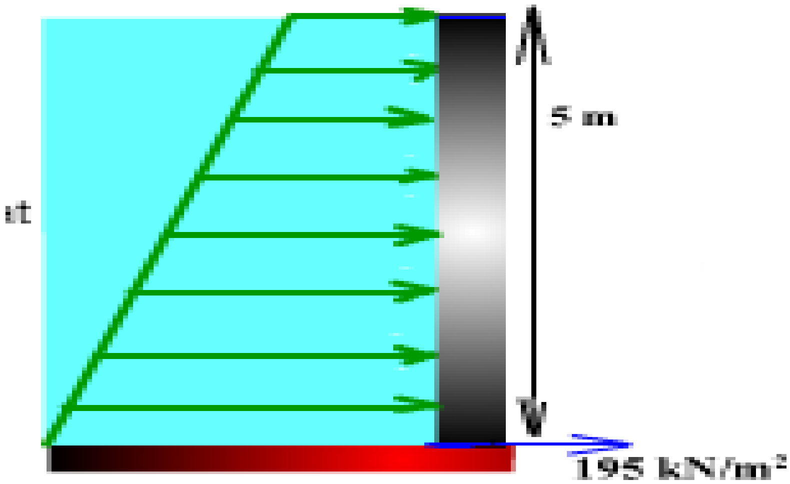


Fig. Ex1

Given data: Area of the gate = 5x3 = 15 m^2

The equivalent height of water which gives a pressure intensity of 195 kN/m^2 at the bottom.

h = P/w =19.87m.

Total force $F = wA\bar{x}$.

And $\bar{x} = 19.87 - 2.5 = 17.37m$

Centre of Pressure $\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$ $[I_G = bd^3/12]$

Answer: 2.56MN and 17.49 m

Example 2 :

A vertical rectangular gate of 4m x 2m is hinged at a point 0.25 m below the centre of gravity of the gate. If the total depth of water is 7 m what horizontal force must be applied at the bottom to keep the gate closed?

Solution

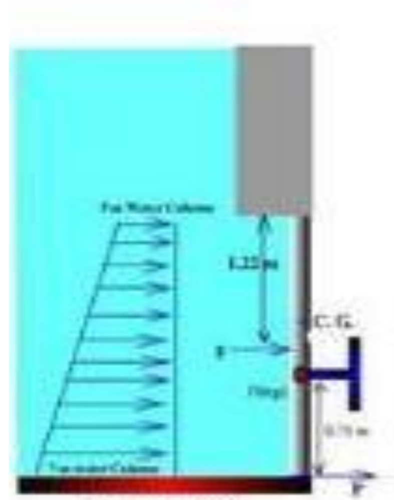


Fig. Ex2

Given data: Area of the gate = 4x2 = 8 m ^{2}

Depth of the water = 7 m

Hydrostatic force on the gate

$$F = wA\bar{x} \qquad \bar{x} = 5+1 = 6 \text{ m}$$

$$= 4.7 \times 10^5 N$$

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}} = 6.22m$$

Taking moments about the hinge we get, $F \times 0.03 = P \times 0.75$

Answer: 18.8 kN.

Buoyancy

Introduction

In our common experience we know that wooden objects float on water, but a small needle of iron sinks into water. This means that a fluid exerts an upward force on a body which is immersed fully or partially in it. The upward force that tends to lift the body is called the buoyant force, F_b.

The buoyant force acting on floating and submerged objects can be estimated by employing hydrostatic principle.

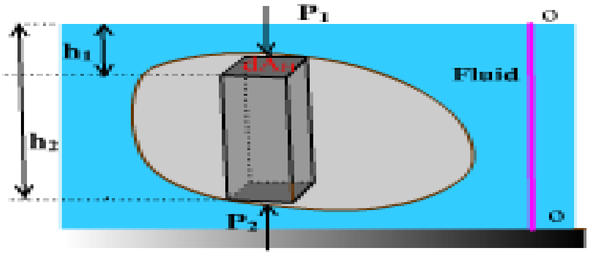


Fig L 10.1 : Buoyant force

With reference to figure(L- 10.1), consider a fluid element of area dA_H . The net upward force acting on the fluid element is

$$dF_B = (P_2 - P_1)dA_H$$
$$= w(h_2 - h_1)dA_H$$

The total upward buoyant force becomes

$$F_B = \int w(h_2 - h_1)dA_H = w(\text{volume of the body})$$ L-10.2

This result shows that the buoyant force acting on the object is equal to the weight of the fluid it displaces.

Center of Buoyancy

The line of action of the buoyant force on the object is called the center of buoyancy. To find the centre of buoyancy, moments about an axis OO can be taken and equated to the moment of the resultant forces. The equation gives the distance to the centeroid to the object volume.

The centeroid of the displaced volume of fluid is the centre of buoyancy, which, is applicable for both submerged and floating objects. This principle is known as the Archimedes principle which states:

"A body immersed in a fluid experiences a vertical buoyant force which is equal to the weight of the fluid displaced by the body and the buoyant force acts upward through the centroid of the displaced volume".

Buoyant force in a layered fluid

As shown in figure (L-10.2) an object floats at an interface between two immiscible fluids of density ρ_1 and ρ_2 .

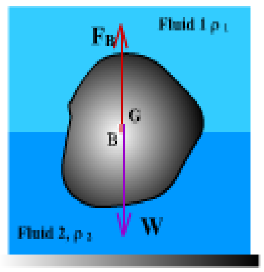


Fig. L-10.2: Buoyant force in a layered fluid

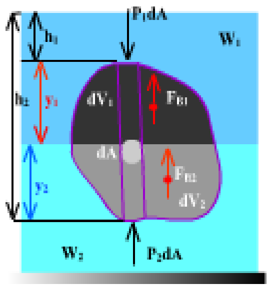


Fig. L-10.3: Element with hydrostatics forces

Considering the element shown in Figure L-10.3, the buoyant force F_B is

$$F_B = \int dF_B = \int \rho_1 g dV_1 + \int \rho_2 g dV_2$$
$$= \sum_1^n \rho_i g \left(\text{displaced volume}\right)_i \qquad \text{L-10.3}$$

where dV_1 and dV_2 are the volumes of fluid element submerged in fluid 1 and 2 respectively. The centre of buoyancy can be estimated by summing moments of the buoyant forces in each fluid volume displaced.

Buoyant force on a floating body

When a body is partially submerged in a liquid, with the remainder in contact with air (as shown in figure), the buoyant force of the body can also be computed using equation (L-10.3). Since the specific weight of the air (11.8 N/m^3) is negligible as compared with the specific weight of the liquid (for example specific weight of water is 9800 kN/m^3),we can neglect the weight of displaced air. Hence, equation (L-10.3) becomes

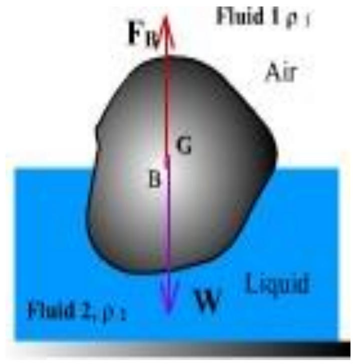


Fig. L-10.4: Partially submerged body

$F_B = \rho g$ (Displaced volume of the submerged liquid)

= The weight of the liquid displaced by the body.

The buoyant force acts at the centre of the buoyancy which coincides with the centeroid of the volume of liquid displaced.

Example 1:

A large iceberg floating in sea water is of cubical shape and its specific gravity is 0.9 If 20 cm proportion of the iceberg is above the sea surface, determine the volume of the iceberg if specific gravity of sea water is 1.025.

Solution:

Let the side of the cubical iceberg be *h*.

Total volume of the iceberg = h^3

volume of the submerged portion is = (h -20) x h^2

Now,
For flotation, weight of the iceberg = weight of the displaced water

$$(h-20)\times h^2\times 1.025\times w = h^3\times 0.9\times w$$
$$or,\ h = 164\ cm$$

The side of the iceberg is 164 cm.

Thus the volume of the iceberg is 4.41m^3

Answer: 4.41m^3

Stability

Introduction

Floating or submerged bodies such as boats, ships etc. are sometime acted upon by certain external forces. Some of the common external forces are wind and wave action, pressure due to river current, pressure due to maneuvering a floating object in a curved path, etc. These external forces cause a small displacement to the body which may overturn it. If a floating or submerged body, under action of small displacement due to any external force, is overturn and then capsized, the body is said to be in unstable. Otherwise, after imposing such a displacement the body restores its original position and this body is said to be in stable equilibrium. Therefore, in the design of the floating/submerged bodies the stability analysis is one of major criteria.

Stability of a Submerged body

Consider a body fully submerged in a fluid in the case shown in figure (Fig. L-11.1) of which the center of gravity (CG) of the body is below the centre of buoyancy. When a small angular displacement is applied a moment will generate and restore the body to its original position; the body is stable.

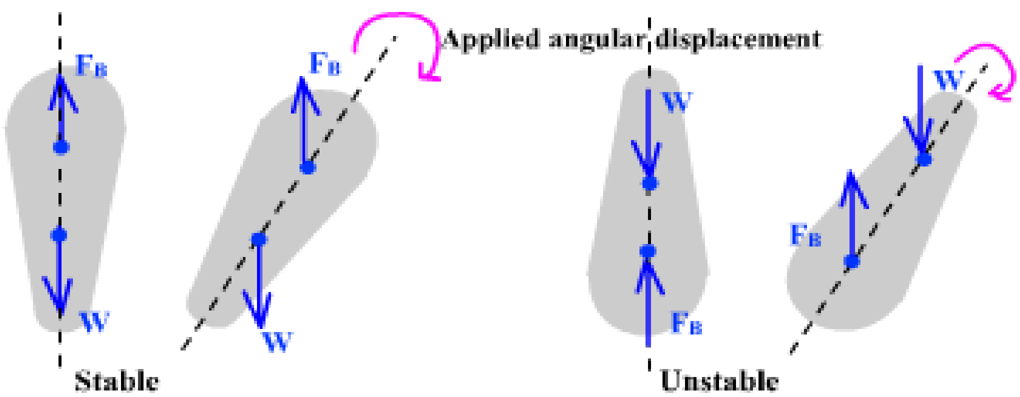


Fig. L-11.1.1 Fig. L-11.1.2

However if the CG is above the centre of buoyancy an overturning moment rotates the body away from its original position and thus the body is unstable (see Fig L-11.2). Note that as the body is fully submerged, the shape of the displaced fluid remains the same when the body is tilted. Therefore the centre of buoyancy in a submerged body remains unchanged.

Stability of a floating body

A body floating in equilibrium ($F_B = W$) is displaced through an angular displacement θ. The weight of the fluid W continues to act through G. But the shape of immersed volume of liquid changes and the centre of buoyancy relative to body moves from B to B 1 . Since the buoyant force F_B and the weight *W* are not in the same straight line, a turning movement proportional to ' $W x \theta$ ' is produced.

In figure (Fig. L-11.2) the moment is a restoring moment and makes the body stable. In figure (Fig. L-11.2) an overturning moment is produced. The point ' *M* ' at which the line of action of the new buoyant force intersects the original vertical through the *CG* of the body, is called the metacentre. The restoring moment

$$= W.x = W.\overline{GM}.\theta$$

Provided θ is small; $\sin\theta = \theta$ (in radians).

The distance *GM* is called the metacentric height. We can observe in figure that

Stable equilibrium: when *M* lies above *G* , a restoring moment is produced. Metacentric height *GM* is positive.

Unstable equilibrium: When *M* lies below *G* an overturning moment is produced and the metacentric height *GM* is negative.

Natural equilibrium: If *M* coincides with *G* neither restoring nor overturning moment is produced and *GM* is zero.

Determination of Meta-centric Height

Experimental method

The metacentric height of a floating body can be determined in an experimental set up with a movable load arrangement. Because of the movement of the load, the floating object is tilted with angle θ for its new equilibrium position. The measurement of θ is used to compute the metacentric height by equating the overturning moment and restoring moment at the new tilted position.

The overturning moment due to the movement of load *P* for a known distance, *x*, is $= P.x$

The restoring moment is $= W.\overline{GM}.\theta$

For equilibrium in the tilted position, the restoring moment must equal to the overturning moment. Equating the same yields

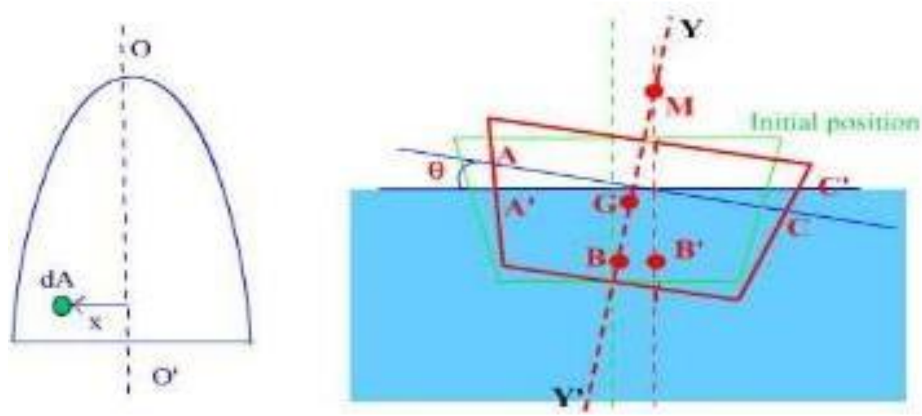
$$P.x = W.\overline{GM}.\theta$$

The metacentric height becomes

$$\overline{GM} = \frac{P.x}{W.\theta}$$

And the true metacentric height is the value of $\overline{GM}$ as $\theta \to 0$. This may be determined by plotting a graph between the calculated value of $\overline{GM}$ for various θ values and the angle θ.

Theoretical method:



For a floating object of known shape such as a ship or boat determination of meta-centric height can be calculated as follows.

The initial equilibrium position of the object has its centre of Buoyancy, B, and the original water line is AC. When the object is tilted through a small angle θ the center of buoyancy will move to new position B'. As a result, there will be change in the shape of displaced fluid. In the new position $A'C'$ is the waterline. The small wedge OCC' is submerged and the wedge OAA'

is uncovered. Since the vertical equilibrium is not disturbed, the total weight of fluid displaced remains unchanged.

Weight of wedge OAA' = Weight of wedge OCC'.

In the waterline plan a small area, *da* at a distance *x* from the axis of rotation *OO* uncover the volume of the fluid is equal to $DD'xda = x\theta da$

Integrating over the whole wedge and multiplying by the specific weight *w* of the liquid,

Weight of wedge $0AA' = \int_{OAA'} w\theta xda$

Similarly,

Weight of wedge $OCC' = \int_{OCC'} w\theta xda$

Equating Equations ( ) and (),

$$W\theta \int_{OAA'} xda = W\theta \int_{OCC} xda$$

$$\int xda = 0$$

in which, this integral represents the first moment of the area of the waterline plane about *OO* , therefore the axis *OO* must pass through the centeroid of the waterline plane.

Computation of the Meta-centric Height

Refer to Figure(), the distance $\overline{BM}$ is

$$BM = BB'/\theta$$

The distance BB' is calculated by taking moment about the centroidal axis YY'.

$$BB'wv_{A'ECCO} = \int_{AA'ECO} xwdv + \int_{OCC'} xwdv - \int_{OAA'} xwdv$$

The integral $\int_{AA'ECO} xwdv$ equals to zero, because YY' axis symmetrically divides the submerged portion $AA'ECO$.

At a distance x , $dv = Lx\tan\theta.dx$

Substituting it into the above equation gives

$$BB'V_{AECCO} = 0 + \int_{OCC} xLx\tan\theta dx - \int_{OAA'} xL(-x\tan\theta)dx$$
$$= \tan\theta \int_{waterline} x^2 dA_{waterline}$$
$$= \tan\theta \, I_o$$

Where I_0 is the second moment of area of water line plane about OO' . Thus,

Distance

$$\overline{BM} = BB'/\theta$$
$$= \frac{I_o \tan\theta}{\theta.V_{AECCO}}$$
$$= \frac{I_o}{V_{AECCO}}$$

Since,

$$\overline{BM} = \overline{GM} + \overline{BG}$$
$$\overline{GM} = \frac{I_o}{V_{submerged}} - \overline{BG}$$

Example:

A large iceberge, floating in seawater, is of cubical shape and its average specific gravity is 0.9. If a 20-cm -high proportion of the iceberg is above the surface of the water, determine the volume of the iceberg if the specific gravity of the seawater is 1.025.

Solution:

Let the side of the cubical iceberg is *h*.

Then volume of the submerged portion is = (h -20) x h^2

Total volume of the iceberg = h^3

Now,

For flotation, weight of the iceberg = weight of the displaced water

$$(h - 20) \times h^2 \times 1.025 = h^3 \times 0.9$$
$$or,\ h = 164$$

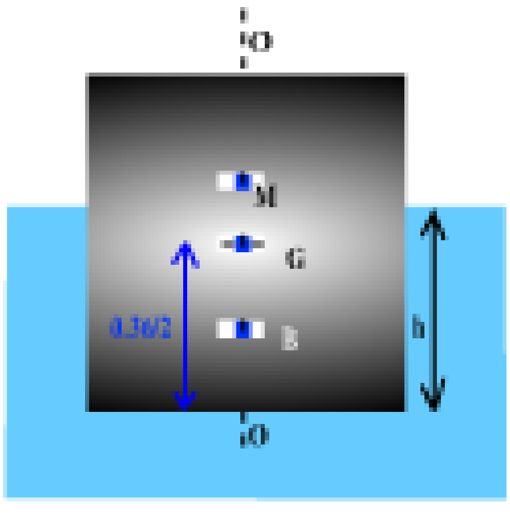
So, the side of the iceberg is 164 cm.

Thus the volume of the iceberg is $4.41m^3$

Example

A log of wood of 1296 cm 2 cross section (square) with specific gravity 0.8 floats in water. Now if one of its edges is depressed to cause the log roll, find the period of roll.

Solution



Let, *h* be the depth of immersion and *L* be the length (perpendicular to the page)

Since the section is square its dimension should be 0.36 m x 0.36 m
For flotation
Weight of water displaced = Weight of the log

$$L \times 0.1296 \times 0.8 = h \times 0.36 \times L$$

Then, *h* = 0.288 m.

$$\overline{BG} = \frac{0.36}{2} - \frac{h}{2} = 0.036$$

$$\overline{BM} = \frac{I_0}{V_{submerged}} = \frac{\frac{1}{12} \times L \times 0.36^3}{0.36 \times 0.288 \times L} = 0.0375$$

$$\overline{GM} = \left(\overline{BM} - \overline{BG}\right) = 0.0015\ m$$

Time period, $T = \frac{2}{\pi}\sqrt{\frac{K_{G^2}}{GM}}$. and we have, $K_{G^2} = \frac{0.36^2}{12}$

Answer: 5.38 second

Example

To find the metacentre of a ship of 10,000 tonnes a weight of 55 tonnes is placed at a distance of 6 m from the longitudinal centre plane to cause a heel through an angle of 3^{0}. What is the metacentre height? Hence find the angle of heel and its direction when the ship is moving ahead and 2.8 MW is being transmitted by a single propeller shaft at the rate of 90 rpm.

Solution

Given data: Weight of the ship, W = 10 7 kg

Angle of heel ? = 3^{0}

Distance of the weight X = 6 m

Weight placed *w* = 5.5 x 10 4

Meta-centric height

$$h = \frac{w.X}{W\ \tan\theta}$$
$$= 0.629\ \text{m}.$$

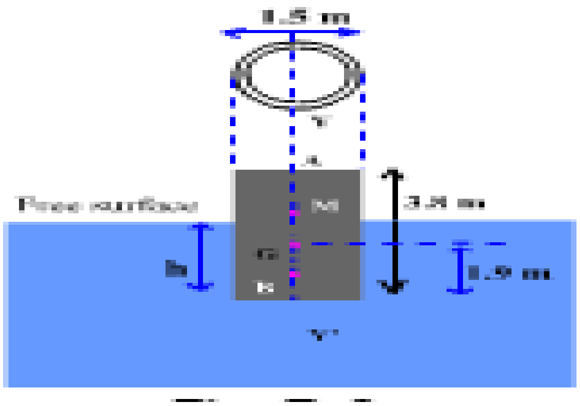
Torque transmitted - $T = P/\omega = 2.97 \times 10^5 N - m.$

$$\omega.h.\tan\theta' = T.$$

Answer:- 0.629 m and 0.27^{0}.

Example

A hollow cylinder closed in both end, of outside diameter 1.5 m and length of 3.8 m and specific weight 75 kN per cubic meter floats just in stable equilibrium condition. Find the thickness of the cylinder if the sea water has a specific weight of 10 kN per cubic meter.



Solution

Given data : Outside diameter 1.5 m
Length L = 3.8 m
Specific weight 75 kN/m ^{3}
Let the thickness t and immersion depth *h* .
For flotation
Weight of water displaced = weight of the cylinder

$$\frac{\pi}{4}(1.5^2\times h)\times 10=\left[\pi\{1.5\times t\}\,3.8+2\times\frac{\pi}{4}\times 1.5^2\times t\right]\times 75$$

Assuming the thickness is very small compared to the diameter

h = 91 t

$$\overline{BM}=\frac{I_0}{V_{submerged}}=\frac{1.545\times10^{-3}}{t} \qquad \text{as we have } I_0=\frac{\pi}{64}\,1.5^4$$

$$\overline{BG}=\left[\frac{L}{2}-h\right]=[\frac{3.8}{2}-\frac{91}{2}t]$$

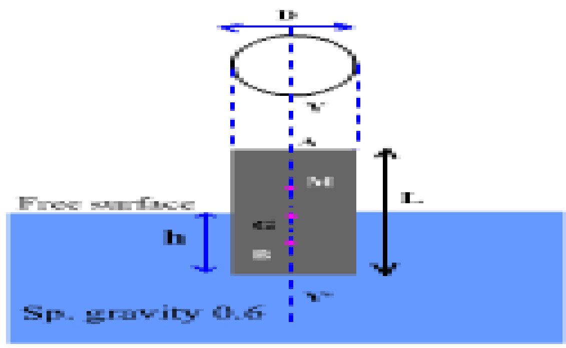
For the cylinder to be in equilibrium $\overline{BM}=\overline{BG}$

Solving for *t* we have *t* = 0.0409 *or* 0.000829m

Answer:- t = 0.83 mm

Example

A wooden cylinder of length L and diameter D is to be floated in stable equilibrium on a liquid keeping its axis vertical. What should be the relation between L and D if the specific gravity of liquid and that of the wood are 0.6 and 0.8 respectively?



Solution

Given data: Specific
gravity of liquid = 0.6
Specific gravity of liquid
= 0.8
If the depth of immersion is h
Weight of water displaced = weight of the cylinder

$$\frac{\pi}{4}D^2L\times0.6=\frac{\pi}{4}D^2h\times0.8$$

The depth of immersion $h=\frac{3}{4}L$.