LEARNING MATERIAL

OF

CONTROL SYSTEM ENGINEERING (6TH SEM)

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BHUBANESWAR

System:- The physical arrangement of components in a specific manner inorder to perform a specific function is called system. eg:- Fan, AC, traffic lights, satellite, missiles. etc.

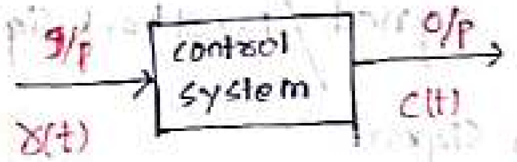
Control system:- If the o/p of the system can be varied by changing i/p of the system then it is called controlled system. eg:- Fan, AC, traffic lights.

control systems are classified into two types:-

1. openloop control system
2. closed loop control system.

1. OpenLoop control system:-

If the output of the system is not taken into consideration for changing the i/p (indirectly o/p). then it is called openloop control system.



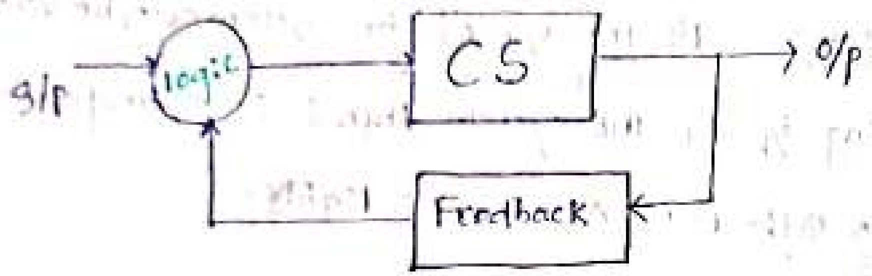
$r(t) \rightarrow$ i/p or excitation

$c(t) \rightarrow$ o/p or Response.

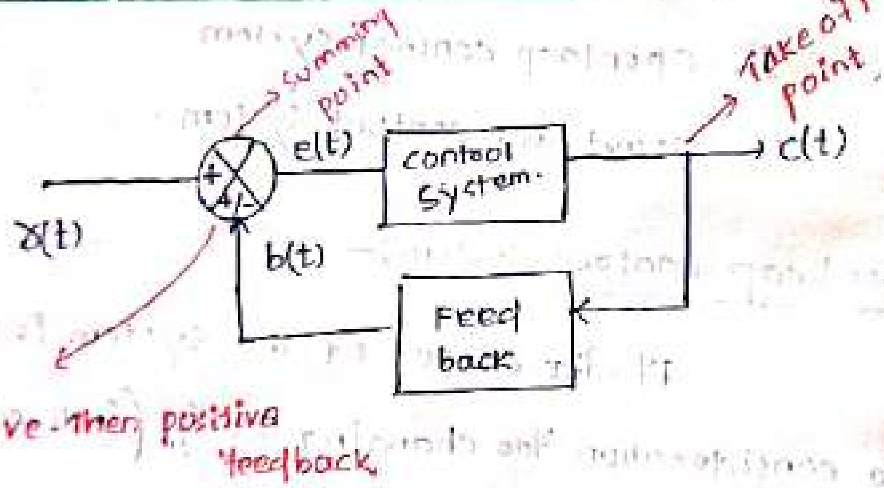
e.g:- Fan, trafficlights without Sensors etc.

2. Closed loop control system :-

The o/p of the system is taken into consideration for changing inputs (indirectly o/p). Then it is called closed loop C.S.



General Representation of closed loop :-



If +ve - then positive feedback

'-' → then negative feedback

b(t) → Base signal / Feedback signal

e(t) → Error signal

e.g:- AC, traffic light with sensors, Missiles, lunching of satellites.

Differences betⁿ openloop & close loop:-

Open Loop	Closed Loop
1. Easy to design.	1. complex to design.
2. cost is less.	2. cost is more
3. Less Accurate.	3. More Accurate.
4. openloop system is more stable.	4. Closed loop system is less stable compared to open loop.
5. Bandwidth is less.	5. Band width is more.
6. openloop gain is more.	6. Gain is less.
7. Easily affected by noise	7. Noise can be elliminated or reduced.

OP-MT-17

Transfer function:-

- The mathematical function which transforms input of the system to o/p is called transfer function.

- Mathematically it is defined as the ratio of laplace transform of o/p to Laplace transform of i/p under zero initial conditions.

$$\text{Transfer function} = \left.\frac{LT(o/p)}{LT(i/p)}\right|_{\text{initial conditions}=0.}$$

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Limitations of Transfer function:-

1. It is applicable to linear systems only.

2. It is defined under zero initial conditions.

3. It is applicable single i/p & single o/p systems only.

4. It gives information about the complete system but not the behaviour of individual component present inside the system.

GATE:-

NOTE:-

⇒ Transfer function of the system depends on System components & their arrangement in the System.

⇒ o/p of the system depends on transfer function & i/p to the system.

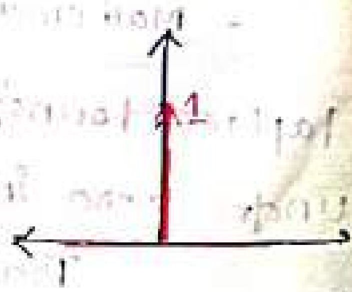
$$LT[o/p] = T.F \; L.T[i/p]$$

⇒ Transfer function in time domain is Impulse Response of the system.

TEST SIGNALS:-

Impulse - $\delta(t) = 1, \; t = 0$

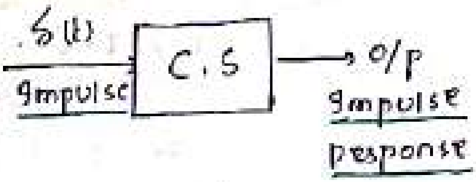
$= 0, \; t \neq 0$



$L.T[\delta(t)] = 1$

$$L.T[O/P] = T.F \times L.T[I/P]$$
$$= T.F \times L.T[\delta(t)]$$
$$= T.F \times 1$$
$$= T.F$$

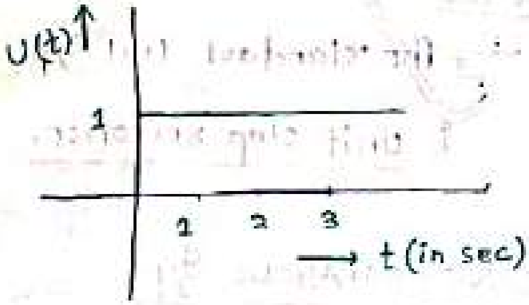
$L.T$ [Impulse Response] = T.F



- Transfer function is also called as impulse Response of the system [time domain]. Applying Impulse input is not possible practically, it is theoritically i/p.

Unit- step function :-

$$U(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



$$\frac{d\,u(t)}{dt} = \frac{\text{change in } u(t)}{\text{change in time}} = \delta(t).$$

$$T.F = L.T[\text{Impulse Response}]$$

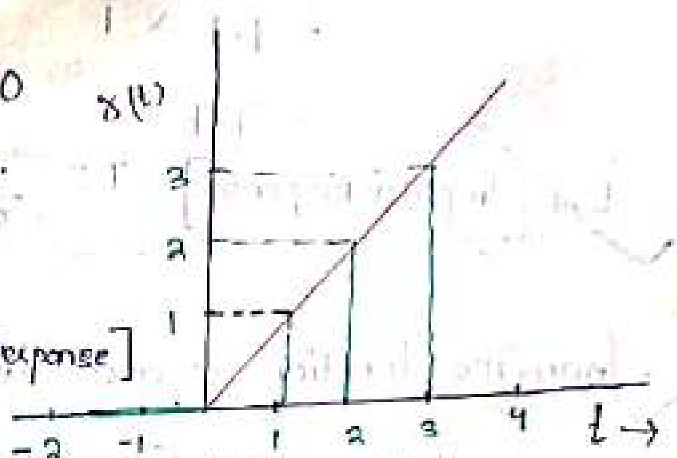
$$= L.T\left[\frac{d}{dt}(\text{step response})\right]$$

- step i/p can be applied practically to electronic/ electrical systems.

Unit Ramp Function:-

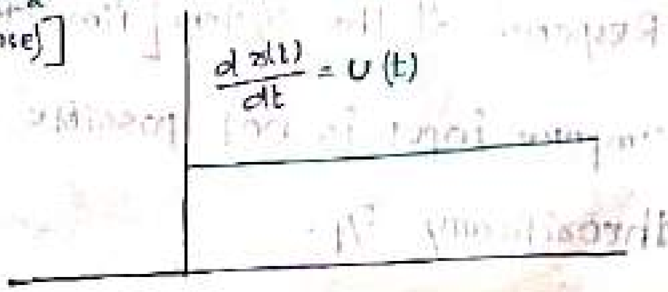
$r(t) = t \; ; \; t \geq 0$

$\quad\;\; 0 \; , \; t < 0 .$



T.F = L.T [Impulse Response]

$= L.T\left[\frac{d^2}{dt^2}(\text{Ramp Response})\right]$



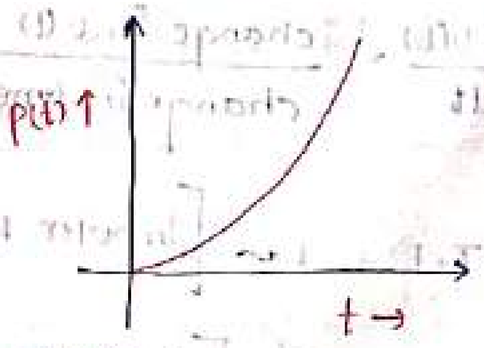
Note:-

→ The standard test i/p for all the control systems is unit step responce.

UNIT PARABOLIC I/P:-

$p(t) = \frac{1}{2}t^2 ; \; t \geq 0$

$\quad\;\; = 0 \; ; \; t < 0$

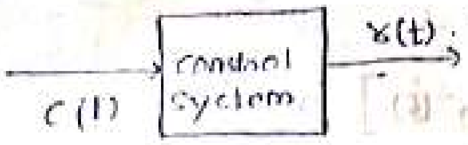


$\delta(t)$

$u(t)$ ↓ downword integration ↑ differentiation.

$r(t)$

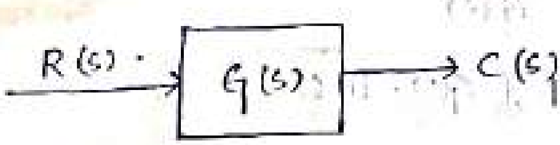
Openloop control system:-



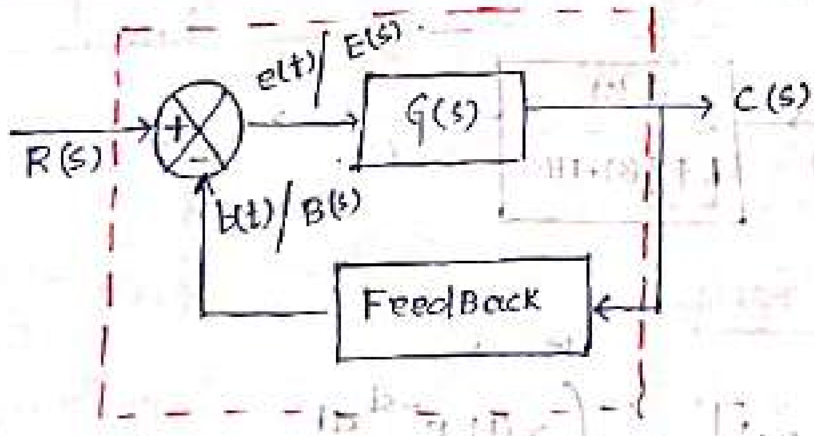
$$TF = \frac{LT[r(t)]}{LT[c(t)]}$$

$$= \frac{C(s)}{R(s)}$$

$$G(s) = \frac{C(s)}{R(s)}$$ = Transfer function of openloop control system.



Closed loop control system:-



Transfer function of Feedback system-

$$H(s) = \frac{B(s) \rightarrow o/p}{C(s) \rightarrow i/p}$$

$$E(s) = R(s) - B(s).$$

$E(s) = R(s) - H(s) \cdot C(s)$

o/p

$\rightarrow C(s) = E(s) \cdot G(s)$

$= [R(s) - H(s) \cdot C(s)] G(s)$

$\rightarrow C(s) + C(s) \cdot H(s) G(s) = R(s) \cdot G(s)$

$\rightarrow C(s) [1 + H(s) \cdot G(s)] = R(s) \cdot G(s)$

$$\rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + H(s) \cdot G(s)}}$$

$$TF_{\text{closed loop}} = \frac{G(s)}{1 \pm G(s) \cdot H(s)}$$

$T(s)$ or $M(s)$

$+ \rightarrow$ -ve feed back

$- \rightarrow$ +ve feed back

$$R(s) \longrightarrow \boxed{\frac{G(s)}{1 + G(s) \cdot H(s)}} \longrightarrow C(s)$$

LAPLACE TRANSFORM :-

$$[L(x(t))] = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= X(s)$$

Def^n :- where s - complex frequency

$= \sigma + j\omega$

σ - region of convergence

(1) $LT[\delta(t)] = 1$

(2) $LT[u(t)] = 1/s$

(3) $LT[t^n] = \frac{n!}{s^{n+1}}$

(4) $LT[e^{-at}u(t)] = \frac{1}{s+a}$

(5) $LT[\cos\omega t] = \frac{s}{s^2+\omega^2}$

(6) $LT[\sin\omega t] = \frac{\omega}{s^2+\omega^2}$

(7) if $x(t) \rightleftharpoons X(s)$

then $e^{-at}x(t) \rightleftharpoons X(s+a)$

e.g. (i) $L[e^{-at}\cos\omega t] = \frac{s+a}{(s+a)^2+\omega^2}$

(ii) $L[e^{-at}\sin\omega t] = \frac{\omega}{(s+a)^2+\omega^2}$

(8) if $x(t) \rightleftharpoons X(s)$.

$x(t-t_0) \rightleftharpoons X(s)e^{-st_0}$

(9) $\frac{dx(t)}{dt} \rightleftharpoons sX(s)$.

Then $\int x(t)\,dt = \frac{X(s)}{s}$.

If $x(t) \rightleftharpoons X(s)$.

Then initial value of $x(t)$.

$$x(0) = \lim_{t \to 0} x(t)$$

$$\boxed{x(0) = \lim_{s \to \infty} s.X(s).}$$

Final value of $x(t)$

$$x(\infty) = \lim_{t \to \infty} x(t).$$

$$\boxed{x(\infty) = \lim_{s \to 0} s \times X(s)}$$

⇒ Final value is also called as steady state value of the system.

(1) $X(s) = \dfrac{s+2}{s(s+3)(s+4)}$ $x(\infty) = 2/12 = 1/16$

v.v. Imp

⇒ NOTE:- → Final value theorem are not applicable when poles of the system are present at right half.

→ when the poles are present at imaginary axis.

$X(s) = \dfrac{s+2}{s(s-3)(s+4)}$ $x(\infty) = -\dfrac{1}{6}$

↳ as poles are present at right half so $x(\infty)$ can't be found.

$X(s) = \dfrac{K}{s(s^2+4)}$ as poles are in imaginary axis so $x(\infty)$ can't be found.

$$X(s) = \frac{A}{s} + \frac{B}{s-3} + \frac{C}{s+4}$$

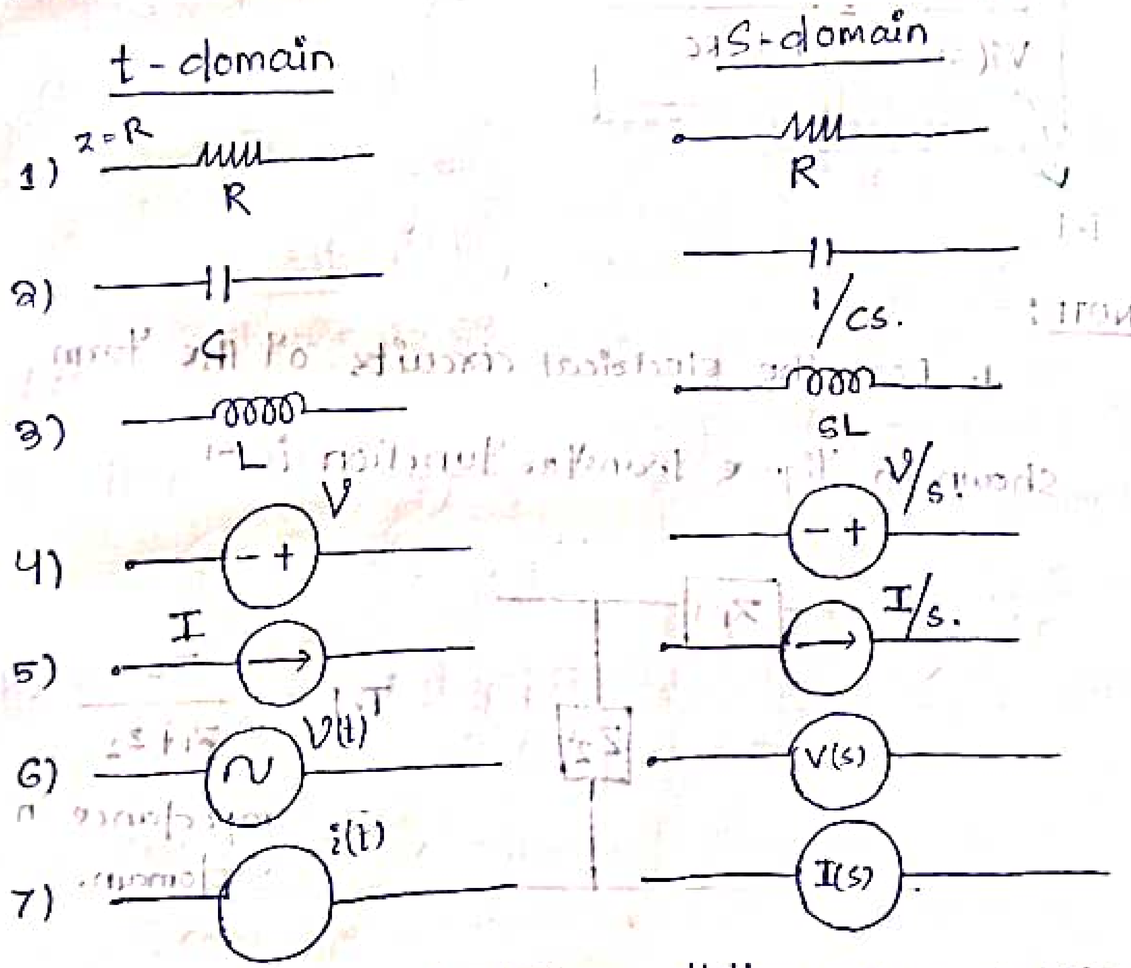
$$x(t) = A\,u(t) + B e^{3t} + C e^{-4t}$$

$A\,u(t)$ ↓ Final value A

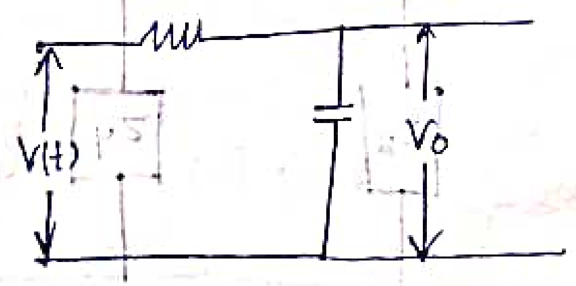
$B e^{3t}$ ↓ But here we can't find final value.

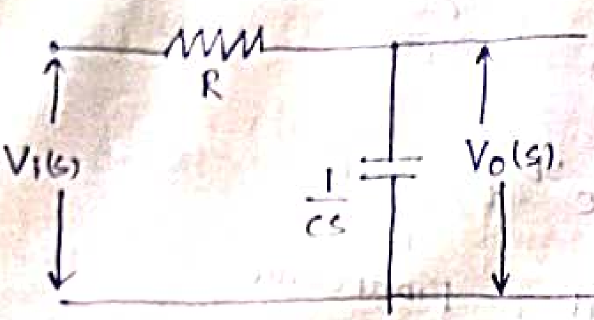
$C e^{-4t}$ ↓ Final value 0

Transfer function of physical system:-



Q. Obtain the transfer function of following electrical N/w.





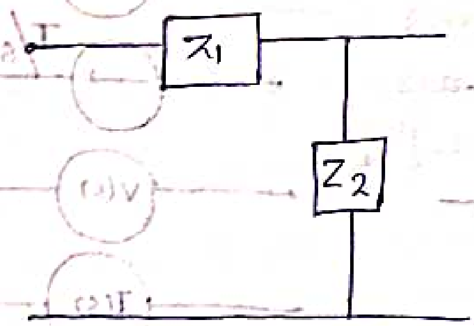
$$V_o(s) = \frac{V_i(s) \times \frac{1}{sC}}{R + \frac{1}{sC}}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sRC}}$$

T.F

NOTE:-

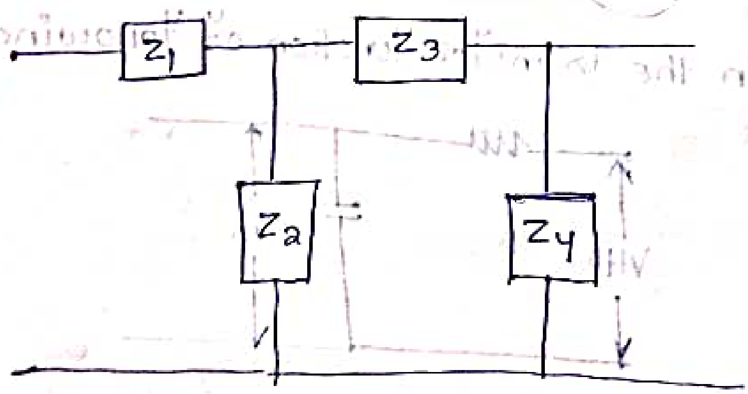
1. For the electrical circuits of the form shown in figure transfer function is →



$$T.F = \frac{Z_2}{Z_1 + Z_2}$$

Z - Impedance in S domain.

2.



Closed loop system is less Sensitive to the change in G(s) but more sensitive to the changes in H(s). Therefor feed back system must be designed carefully, to make the system insensitive to the changes in component values.

NOTE:-

(i) If openloop transfer function,

$G(s) = \frac{N(s)}{D(s)}$ & Feedback is H(s) then closed loop transfer function.

Imp

$$\boxed{T(s) = \frac{N(s)}{D(s) + N(s) \cdot H(s)}} \quad (\text{-ve feedback})$$

(ii) If the closed Loop transfer function

$T(s) = \frac{N(s)}{D(s)}$ & feed back is H(s).

Imp

$$\boxed{G(s) = \frac{N(s)}{D(s) - N(s) \cdot H(s)}} \quad (\text{-ve feedback})$$

eq:- (i) $G(s) = \frac{2}{s(s+2)}$

$H(s) = 4$

$$T(s) = \frac{2}{s^2+2s+8}$$

(ii) $T(s) = \frac{2}{s^2+2s+8}$ $H(s) = 4$

Q) In control system is described by the differential

eq^n. $6\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 4y = 5\frac{dx}{dt} + 4x$

(OP-19-11-2017)

where x - i/p & Y - o/p of the system what is the

transfer function.

Sol^n:- $T.F = \frac{Y(s)}{X(s)}$.

Apply in Laplace

$$6s^2Y(s) + 3sY(s) + 4Y(s) = 5sX(s) + 4X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{5s+4}{6s^2+3s+4} \text{ (Ans)}$$

Q) If the impulse response of a system is $[e^{-t} - e^{-2t}]u(t)$

$T.F = LT$ [Impulse response]

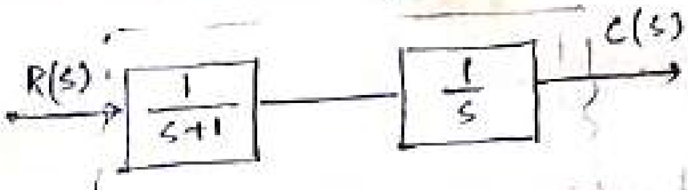
$$= LT[e^{-t} - e^{-2t}]$$

$$= LT[e^{-t}] - L.[e^{-2t}]$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$= \frac{1}{(s+1)(s+2)}$$

Q) What is the unit impulse Response for the system shown in figure.



$$T.F = \frac{1}{s(s+1)}$$

$$= \frac{A}{s} + \frac{B}{s+1}$$

$$= \frac{1}{s} + \frac{-1}{s+1}$$

Impulse Response = $L^{-1}\left[\frac{1}{s}\right] + L^{-1}\left[\frac{-1}{s+1}\right]$

$$= u(t) + u(t)(-1)e^{-t}$$

$$= (1-e^{-t})\,u(t).$$

Q) The impulse response of initially relaxed system is $e^{-at}.u(t)$. To produce a response of $t.e^{-at}u(t)$ the i/p of the system must be equal to ?

Sol:- IR - $e^{-at}u(t)$ o/p = $t.e^{-at}u(t)$

TF = LT [IR]   i/p = ?

$$= LT\left[e^{-at}u(t)\right]$$

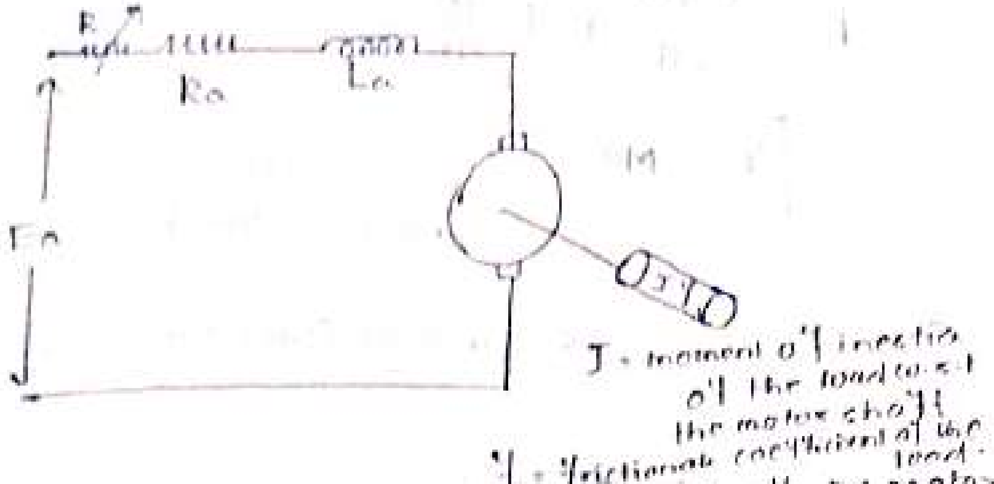
$$\frac{1}{s+a}$$

LT [o/p] = TF×L[i/p]

$$\Rightarrow LT\left[e^{-at}t\,u(t)\right] = \frac{1}{s+a}\cdot LT\left[i/p\right]$$

Mathematical modelling of armeture control d c servo motor:-

The set of differential equation that describe the dynamic behaviour of the system is called matematical modelling.



As we control the armature current of DC motor so it is called armature control of dc motor.

$T \propto \phi_f I_a$.

I_f - constant.

if I_f is constant then ϕ_f is constant.

so we can only vary I_a.

so $T = K_T I_a$ (as ϕ_f is constant)

Analysis → matematical eq^n → control eq^n which represents the analysis

<u>Applying KVL</u>:-

$$E_a - i_a R_a - L_f \frac{di_a}{dt} - E_b = 0 \quad \text{—(i)}.$$

$$T = K_T i_a \quad \text{—(ii)}$$

$E_b \propto \frac{d\theta}{dt}$. θ - rotational displacement

$$\Rightarrow E_b = K_b \frac{d\theta}{dt} \quad \text{—(iii)}$$

as backemf amount is directly proportional to the speed.

$$T = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} \quad -(iv)$$

$$\boxed{T = Ma + Bv}$$

(newton's 2nd law)

08 Jan 2015 (2nd class) (R.S)

Mathematical Modelling :-

$$E_a(t) - E_b(t) = R_a i(t) + L_a \frac{d i(t)}{dt} \quad -(i)$$

$$T(t) = K_T i_a(t) \quad -(ii)$$

$$E_b(t) = k_b \frac{d\theta}{dt} \quad -(iii)$$

$$T(t) = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} \quad -(iv).$$

$$E_a(s) - E_b(s) = R_a I_a(s) + sL_a I_a(s) \quad -(i)$$

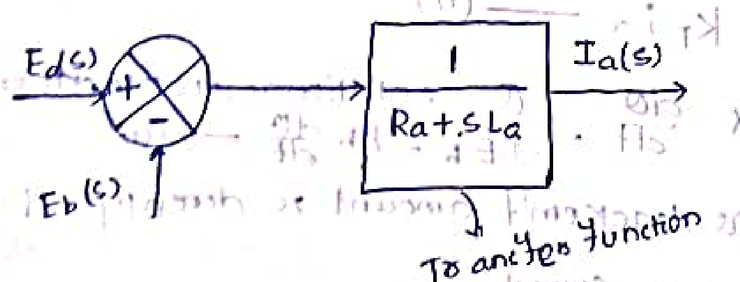
$$T(s) = K_T I_a(s) \quad -(ii)$$

$$E_b(s) = K_b\, s\, \theta(s) \quad -(iii)$$

$$T(s) = J s^2 \theta(s) + f s\, \theta(s) \quad -(iv)$$

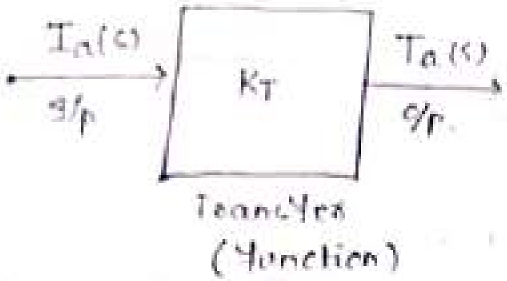
from eqn -(i). Transfer function = $\frac{\text{output}}{\text{input}}$

$$\Rightarrow \frac{I_a(s)}{E_a(s) - E_b(s)} = \frac{1}{R_a + sL_a}$$

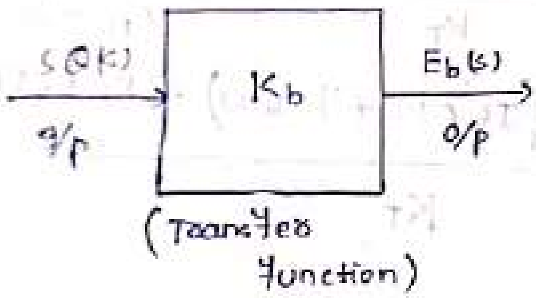


$T(s) = K_T I_a(s)$

$\Rightarrow \quad K_T = \dfrac{T_a(s)}{I_a(s)}.$



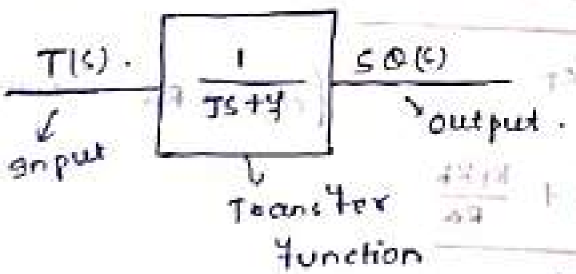
for eq^n(3) $\rightarrow$

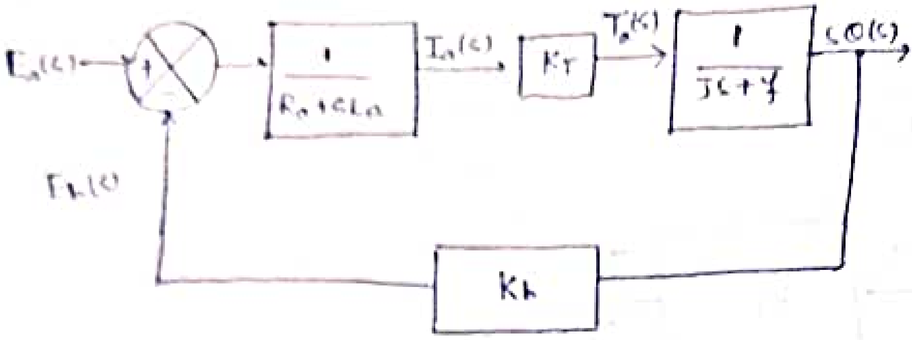


for eq^n-(4) $\rightarrow$

output - $s\theta(s)\left[Js+f\right]$

input - $T(s)$.





$\Rightarrow E_a(s) - E_b(s) = (R_a + sL_a)\, I_a(s).$

$\Rightarrow E_a(s) - K_b\, s\theta(s) = (R_a + sL_a)\dfrac{T(s)}{K_T}$ (replace E_b value from 3rd eqⁿ & I_a from 2nd eqⁿ)

$$\Rightarrow E_a(s) = \frac{(R_a + sL_a)\left(Js^2\theta(s) + Ys\theta(s)\right)}{K_T} + K_b s\theta(s).$$

(put $T_a(s)$ from 4th eqⁿ)

$$\Rightarrow E_a(s) = \frac{(R_a + sL_a)\left(Js^2\theta(s) + Ys\theta(s)\right) + K_T K_b s\theta(s).}{K_T.}$$

$$\Rightarrow \frac{E_a(s)}{s\theta(s)} = \frac{(R_a + sL_a)(Js + Y) + K_T K_b}{K_T}$$

(L_a value is much more less than R_a so L_a value is neglected)

$$\Rightarrow \frac{E_a(s)}{s\theta(s)} = \frac{R_a(Js + Y) + K_T K_b}{K_T}$$

(divide R_a in both Nr & Dr)

$$\Rightarrow \frac{E_a(s)}{s\theta(s)} = \frac{(Js + Y) + \frac{K_T K_b}{R_a}}{K_T/R_a.}$$

$$\Rightarrow \frac{s\theta(s)}{E_a(s)} = \frac{(K_T/R_a)}{\left[Js + \left(Y + \frac{K_T K_b}{R_a}\right)\right]}$$

order of a system respresents the highest power of Dr in transfer function.

$$\Rightarrow \frac{sQ(s)}{E_a(s)} = \frac{K_T/R_a}{(Js + Y_0)} \quad \left[\text{Let } Y_0 = \left(Y + \frac{K_T K_b}{R_a}\right)\right]$$

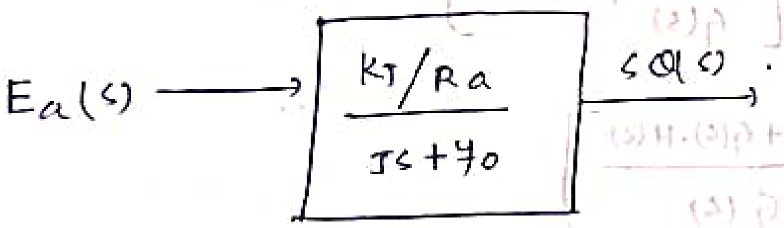
$Q(t) \to Q(s)$ = displacement

$\frac{dQ}{dt} \to sQ(s)$ = speed.

$\frac{d^2Q}{dt^2} \to s^2Q(s)$ = acceleration

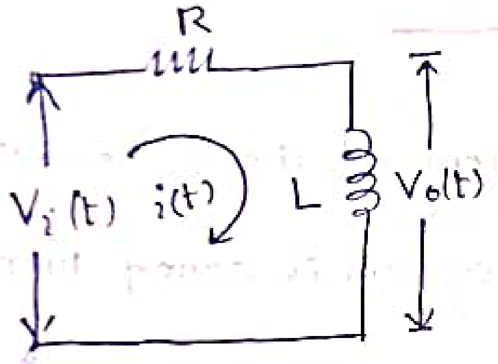
$$\frac{Q(s)}{E_a(s)} = \frac{(K_T/R_a)}{s(Js+Y_0)}.$$

is 2nd order system as power of the Dr is '2'.



(overall transfer function)

Mathematical Modelling of electrical system:-



$$V_o(t) = L\, di/dt \quad \text{— (2)}$$

$$V_i(t) - i(t)\,R - L\, di/dt = 0 \quad \text{— (1)}$$

Transfer function is only applicable for linear & time invariant System.

s - domain equation Laplace transformation ~~from~~ of eq^n (1) & (2).

$$V_i(s) - I(s)\cdot R - Ls\,I(s) = 0$$

$$V_o(s) = Ls\,I(s).$$

Transfer function:-

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{SL\,I(s)}{RI(s)+sLI(s)} = \frac{SL}{R+SL}$$

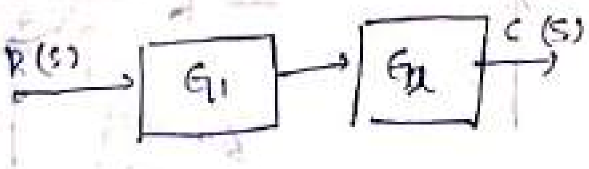
procedure for calculation of Transfer function:-

(1) Block diagram reduction

(2) signal flow graph.

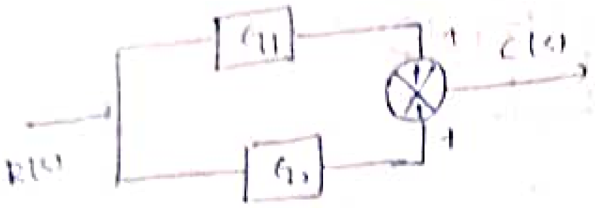
(1) Block diagram reduction:-

Rule1:-



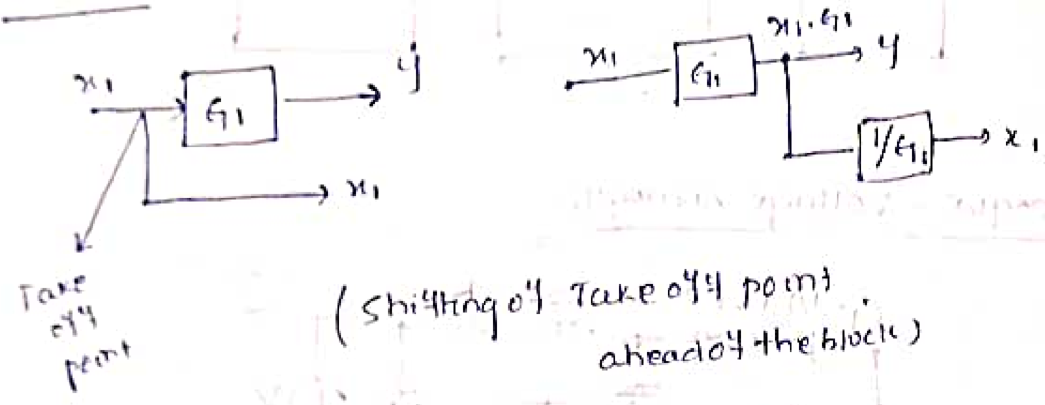
$$\frac{C(s)}{R(s)} = G_1 \times G_2$$

Rule-2



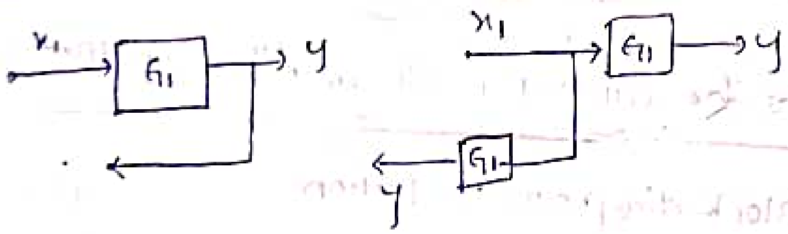
$$\frac{C(s)}{R(s)} = G_1 + G_2$$

Rule-3

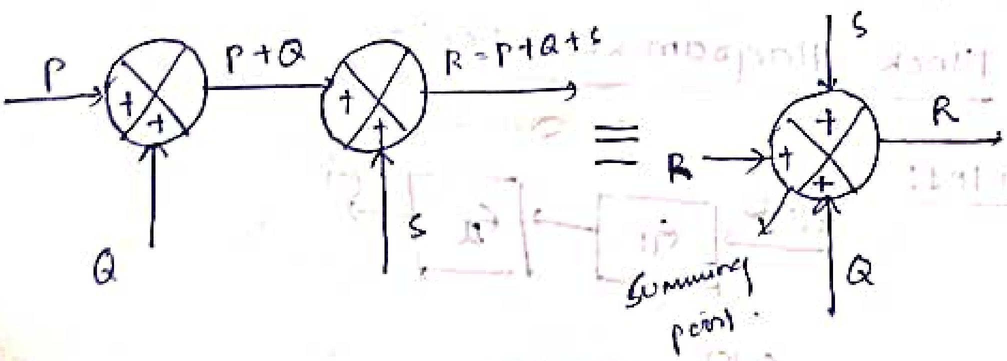


(shifting of Take off point ahead of the block)

Rule-4:-



Rule-5

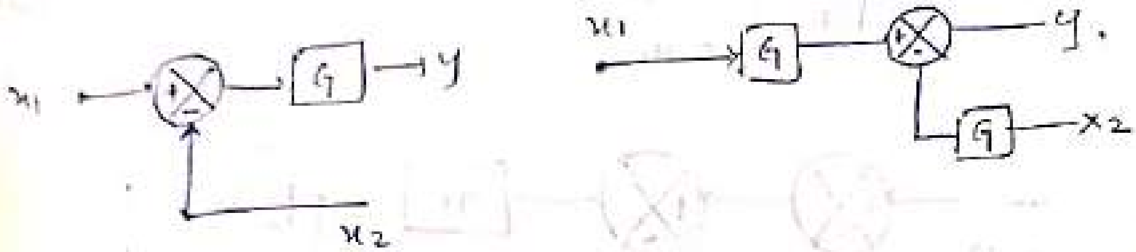


Rule - 6

$y = x_1 G - x_2$

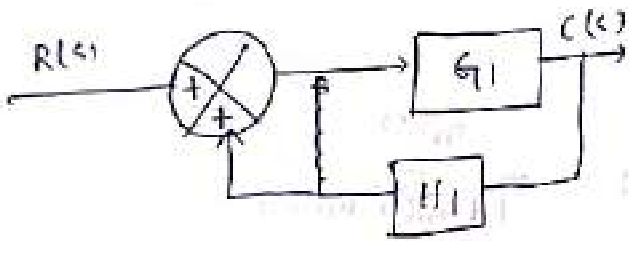
$x_1 G - x_2$

Rule 7



$y = (x_1 - x_2) G$

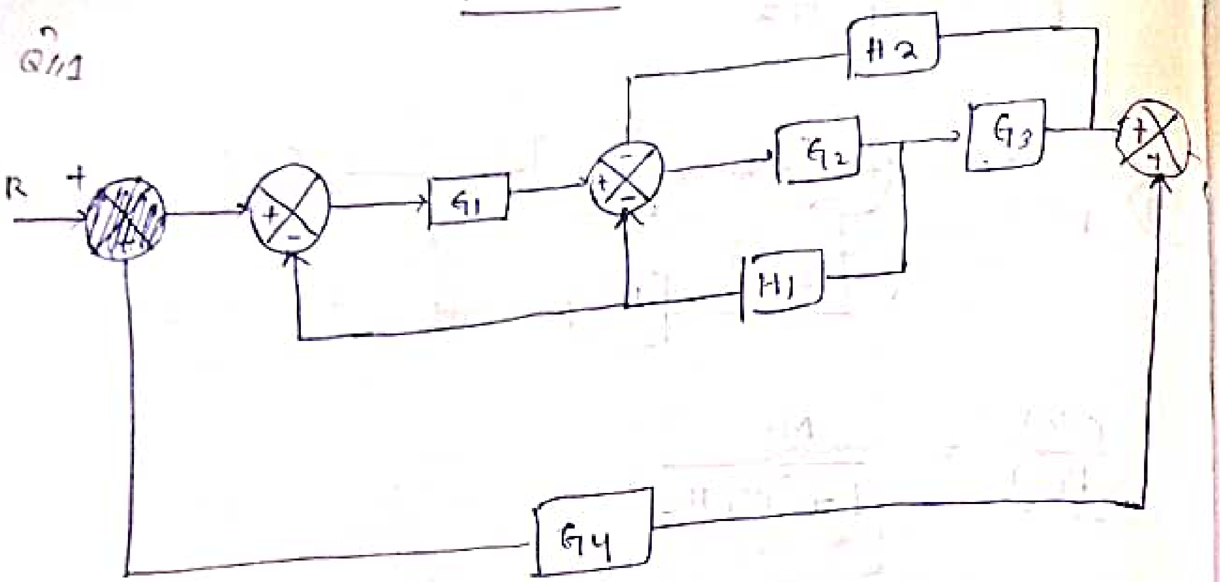
Q. 1 Findout the transfer function using blockdiagram reduction rule.



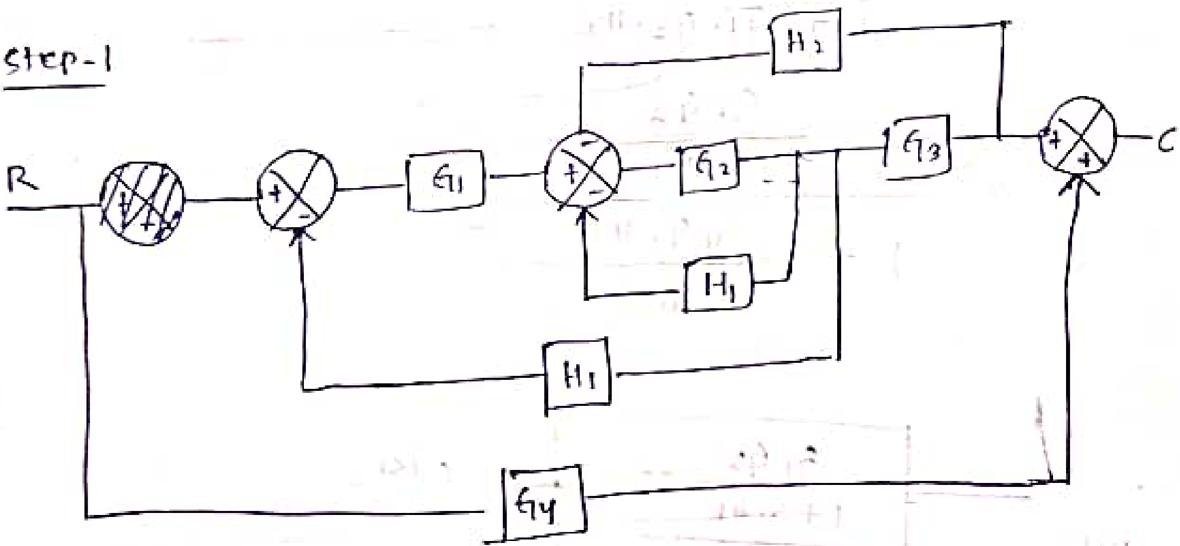
$$\frac{C(s)}{R(s)} = \frac{G_1}{1 - G_1 H_1}$$

16 Jan 15 (7th class)

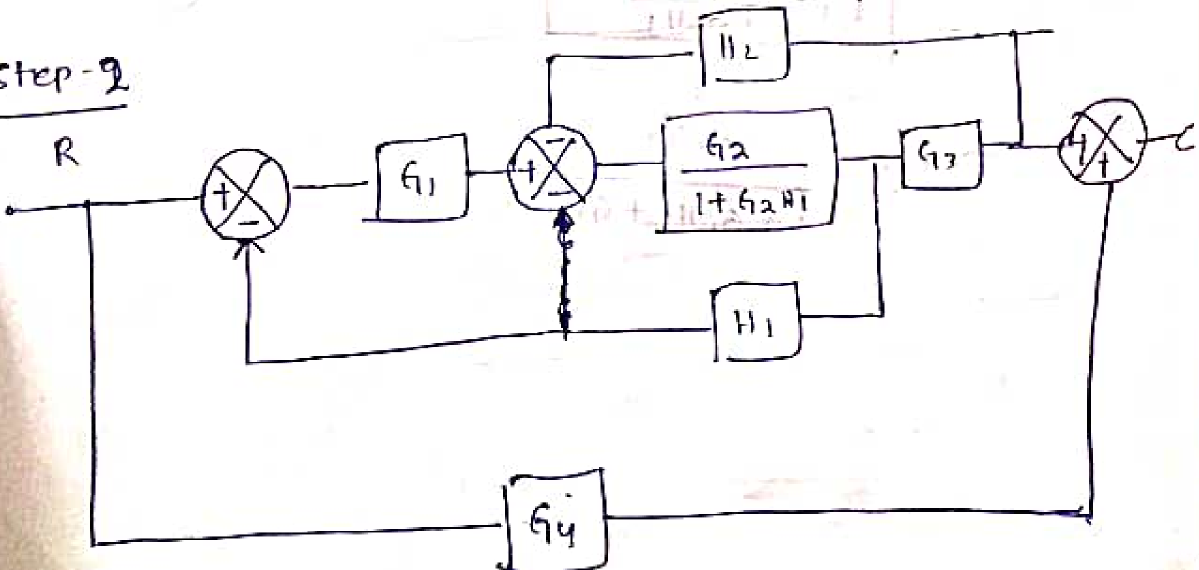
Q. 11



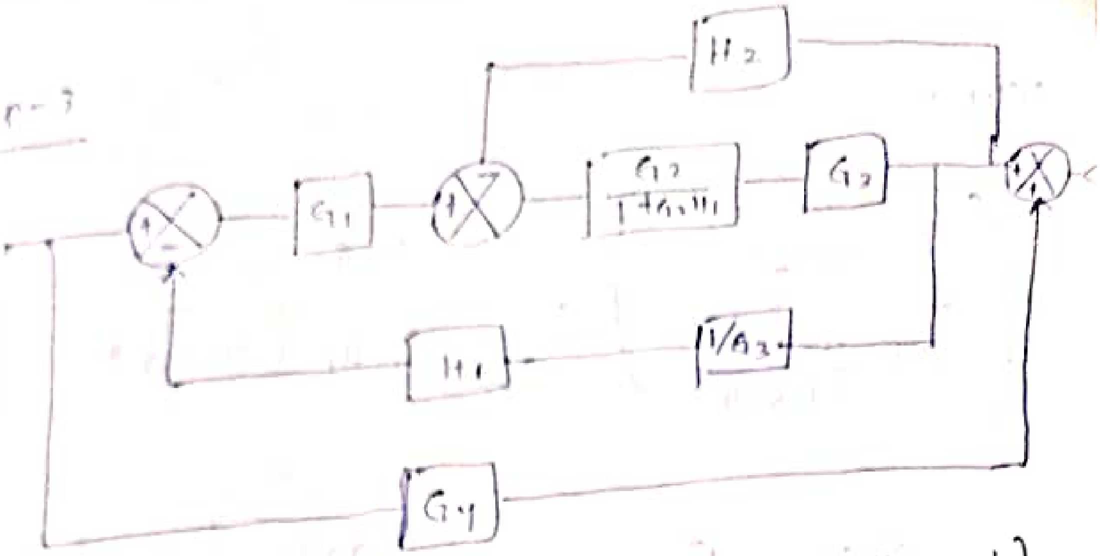
Step-1



Step-2

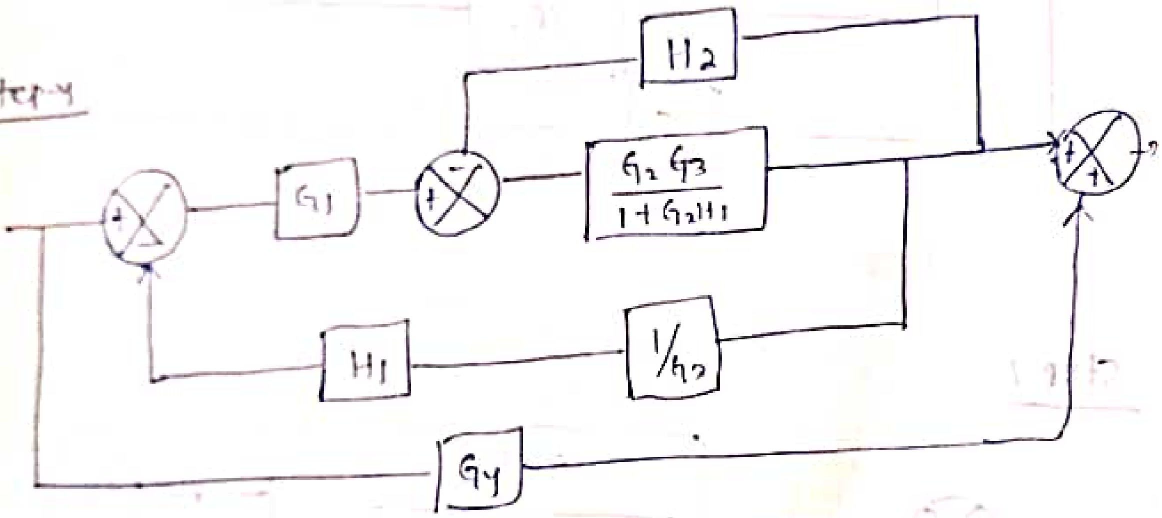


step-3

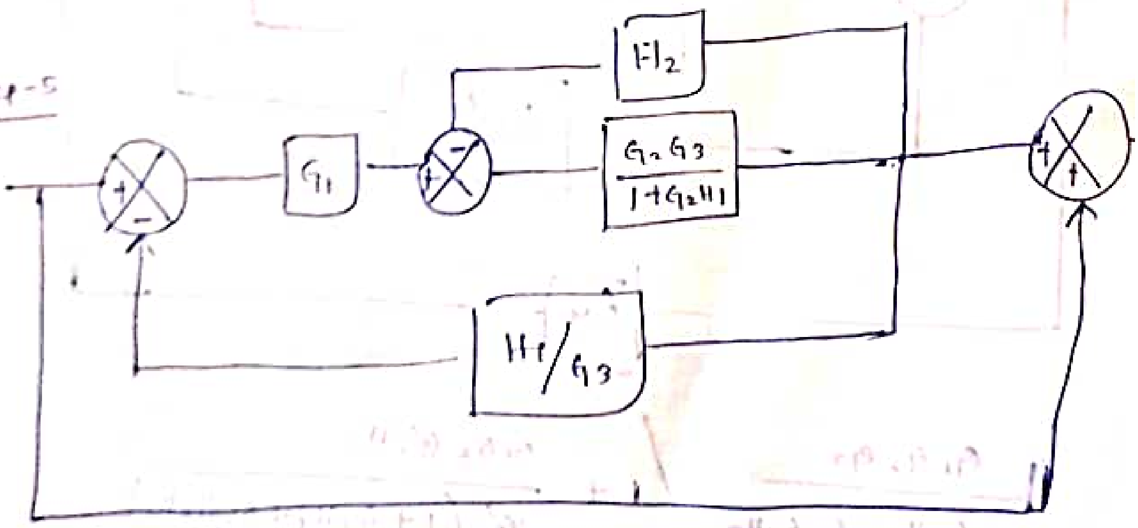


(shifting of take off point).

step-4

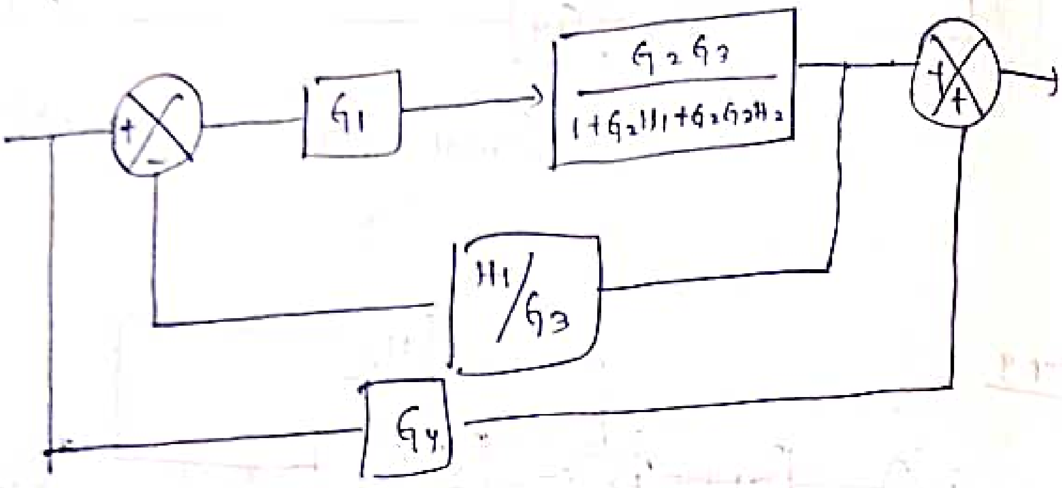


step-5

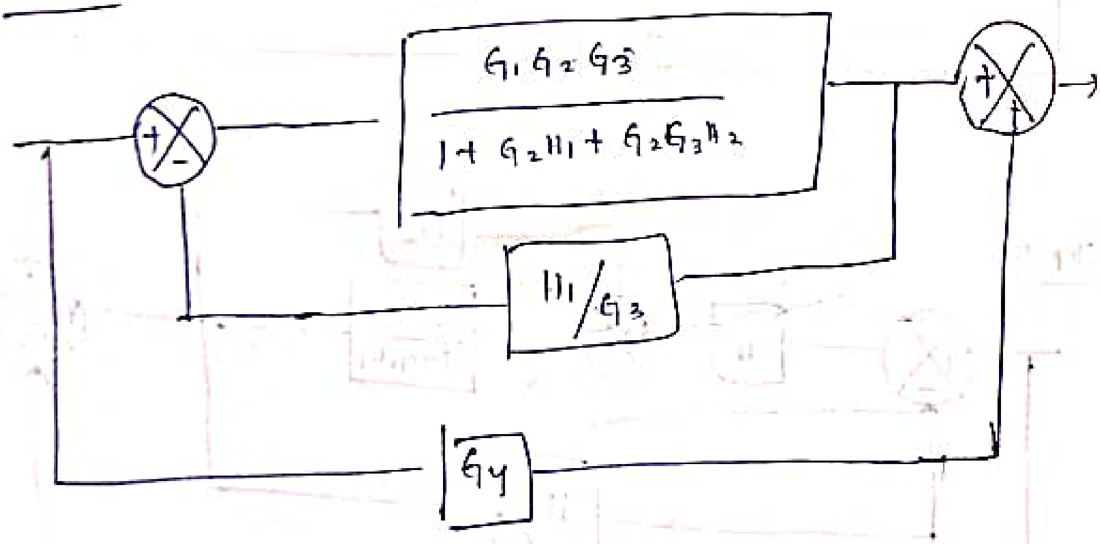


(Step-6)

$$\left(\frac{\dfrac{G_2 G_3}{1+G_2 H_1}}{1+\dfrac{G_2 G_3 H_2}{1+G_2 H_1}}\right) = \frac{G_2 G_3}{1+G_2 H_1 + G_2 G_3 H_2}$$

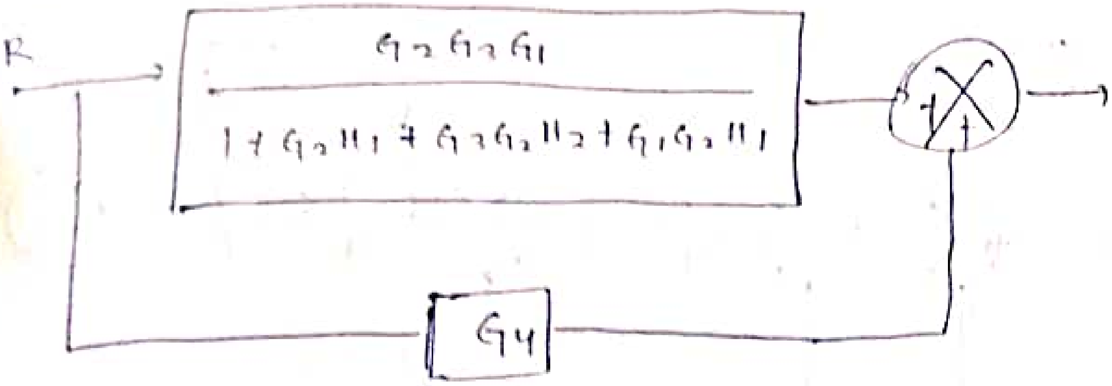


Step-7

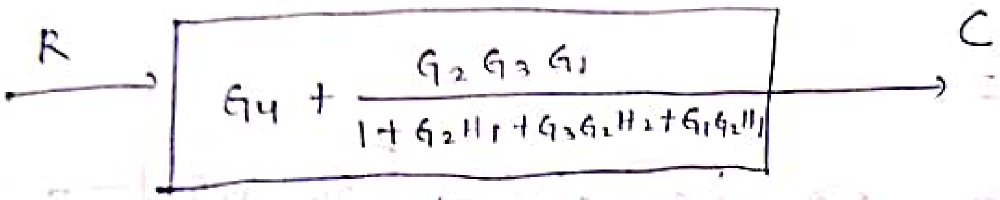


$$\frac{G_1 G_2 G_3}{1+G_2 H_1 + G_2 G_3 H_2} \Big/ 1 + \frac{G_1 G_2 \cancel{G_3} H_3}{\cancel{G_3}\left(1+G_2 H_1 + G_2 G_3 H_2\right)}$$

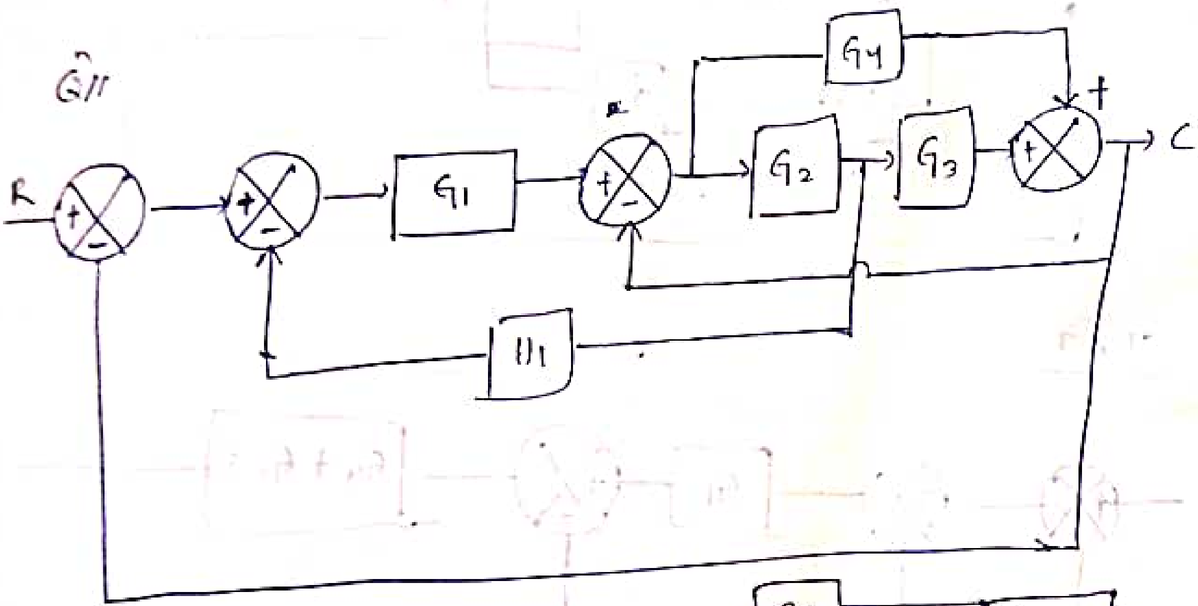
step-8



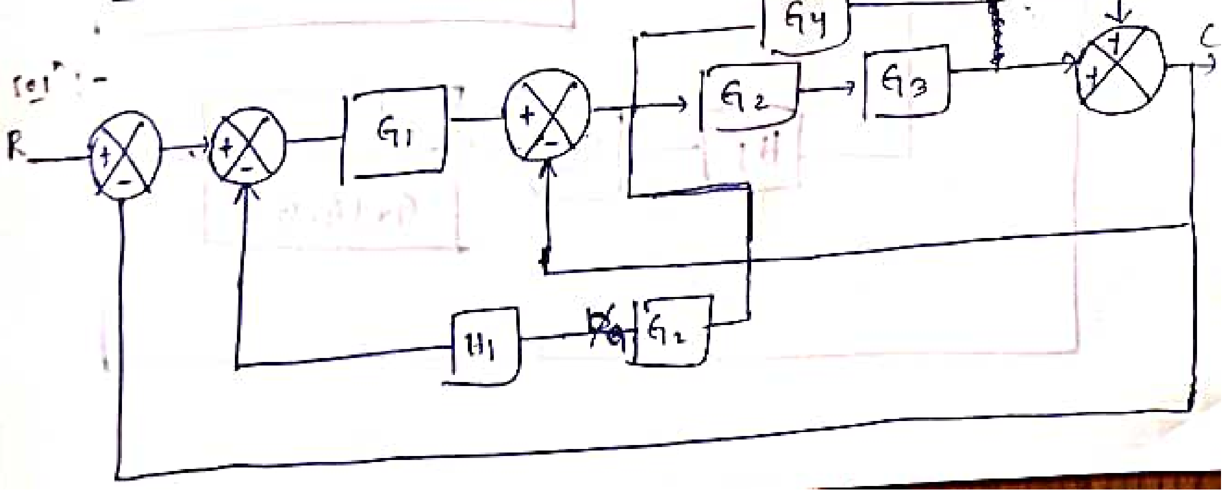
step-9



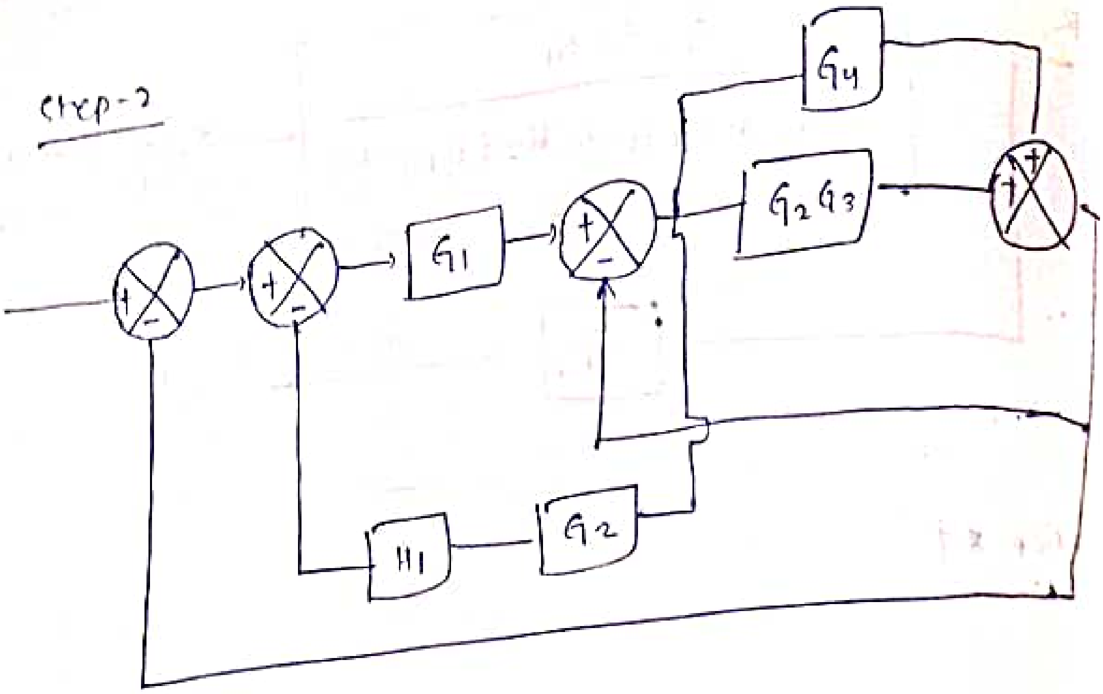
Q11



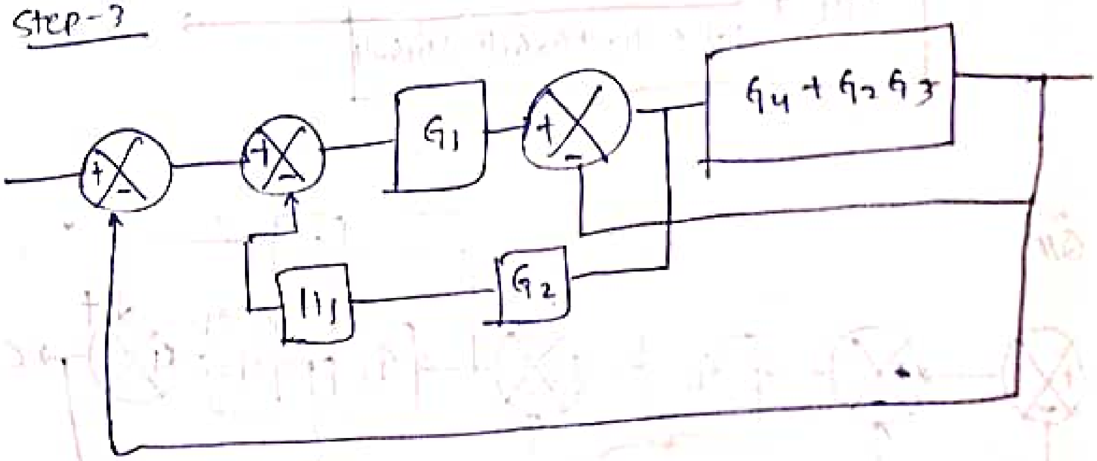
sol:-



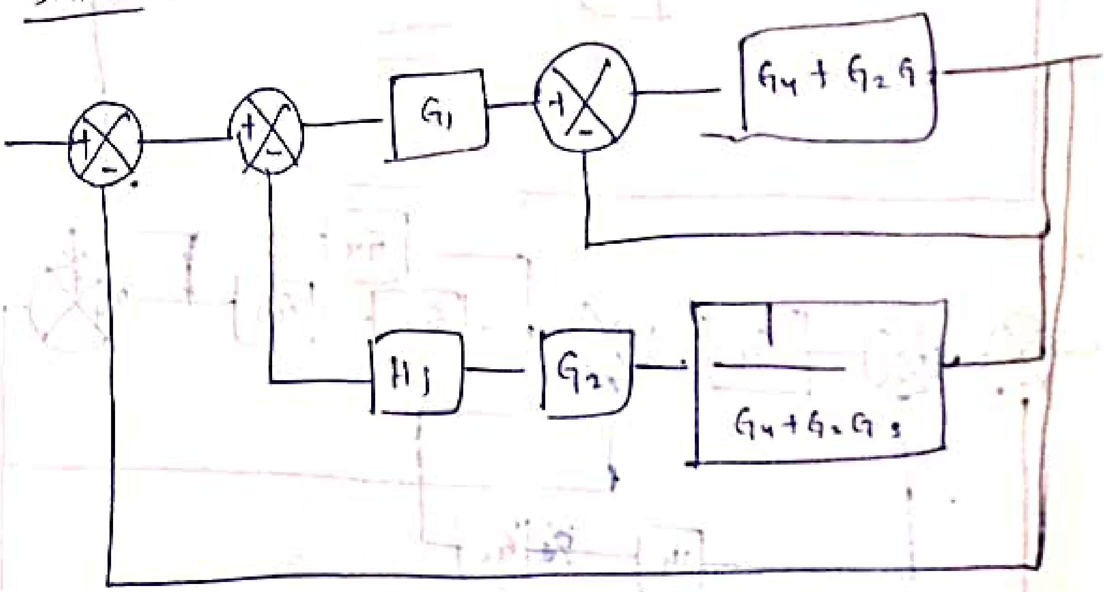
step-2



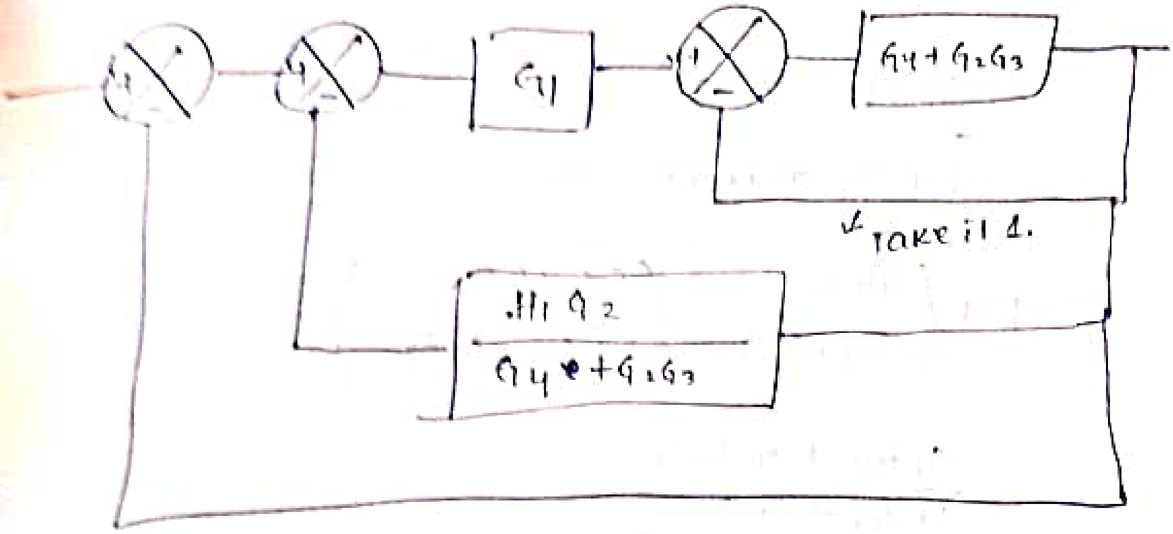
step-3



step 3



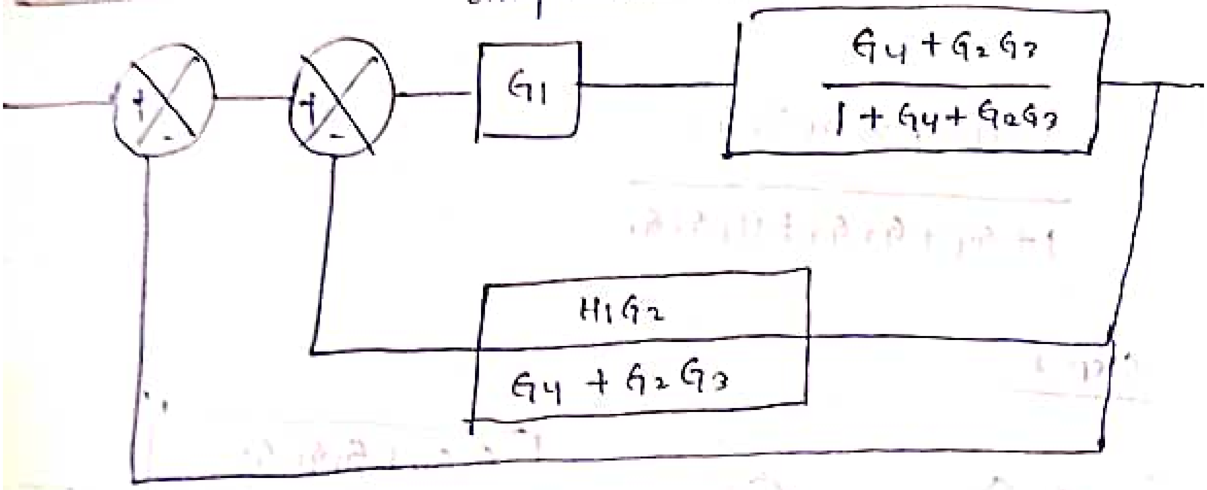
step 4



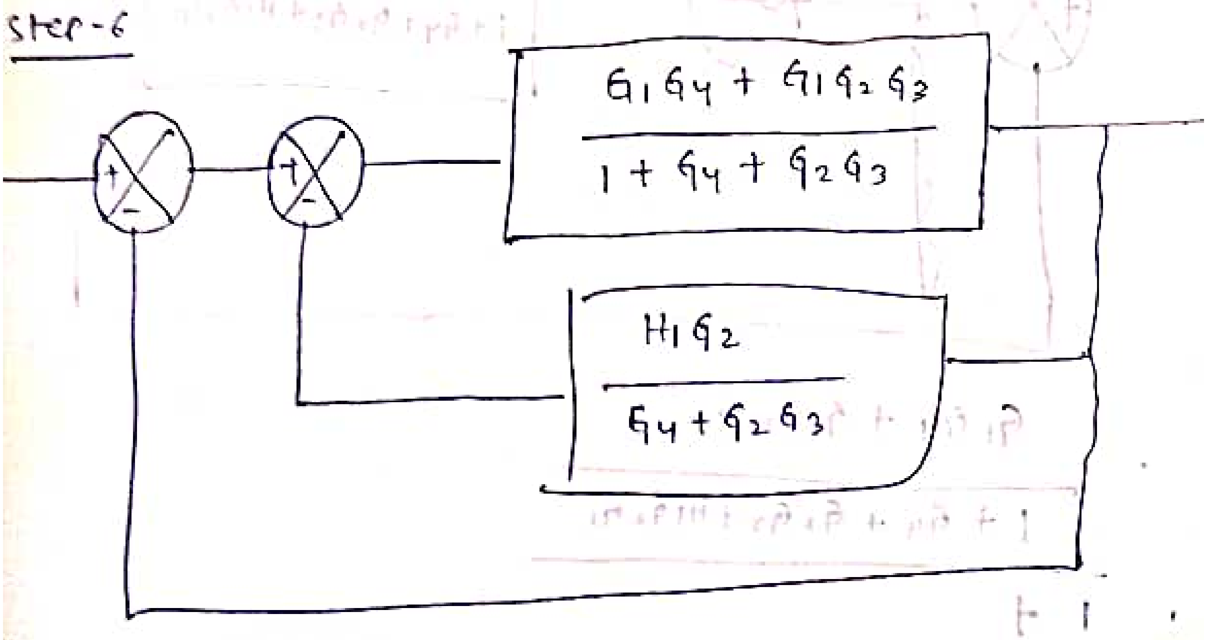
$$\frac{G_4+G_2G_3}{1+(G_4+G_2G_3)\times 1}$$

(if the feedback is one it is called unity feedback)

step 5



step-6



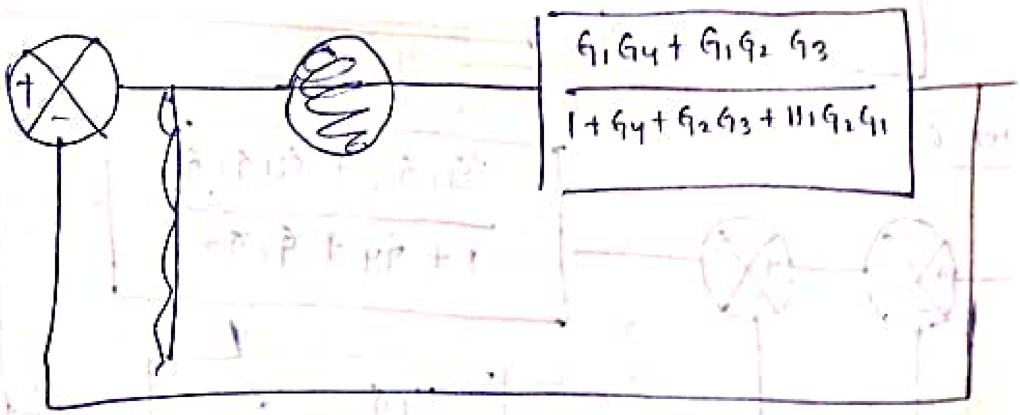
Step-7

$$\frac{\dfrac{G_1G_4 + G_1G_2G_3}{1+G_4+G_2G_3}}{1+\left(\dfrac{H_1G_2}{G_4+G_2G_3}\right)\left(\dfrac{G_1G_4+G_1G_2G_3}{1+G_4+G_2G_3}\right)}$$

$$= \frac{\dfrac{G_1G_4 + G_1G_2G_3}{1+G_4+G_4G_3}}{1+\dfrac{H_1G_2G_1}{1+G_4+G_2G_3}}$$

$$= \frac{G_1G_4 + G_1G_2G_3}{1+G_4+G_2G_3+H_1G_2G_1}$$

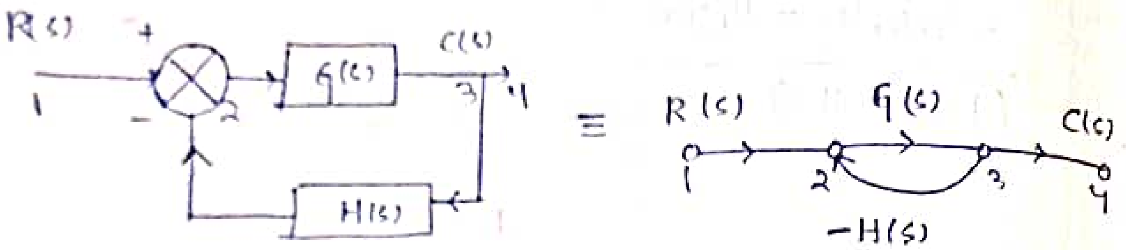
Step 8

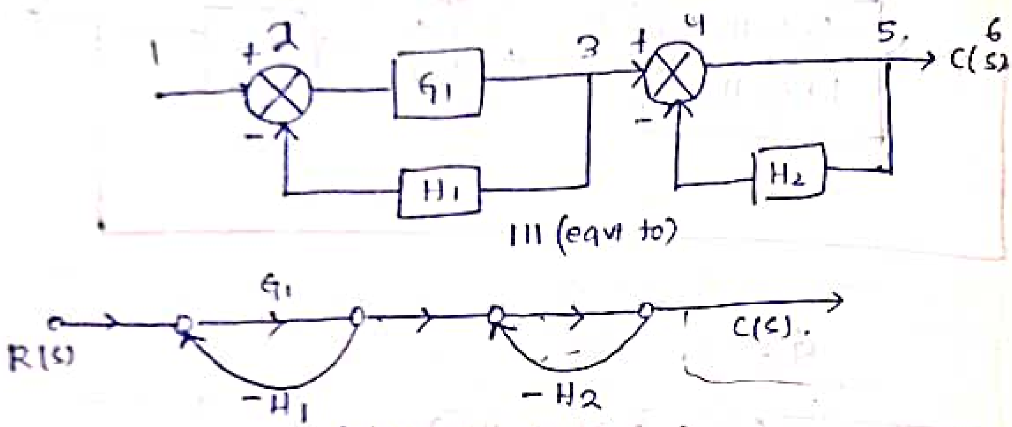


$$\frac{\dfrac{G_1G_4 + G_1G_2G_3}{1+G_4+G_2G_3+H_1G_2G_1}}{1+}$$

20 Jan, 2015 (RC) (9th class)

Signal flow graph:- It is the graphical representation of any physical system.





(figure-2)

Represent each i/p, ~~to~~ o/p, summing point, take off point by ~~toa~~ node.

→ If block is absent then unity gain.

procedure to calculate transfer function:-

$$T(s) = \frac{1}{\Delta} \sum_{K} P_K \Delta_K \text{. (Mason's gain formula)}$$

Δ - determinant of the signal flow graph.

K - it represent no of forward path in signal flow graph.

P_K - Kth forward path

Δ_K - determinant relating to forward path P_K.

Step-1:-

In step-1, calculate no of forward path.

Defn of forward path:-

forward path is the path from i/p to o/p node without tracing any node twice.

fig-2 have one forward path.

Step-2:-

Find out no of loops.

fig-2 have 2 loops.

Step-3:-

calculate the forward path gain.

(By multiplying all the gains)

fig-2 have- $1 \times G_1 \times 1 \times 1 \times 1 = G_1$

Step-4:-

calculate the loop gain of each individual loop.

for fig-2 - gains are $-G_1H_1$ & $-H_2$ respectively.

Step. 5:-

calculate ΔK,

$\Delta K = 1$ (when the forward path K touches all individual loop)

$\Delta K = 1 -$ (Loop gain of non-touching loop)

Step-6:-

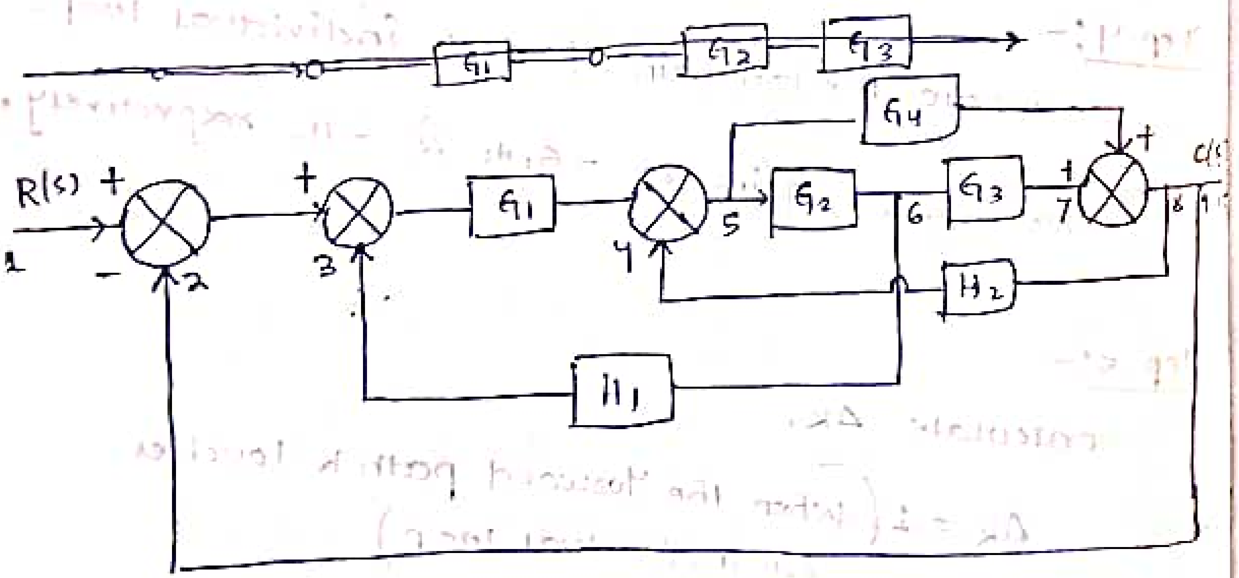
Calculate 'Δ'

$\Delta = 1 -$ (Summation of all individual loop gain)
+ (summation of gain product of two nontouching loop)
− (Summation of gain product of three non-touching loop)

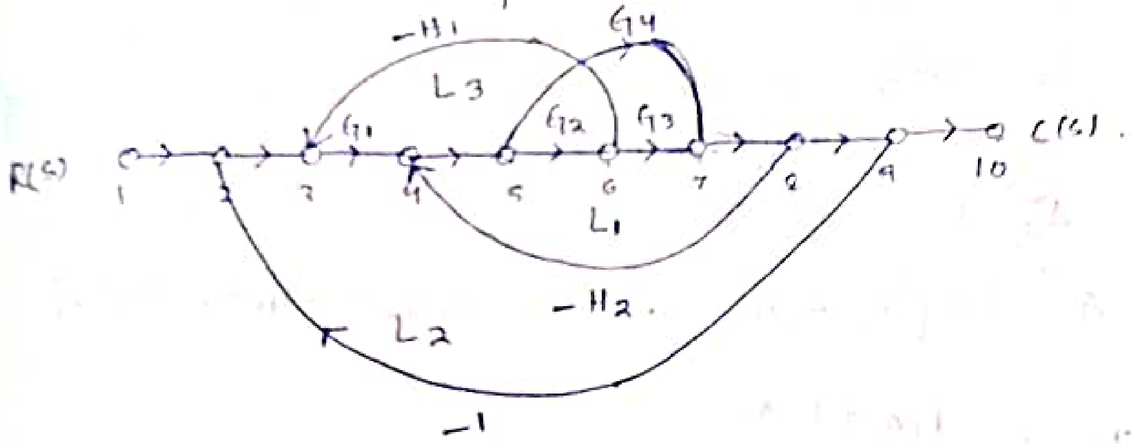
two non-touching loop:-

If there is no common node betn two individual loop this ~~foo~~ two loops are called non-touching loop.

Qn 1.



connect each nodes by the signal:-



step-1

No of forward path $K=2$.

step-2

No of loops = 8

step-3

The forward path gain -

$$P_1 = G_1G_2G_3$$

$$P_2 = G_1G_4.$$

$L_1 = -G_2G_3H_2$

$L_2 \rightarrow -G_1G_2G_3$

$L_3 \rightarrow -G_1G_2H_1$

$L_4 \rightarrow -G_1G_4$ (2, 3, 4, 5, 7, 9, 2)

$L_5 \rightarrow -G_4H_2$ (4, 5, 7, 8, 4)

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_4$

$\Delta_1 = 1$

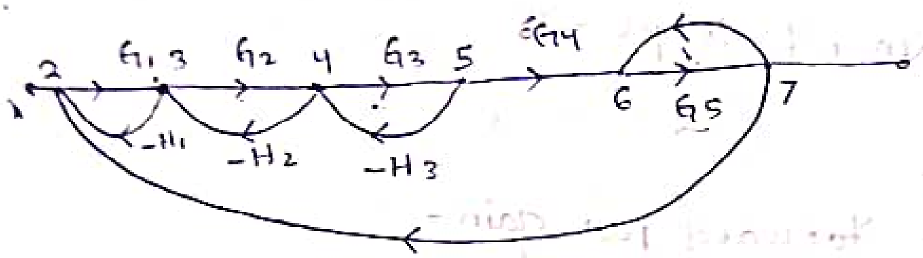
$\Delta_2 = 1$

$\Delta = 1 - (G_3 G_2 H_2 - G_1 G_2 G_3 - G_1 G_2 H_1 - G_1 G_4 - G_4 H_2)$

$$T(s) = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_3 G_2 H_2 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_1 G_4 + G_4 H_2}$$

Q 2.



$P_1 = G_1 G_2 G_3 G_4 G_5 \quad -1$

$L_1 = -G_1 H_1$

$L_2 = -G_2 H_2$

$L_3 = -G_3 H_3$

$L_4 = -G_5 H_5$

$L_5 = -G_1 G_2 G_3 G_4 G_5$

- No of fwd path & gain.
- No of loop & gain
- No of two non-touching loop & gain
- No of three non touching loop & gain.
- Δ_k.

No of two nontouching loops - 4

$L_1 L_3 = G_1 H_1 G_3 H_3$

$L_2 L_4 = G_2 H_2 G_5 H_5$

$L_3 L_4 = G_3 H_3 G_5 H_5$

$L_1 L_4 = G_1 H_1 G_5 H_5$

Number of three-Nontouching loop = 1

$$L_1 L_3 L_4 = -G_1 H_1 G_3 H_3 G_5 H_5$$

$$\Delta_K = \Delta_1 = 1$$

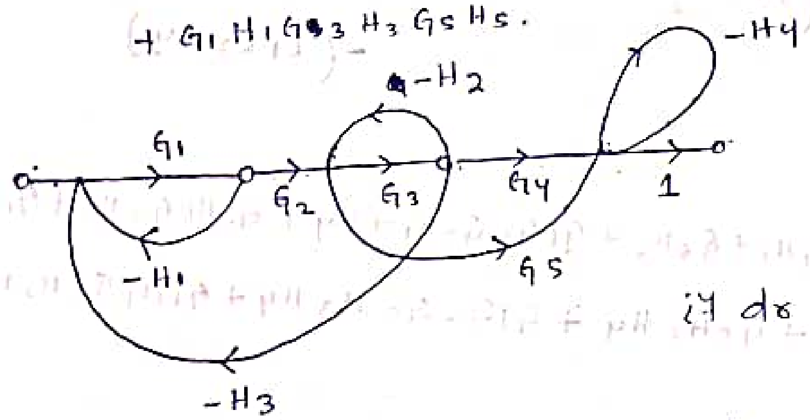
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_3 + L_1 L_4 + L_3 L_4 + L_2 L_4)$$

$$-(L_1 L_3 L_4)$$

$$T(s) = \frac{1}{\Delta}(P_1 \Delta_1)$$

$$= \frac{G_1 G_2 G_3 G_4 G_5}{1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_5 H_5 + G_1 G_2 G_3 G_4 G_5 + G_1 H_1 G_3 H_3 + G_2 H_2 G_5 H_5 + G_3 H_3 G_5 H_5 + G_1 H_1 G_5 H_5 + G_1 H_1 G_3 H_3 G_5 H_5}$$

Q//



if dr is +ve then system is stable

if -ve system unstable.

No of forward path = 2

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_2$ [redacted] G_5 [redacted]

No of loop -

$L_1 = -G_1 H_1$

$L_2 = -G_3 H_2$

$L_3 = -G_1 G_2 G_3 H_1$

$L_4 = -H_4$

No of two non-touching loop - 4

$L_1 L_2 = G_1 H_1 G_3 H_2$

$L_1 L_4 = G_1 H_1 H_4$

$L_2 L_4 = G_3 H_2 H_4$

$L_3 L_4 = G_1 G_2 G_3 H_3 H_4$

No of 3 Non-touching loop = 1

$L_1 L_2 L_4 = - G_1 H_1 G_3 H_2 H_4$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2 + L_1 L_4 + L_2 L_4 + L_3 L_4) - (L_1 L_2 L_4)$$

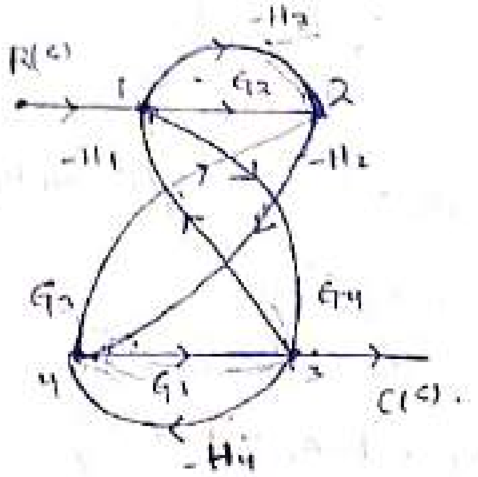
$$= 1 + G_1 H_1 + G_3 H_2 + G_1 G_2 G_3 H_1 + H_4 + G_1 H_1 G_3 H_2 + G_1 H_1 H_4 + G_3 H_2 H_4 + G_1 G_2 G_3 H_3 H_4 + G_1 H_1 G_3 H_2 H_4$$

$$T(s) = \frac{1}{\Delta}\left[P_1 \Delta_1 + P_2 \Delta_2\right]$$

$$= \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_5}{1 + G_1 H_1 + G_3 H_2 + G_1 G_2 G_3 H_1 + H_4 + G_1 H_1 G_3 H_2 + G_1 H_1 H_4 + G_3 H_2 H_4 + G_1 G_2 G_3 H_3 H_4 + G_1 H_1 G_3 H_2 H_4}$$

(Ans)//

Q4.



No of forward paths & gain -

$P_1 = -G_2H_2G_1$

$P_2 = H_3H_2G_1$

$P_3 = G_4$

No of Loop & gain -

$L_1 = -G_2H_2$

$L_2 = -G_4H_1$

$L_3 = -G_1H_4$

$L_4 = G_2H_2G_1H_1$

$L_5 = -H_3H_2G_1H_1$

No of two non-touching loop - 1

$L_1L_2 = G_2G_4H_1H_2$

$\Delta_1 = P_1 = 1$

$P_2 \longrightarrow \Delta_2 = 1$

$P_3 \longrightarrow \Delta_3 = 1 + G_3H_2$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + L_1 L_2$$

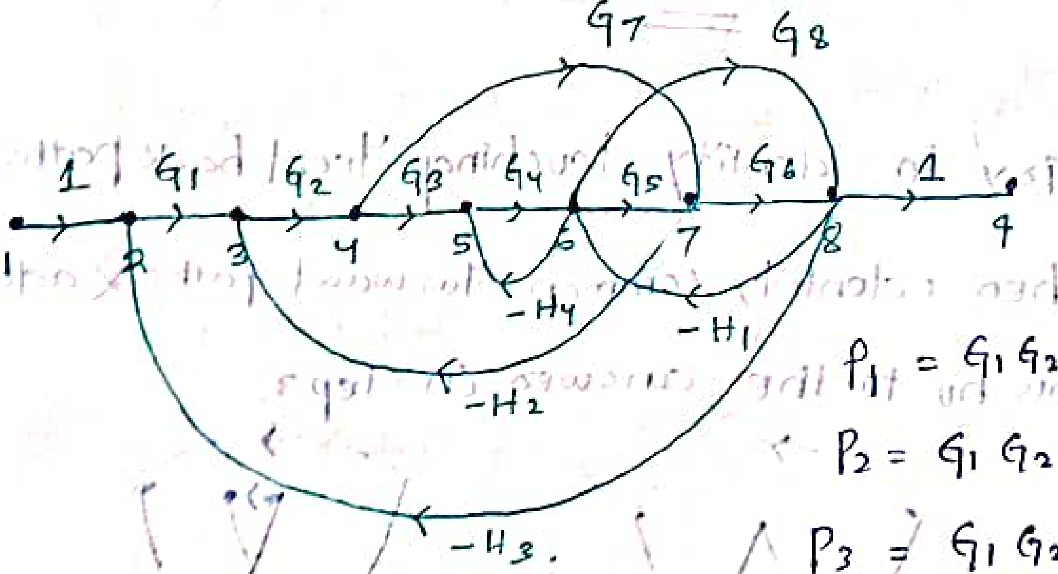
$$= 1 + G_3 H_2 + G_4 H_1 + G_1 H_4 - G_2 H_2 G_1 H_1 + H_3 H_2 G_1 H_1 + G_3 H_2 G_4 H_1$$

$$T(s) = \frac{1}{\Delta}\left(P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3\right)$$

$$= \frac{-G_2 H_2 G_1 + H_3 H_2 G_1 + G_4 + G_3 G_4 H_2}{1 + G_3 H_2 + G_4 H_1 + G_1 H_4 - G_2 H_2 G_1 H_1 + H_3 H_2 G_1 H_1 + G_3 H_2 G_4 H_1}$$

(Ans)

Q^n// In the following graph find the no of fwd path & no of individual loops.



$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$

$P_2 = G_1 G_2 G_7 G_6$

$P_3 = G_1 G_2 G_3 G_4 G_8.$

procedure to find individual loop:-

Step-1:- Identify the no of feed back paths

$-H_1$

$-H_2$

$-H_3$

$-H_4$

Step-2 Identify the no of fwd path for each feed back path.

$-H_1 \longrightarrow 2$

$-H_2 \longrightarrow 2$

$-H_3 \longrightarrow 3$

$-H_4 \rightarrow 1$

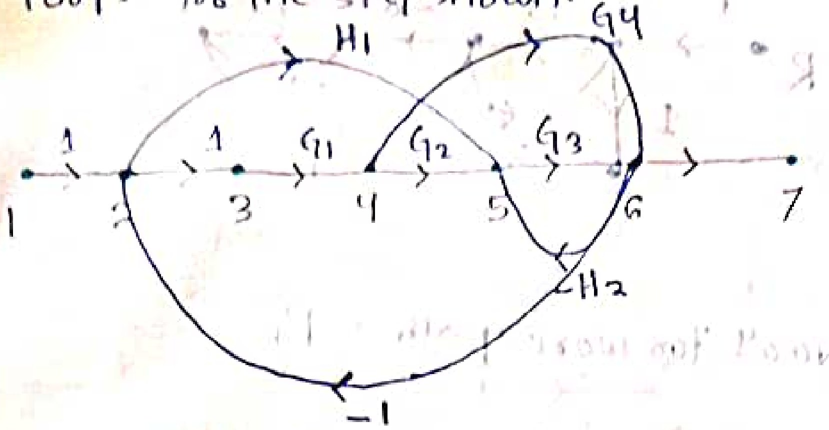
Step-3 Add all the no of forward path to get total no of individual loops.

Ans: 8

Step-4 Try to identify touching feed back paths then identify common forward paths & add this no to the answer in step 3.

(Touching feed back loops)

Q) calculate the no of fwd path, individual loops for the SFG shown.

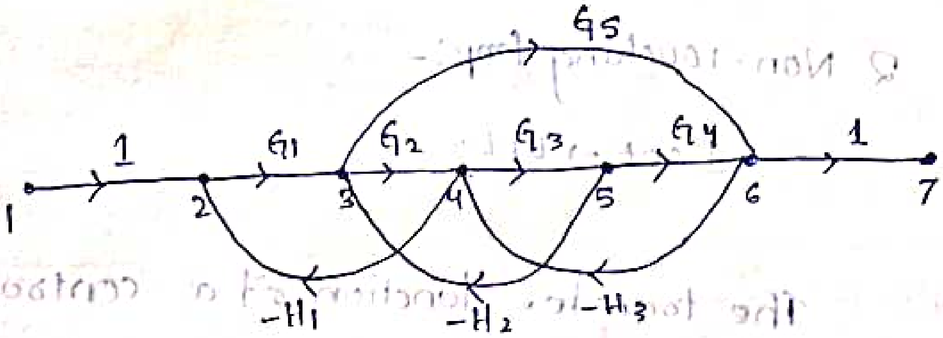


Forward path = 3

Individual loop → $-1 \rightarrow 3$

$-H_2 \rightarrow 1$

4 loops

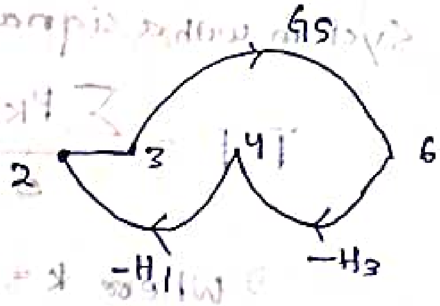


Fwd path = 2.

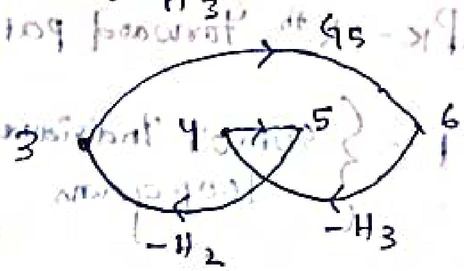
Individual loops → $-H_1$ − 1

$-H_2$ − 1

$-H_3$ − 1

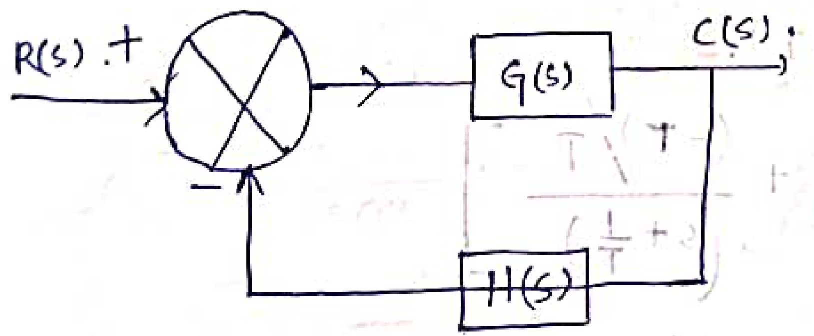


1·



Δ 1
1
1
3
+ 2 (Step-4)
5.

Time Response of first order system:-



derivation

$$G(s) = \frac{1}{sT}, \quad R(s) = 1/s \text{ (unit step input)}$$

$$U(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0. \end{cases}$$

$$H(s) = 1$$

$$T(s) = \frac{G(s)}{1 + G(s) \cdot H(s)} = \frac{1}{sT+1}$$

$$C(s) = R(s) \cdot T(s)$$

$$= \frac{1}{s} \times \left(\frac{1}{sT+1}\right)$$

$$= \frac{1}{s(sT+1)}$$

$$c(t) = L^{-1}\left[\frac{1}{s(sT+1)}\right]$$

$$c(t) = L^{-1}\left[\frac{A}{s} + \frac{B}{sT+1}\right]$$

$$\frac{1}{s(sT+1)} = \frac{A}{s} + \frac{B}{sT+1}$$

$$\Rightarrow 1 = A(sT+1) + Bs$$

$$s = 0, \quad 1 = A + 0$$

$$s = 1, \quad 1 = 1(T+1) + B$$

$$\Rightarrow B = (-T).$$

$$c(t) = L^{-1}\left[\frac{1}{s} + \frac{(-T)/T}{\left(s + \frac{1}{T}\right)}\right]$$

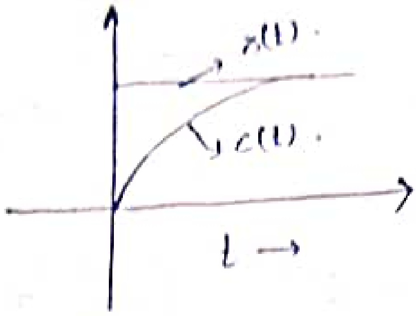
$$\Rightarrow c(t) = 1 - e^{-t/T}$$

$$e(t) = \overset{i/p}{r(t)} - \overset{o/p}{c(t)}.$$

$$= 1 - \left(1 - e^{-t/T}\right)$$

$$e(t) = e^{-t/T}$$

$$e_{ss}(t) = \lim_{t \to \infty} e^{-t/T} = 0$$

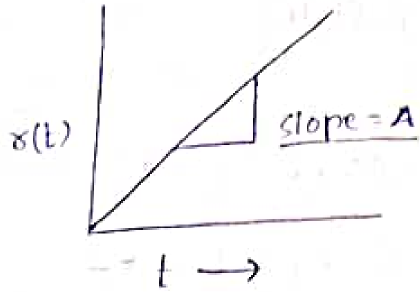


30 Jan, 2015 (PC) (Minchu)

Time-Responce of first order system to ramp input:-

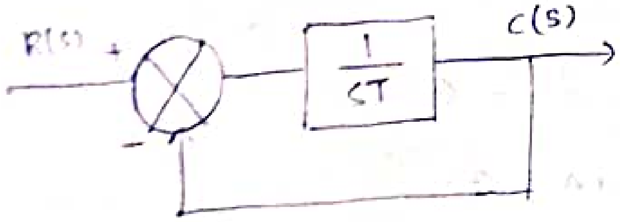
$$r(t) = At, \quad t \geq 0$$
$$= 0, \quad t < 0.$$

unit ramp function $A = 1$



$$r(t) = t, \quad t \geq 0$$
$$0, \quad t < 0$$

$$R(s) = 1/s^2$$



$$H(s) = 1$$

$$G(s) = 1/sT$$

$$\frac{C(s)}{R(s)} = T(s) = \frac{1}{sT + 1}$$

$$C(s) = R(s) \cdot T(s).$$

$C(s) = R(s) \cdot T(s)$

$= \frac{1}{s^2} \left(\frac{1}{sT+1} \right)$

$c(t) = L^{-1} \left[\frac{1}{s^2} \times \frac{1}{sT+1} \right]$

$= L^{-1} \left[\frac{As+B}{s^2} + \frac{C}{sT+1} \right]$

$\frac{1}{s^2(sT+1)} = \frac{As+B}{s^2} + \frac{C}{sT+1}$

$\Rightarrow (As+B)(sT+1) + Cs^2 = 1$

$s = 0, \quad B = 1$

$\cancel{s = 1 \quad A(T+1) + C = 1}$

$\cancel{s = -1 \quad -A(-T+1) + C = 1}$

$\cancel{(+) \quad (-) \quad (-)}$

$\cancel{A(T+1) + A(-T+1) = 0.}$

$\Rightarrow AT + A - AT + A = 0$

$\Rightarrow 2A =$

$1 = (A+1)(T+1) + C \quad \text{at } s = 1$

$1 = (-A+1)(-T+1) + C \quad \text{at } s = -1$

$A = -T$

$C = T^2.$

$$c(t) = L^{-1}\left[\frac{As+B}{s^2} + \frac{C}{sT+1}\right]$$

$$= L^{-1}\left[\frac{-Ts+1}{s^2} + \frac{T^2}{sT+1}\right]$$

$$= L^{-1}\left[-\frac{T}{s} + \frac{1}{s^2} + \frac{T}{s+1/T}\right]$$

$$= -T + t + Te^{-t/T}$$

error $= e(t) = r(t) - c(t)$

$$= t - \left[-T + t + Te^{-t/T}\right]$$

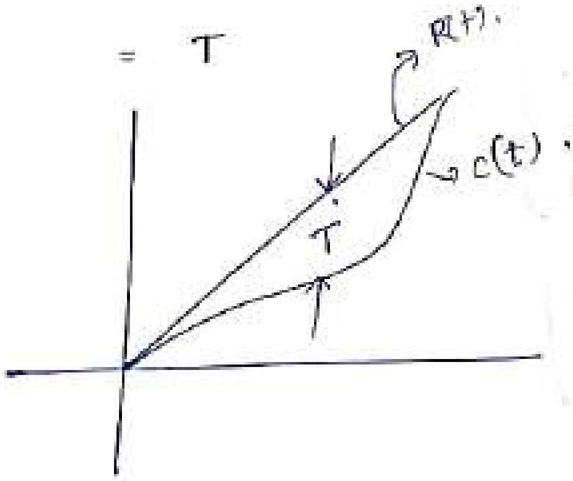
$$= t + T - t - Te^{-t/T}$$

$$= T\left(1 - e^{-t/T}\right)$$

$$e_{ss}(t) = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty} T\left(1 - e^{-t/T}\right)$$

$$= T$$



Time response of a 1st order system to unit impulse i/p:-

impulse, $\delta(t) = A$ when $t = 0$

$= 0, \ t < 0$

$= 0, \ t > 0.$

when $A = 1$, $\delta(t) = 1 \rightarrow$ unit impulse function.

$$\delta(t) = \frac{d}{dt}\left[u(t)\right]$$

$$\delta(s) = s \times 1/s = 1$$

$$G(s) = 1/sT$$

$$H(s) = 1$$

$$\frac{C(s)}{R(s)} = T(s) = \frac{1}{sT+1}$$

$$C(s) = R(s) \cdot T(s).$$

$$= 1 \cdot \left(\frac{1}{sT+1}\right)$$

$$c(t) = L^{-1}\left(\frac{1}{sT+1}\right)$$

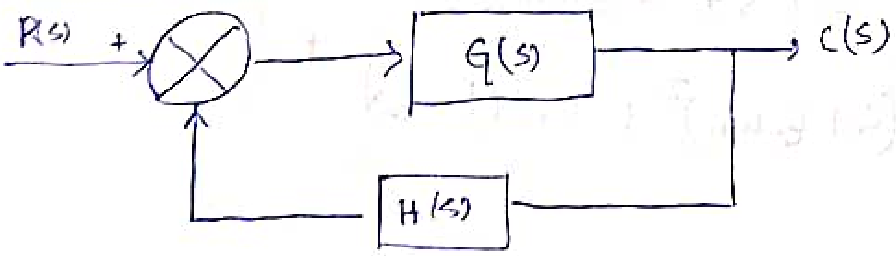
$$= L^{-1}\left[\frac{1}{T\left(s + 1/T\right)}\right]$$

$$= L^{-1}\left[\frac{1/T}{s + 1/T}\right]$$

$$= \frac{e^{-t/T}}{T}$$

Time - Response of a ~~signal~~ second order system to

Unit step i/p :-



$$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$ undamped ce

$$H(s) = 1$$

$$T(s) = \frac{G(s)}{1+G(s)\cdot H(s)}$$

$$= \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}$$

$$\boxed{T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

$$C(s) = R(s)\,T(s)$$

$$s^2+2\xi\omega_n s+\omega_n^2 = (s^2+2\xi\omega_n s+\xi^2\omega_n^2 - \xi^2\omega_n + \omega_n^2)$$

$$= (s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)$$

$$C(s) = \frac{1}{s}\cdot\frac{\omega_n^2}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi)^2}$$

$$= \frac{-(s+2\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)} + \frac{1}{s}$$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

31 Jan, 2015 (RS) (15th class)

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2 + (\omega_n\sqrt{1-\xi^2})^2} - \frac{\xi\omega_n\,\omega_d}{\omega_d\left\{(s+\omega_d\cdot s)^2 + (\omega_n\sqrt{1-s^2})^2\right\}}$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{s+\xi\omega_n}{\{(s+\xi\omega_n)^2 + \omega_d^2\}} - \frac{\xi\omega_n\omega_d}{(\omega_n\sqrt{1-\xi^2})\{(s+\xi\omega_n)^2 + \omega_d^2\}}\right.$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi}}(\sin\omega_d t)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cdot\cos\omega_d t + \xi\sin\omega_d t\right]$$

if $\xi = \cos\varphi$, $\sqrt{1-\xi^2} = \sin\varphi$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left\{\sin\varphi\cdot\cos\omega_d t + \cos\varphi\cdot\sin\omega_d t\right\}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \varphi) \qquad \boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$$

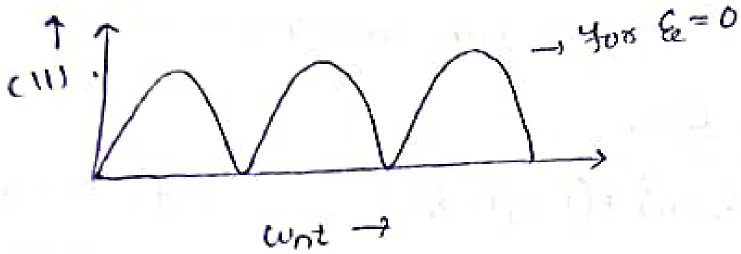
when $\xi = 0$, $\cos\varphi = 0$

$\Rightarrow \varphi = 90°$

undamped system.

$$c(t) = 1 - \frac{e^{-0t}}{\sqrt{1-0}} \sin(\omega_n t + 90)$$

$$= (1 - \cos \omega_n t)$$



Case-II

When $\xi \to 1$

$$\lim_{\theta \to 0} \sin\theta = 0$$

$$\lim_{\theta \to 0} \cos\theta = 1$$

$$\lim_{\xi \to 1} c(t) = \lim_{\xi \to 1} \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n \sqrt{1-\xi^2}\, t + \phi\right)\right]$$

$$= \lim_{\xi \to 1} \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \left\{ \sin \omega_n \sqrt{1-\xi^2}\, t \cdot \cos\phi + \sin\phi \cdot \cos \omega_n \sqrt{1-\xi^2}\, t \right\}\right]$$

$$= \lim_{\xi \to 1} \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \left\{ \sin \omega_n \sqrt{1-\xi^2}\, t \cdot \xi + \sqrt{1-\xi^2} \cdot \cos \omega_n \sqrt{1-\xi^2}\, t \right\}\right]$$

$$\lim_{\xi \to 1} \sin \omega_n \sqrt{1-\xi^2}\, t = \sin \omega_n t$$

$$\lim_{\theta \to 0} \sin\theta = \theta$$

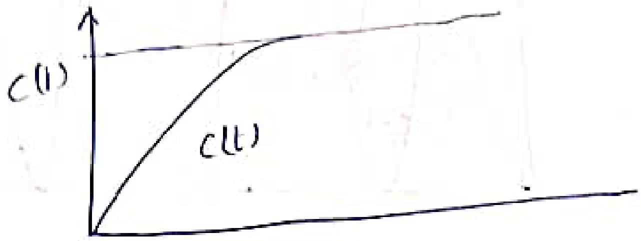
$$\lim_{\theta \to 0} \cos\theta = 1$$

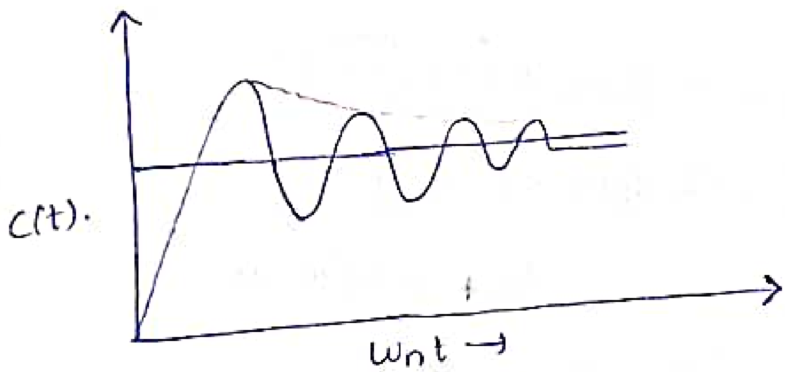
$$\lim_{\xi \to 1} \cos \omega_n \sqrt{1-\xi^2}\, t = 1$$

$$c(t) = \lim_{\xi_0 \to 1} \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi_0^2}} \left\{ \omega_d t \cdot \xi_0 + \sqrt{1-\xi_0^2} \cdot 1 \right\} \right]$$

$$= \lim_{\xi_0 \to 1} \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi_0^2}} \left\{ \omega_n \sqrt{1-\xi_0^2}\, t \cdot \xi_0 + \sqrt{1-\xi_0^2} \right\} \right]$$

$$c(t) = \left[1 - e^{-\xi_0 \omega_n t} \left(\xi_0 \omega_n t + 1 \right) \right]$$



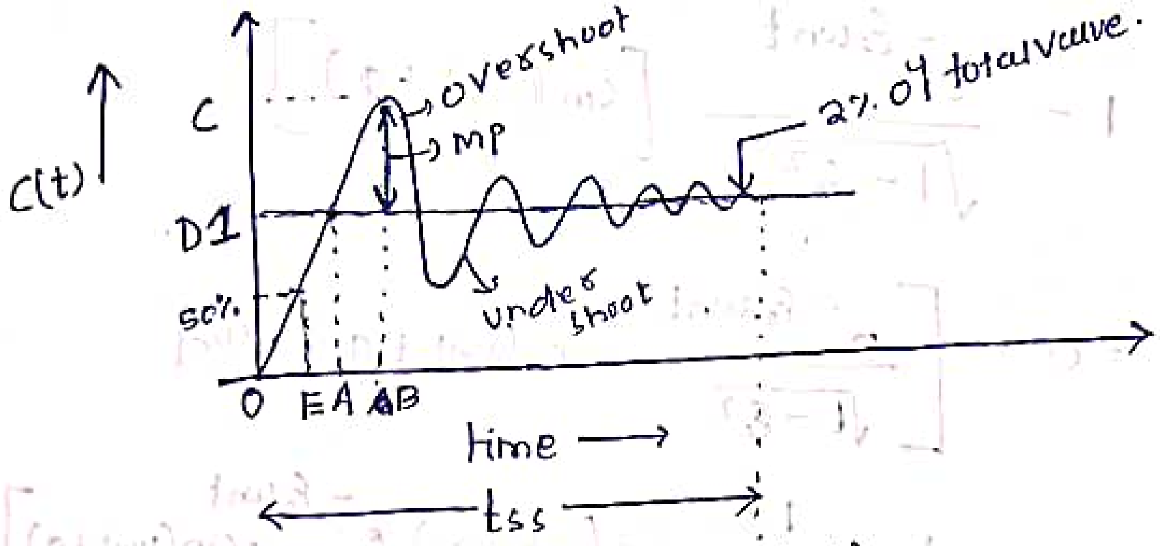


$$c(t) = 1 - \frac{e^{-\xi_0 \omega_n t}}{\sqrt{1-\xi_0^2}} \sin(\omega_d t + \phi)$$

When $\xi_0 < 1$, the system is underdamped system & the respose is exponentially decreasing sinusoidal function.

→ When $\xi_0 = 0$, the system is undamped system, & the response is $c(t) = 1 - \cos\omega_n t$. The responce is oscillating in nature.

Time Response of second order system to step input :-



overshoot :- (Mp) (max^m peak overshoot)

- The max^m positive peak deviation from the desired value at very 1st instance. [CD]

peak Time (tp) :- The time needed to reach the max^m peak is called peak time. (OB).

Rise time (t_r) :-

The time needed to reach 100% of the final value or desired value is called the rise time. (OA).

Delay time :-

The time needed to reach 50% of the desired value is called the delay time t_d. (OE).

Settling time :-

t_{ss} - The time required to reach 2-5% of the final value.

Time domain specification :-

t_p = slop at point (peak) C

$$\frac{dc(t)}{dt} = 0$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \varphi)\right]$$

$$\frac{dc(t)}{dt} = 0 - \left[\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \cos(\omega_d t + \varphi) \cdot \omega_d + \frac{1}{\sqrt{1-\xi^2}} \cdot (-\xi\omega_n)\, e^{-\xi\omega_n t} \cdot \sin(\omega_d t + \varphi)\right]$$

$$\left[\because \frac{dc(t)}{dt} = 0\right]$$

$$\Rightarrow \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \cos(\omega_d t + \varphi)\, \omega_d = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \varphi)\,(\xi\omega_n)$$

$$\Rightarrow \frac{\cos(\omega_d t + \varphi)}{\sin(\omega_d t + \varphi)} = \frac{\xi\omega_n}{\omega_d} \qquad \cos\varphi = \xi, \quad \sin\varphi = \sqrt{1-\xi^2}$$

$$\Rightarrow \cot(\omega_d t + \varphi) = \cot\varphi$$

$$\tan(\omega_d t + \varphi) = \tan\varphi$$

$w_d t = n\pi$

At very first instant $n = 1$

$w_d t_p = \pi$

$$\Rightarrow \boxed{t_p = \frac{\pi}{w_d}}$$

$$c(t_p) = 1 - \frac{e^{-\xi w_n \times \frac{\pi}{w_d}}}{\sqrt{1-\xi^2}} \cdot \left[\sin\left(w_d \times \frac{\pi}{w_d} + \varphi\right)\right]$$

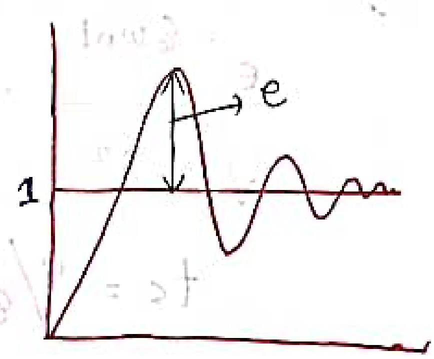
$$= 1 - \frac{e^{-\xi w_n \frac{\pi}{w_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot \left[\sin(\pi + \varphi)\right]$$

$$= 1 + \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} (\sin\varphi)$$

$$= 1 + \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \sqrt{1-\xi^2}$$

$$= 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$= 1 + e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$



$$\% M_p = \left[\frac{c(t_p) - 1}{1}\right] \times 100$$

(percentage ~~rate in~~ max^m peak overshoot)

$$\% M_p = e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)} \cdot \times 100$$

<u>t_r :-</u>

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

At t_r, $c(t) = 1$.

$$\sin(\omega_d t + \phi) = 0$$

$$= \sin n\pi$$

$$= \sin \pi \quad (\text{when } n = 1)$$

$$\omega_d t + \phi = \pi$$

$$t_r = \left(\frac{\pi - \phi}{\omega_d}\right)$$

<u>Settling time (t_{ss}) :-</u>

$$\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} = 0.02$$

$$t_s = 4/\xi\omega_n.$$

$$\ln e^{-\xi \omega_n t} = \ln\left[(0.02)\sqrt{1-\xi^2}\right]$$

$$-\xi\omega_n t = \ln\left[(0.02)\sqrt{1-\xi^2}\right]$$

$$\xi\omega_n t = \ln\left[0.02\sqrt{1-\xi^2}\right]$$

$$\Rightarrow \xi\omega_n t = 4$$

$$\rightarrow \boxed{t_s = \frac{4}{\xi\omega_n}.}$$

for 5% $\rightarrow$ $\boxed{t_s = 3/\xi\omega_n}$

Date: 5, Feb, 2015 (17th class) (RS)

Steady state Error:-

$$e_{ss} = \lim_{t\to\infty} e(t).$$

$$= \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{s \times R(s)}{[1 + G(s)H(s)]}$$

Q11 A unity feedback system is characterised by an open loop transfer function -

$$G(s) = \frac{K}{s(s+10)}$$

determine the gain K so that the system yield will have a damping ratio of 0.5 & for this value of 'k' determine t_p, M_p, & max^m positive deviation in percentage, for a unit step i/p.

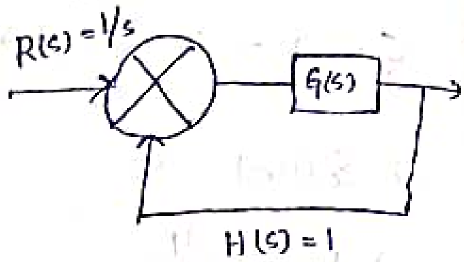
Sol^n :-

Because of unit step function,

$$R(s) = 1/s.$$

$$H(s) = 1$$

$$\xi = 0.5$$



$$T(s) = \frac{G(s)}{1 + G(s) \cdot H(s)}.$$

$$= \frac{\frac{K}{s(s+10)}}{1 + \frac{K}{s(s+10)} \times 1}$$

$$= \frac{K}{s(s+10) + K}$$

$$= \frac{K}{s^2 + 10s + K}. \quad - (1)$$

Standard form of 2nd order transfer function -

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad - (2)$$

$$\boxed{\text{damping ratio} = \frac{\text{Actual damping}}{\text{Critical damping}}}$$ (1 always)

Equating coefficient of 's' we get →

$$2\xi\omega_n = 10$$

$$\Rightarrow 2 \times 0.5 \times \omega_n = 10$$

$$\Rightarrow \boxed{\omega_n = 10}$$

$$wd = w_n\sqrt{1-\xi^2}$$

$$= 10\sqrt{1-0.5^2}$$

$$= 8.66$$

$$t_p = (\text{peak time}) = \frac{\pi}{wd}$$

$$= \frac{\pi}{8.66}$$

$$= 0.362 \text{ sec.}$$

$$M_P = e\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]$$

$$= e\left(\frac{-0.5\times\pi}{\sqrt{1-0.5^2}}\right)$$

$$= 0.1630$$

$$\% M_P = 16.3\%$$

comparing equation (1) with (2) we get →

$$K = w_n^2$$

$$= (10)^2$$

$$= 100. \text{ (Ans)}//.$$

Q. 2. The openloop transfer function of a unity feed back system is given by $G(s) = \frac{K}{s(sT+1)}$, where K & T are positive constants. By what factor should the amplifier gain be reduced so that the peak overshoot of a unit step responce of the system is reduced from 75% to 25%.

sol:-

$$G(s) = \frac{K}{s(sT+1)}$$

$$H(s) = 1$$

$$R(s) = 1/s$$

$$T(s) = \frac{G(s)}{1+G(s)\cdot H(s)}$$

$$= \frac{\frac{K}{s(sT+1)}}{1+\frac{K}{s(sT+1)}\times 1}$$

$$= \frac{K}{s(sT+1)+K} = \frac{K/T}{s^2+s/T+K/T}$$

standard form - $\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$2\xi\omega_n = 1/T \Rightarrow \omega_n = \dfrac{1}{2\xi T} \Rightarrow \omega_n^2 = \dfrac{1}{4\xi^2T^2}$ — ①

$\omega_n^2 = K/T$

$\Rightarrow \omega_n^2 = K/T$ — ②

$$\omega_n = \sqrt{K/T}$$

$$\Rightarrow \xi_e = \frac{1}{2\omega_n T}$$

$$= \frac{1}{2\sqrt{\frac{K}{T}} \cdot T}$$

$$\Rightarrow \xi_e = \frac{1}{2\sqrt{KT}}$$

$$MP_1 = 0.75$$

$$= e^{\frac{-\xi_{e1}\pi}{\sqrt{1-\xi_{e1}^2}}}$$

$$MP_2 = 0.25$$

$$= e^{\frac{-\xi_{e2}\pi}{\sqrt{1-\xi_e^2}}}$$

$$MP_1 = 0.75$$

$$\Rightarrow e^{\frac{-\xi_1\pi}{\sqrt{1-\xi_1^2}}} = 0.75$$

$$\Rightarrow -\frac{\xi_1\pi}{\sqrt{1-\xi_1^2}} = \ln(0.75)$$

$$\Rightarrow \frac{-\xi_1}{\sqrt{1-\xi_1^2}} = \frac{\ln(0.75)}{\pi}$$

$$\Rightarrow \frac{\xi_1}{\sqrt{1-\xi_{e1}^2}} = 0.0915$$

$$\Rightarrow \xi_1^2 = (1-\xi_1^2)\, 0.0915$$

$$\Rightarrow 1.0915\,\xi_1^2 = 0.0915 \Rightarrow \xi_1 = 0.09.$$

Similarly $\xi_2 = 0.403.$

$$\xi \propto \frac{1}{\sqrt{KT}}$$

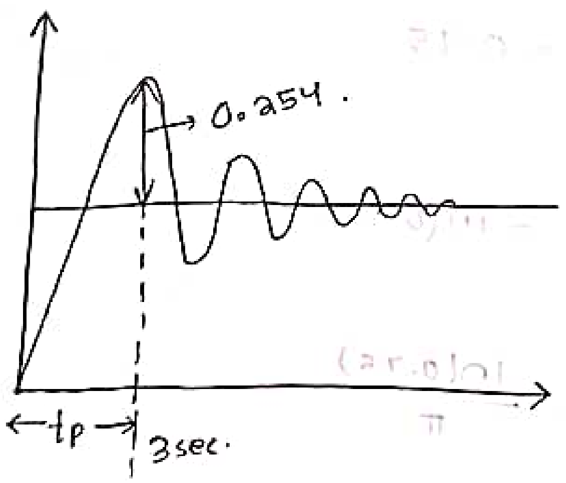
$\Rightarrow \frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}}$ (T = is constant for each case)

$$\Rightarrow \left(\frac{\xi_1}{\xi_2}\right)^2 = \frac{K_2}{K_1}$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{\xi_2}{\xi_1}\right)^2 = 20.05 \text{ (Ans)}$$

→ ~~K1~~

Q. 3. A unity feedback system having forward transfer function $G(s) = \frac{K}{s(sT+1)}$ is subjected to a unit step input determine the value of K & T from the output response curve



$$T(s) = \frac{K/T}{s^2 + s/T + K/T} \quad (\text{prev } Q^n)$$

$$\text{Standard form} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$\Rightarrow 3 = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow 0.254 = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow \ln(0.254) = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow -1.3704 = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \frac{\xi}{\sqrt{1-\xi^2}} = \frac{1.3704}{\pi}$$

$$\Rightarrow \xi^2 = (1-\xi^2)\,0.4362$$

$$\Rightarrow \left(1 + \overset{0.19027}{\cancel{0.4362}}\right)\xi^2 = \cancel{0.4362}\ 0.19027$$

$$\cancel{\Rightarrow \xi^2 = \frac{0.4362}{1.4362}}$$

$$\cancel{\Rightarrow \xi = 0.5511}$$

$$\Rightarrow \boxed{\xi = 0.4}$$

$$\Rightarrow \omega_n = \frac{\pi}{3\sqrt{1-\xi^2}}$$

$$= \frac{\pi}{3\sqrt{1-(0.4)^2}} = 1.142 \text{ rad/sec.}$$

standard form -

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{(1.142)^2}{s^2 + 2 \times 0.4 \times 1.142 \times s + (1.142)^2}$$

$$= \frac{1.30416}{s^2 + 0.9136\, s + 1.30416}$$

$$1/T = 2\xi\omega_n$$

$$\Rightarrow 1/T = 0.9136$$

$$\Rightarrow T = 1/0.9136 = \underline{\underline{1.094}}\ \cancel{sec}\ \underline{sec}$$

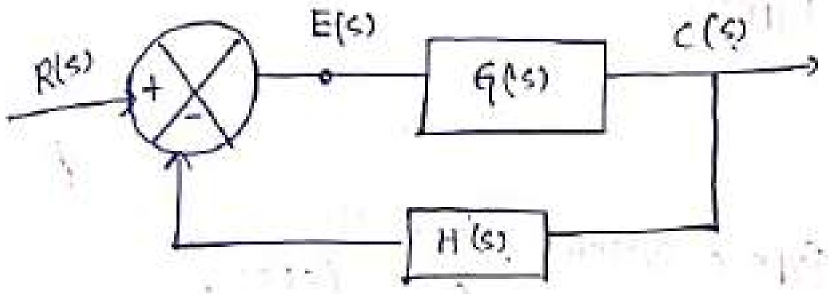
$$\omega_n^2 = K/T$$

$$\Rightarrow 1.30416 = \frac{K}{1.094}$$

$$\Rightarrow K = \underline{\underline{1.4267}}\ (Ans).$$

(or)(10th class)

Steady state error:-



$$T(s) = \frac{C(s)}{R(s)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s).H(s)} \Rightarrow C(s) = \frac{G(s).\ R(s)}{1 + G(s).H(s)}$$

$$E(s).\ G(s) = C(s).$$

$$\Rightarrow E(s).\ G(s) = \frac{R(s).\ G(s)}{1 + G(s).H(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s).H(s)}$$

$$e_{ss}(s) = \lim_{s \to 0} s\,E(s)$$

$$= \lim_{s \to 0} \left[\frac{s \times R(s)}{1 + G(s).H(s)}\right]$$

For unity feedback, $H(s) = 1$

$$e_{ss}(s) = \lim_{s \to 0} \frac{s\ R(s)}{1 + G(s)}$$

$$G(s) = \frac{K(s+z_1)(s+z_2)\ldots\ldots}{s^n(s+p_1)(s+p_2)\ldots\ldots}$$

where n = type of the system.

$n = 0$, type 0

$n = 1$, type 1

$n = 2$, type 2.

Q: Find out the steady state error for a type-0 system

e.g of Type-0 system. $\dfrac{2(s+1)}{s^0(s+4)(s+5)}$.

→ Type of a system define no of poles at origin of s-plane.

For type-0 system →

$$G(s) = \frac{K(s+z_1)\ldots}{(s+P_1)\ldots}$$

For step i/p, $R(s) = 1/s$

For type-0 system,

$$e_{ss} = \lim_{s \to 0} \left[\frac{s \cdot R(s)}{1 + G(s) \cdot H(s)}\right]$$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{s \times 1/s}{1 + K_p} = \frac{1}{1+K_p}}$$

K_p = position error constant.

$$\boxed{K_p = \lim_{s \to 0} G(s) \cdot H(s)}$$

→ Type-0 system, ramp input →

$$R(s) = 1/s^2 .$$

$$\lim_{s \to 0} \frac{s \times 1/s^2}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s(1 + G(s) \cdot H(s))}$$

$$\boxed{e_{ss} = \frac{1}{K_v}}$$

$$\boxed{K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s) .}$$

velocity error constant

→ Type-0 system, unit parabolic input →

$$R(s) = \frac{1}{2} \frac{2!}{s^{2+1}}$$

$$= 1/s^3 .$$

$$e_{ss} = \lim_{s \to 0} \frac{s \times 1/s^3}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2(1 + G(s) \cdot H(s))}$$

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

$$K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s) .$$

acceleration error constant.

For unity feedback system,

For a unit step (type 0)

Error constants:

$$K_p = \lim_{s \to 0} G(s). \qquad e_{ss} = \frac{1}{1+K_p}$$

$$K_v = \lim_{s \to 0} s\,G(s) \qquad e_{ss} = \frac{1}{K_v} \text{ (ramp)}$$

$$K_a = \lim_{s \to 0} s^2 G(s) \qquad e_{ss} = \frac{1}{K_a} \text{ (parabolic)}$$

→ Q. What is the error type-1 system for unit step i/p.

For type-1:-

$$G(s) = \frac{K(s+z_1)\ldots}{s(s+p_1)(s+p_2)\ldots}$$

For unit step i/p, $R(s) = 1/s$.

$$e_{ss}(s) = \lim_{s \to 0} \frac{s \times 1/s}{1 + G(s).H(s)}.$$

at $H(s) = 1$ (for unity feedback)

$$e_{ss}(s) = \lim_{s \to 0} \frac{1}{1+G(s)}.$$

$$= 1/\infty$$

$$\boxed{e_{ss}(s) = 0.}$$

For ramp i/p - $R(s) = 1/s^2$.

$$e_{ss}(s) = \lim_{s \to 0} \frac{s \times 1/s^2}{1 + G(s).H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s\left(1 + G(s)\cdot H(s)\right)}$$

$$= \lim_{s \to 0} \frac{1}{s + s\cdot G(s)\cdot H(s)}$$

$$= \frac{1}{K_v}$$

$$\boxed{K_v = \lim_{s \to 0} s\, G(s)\cdot H(s)}$$

For unit parabolic i/p:-

$$R(s) = 1/s^3$$

$$E_{ss} = \lim_{s \to 0} \frac{s \times R(s)}{1 + G(s)\cdot H(s)}$$

$$= \lim_{s \to 0} \frac{s \times 1/s^3}{1 + G(s)\cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)\cdot H(s)}$$

$$= 1/0 \quad \text{as} \quad s^2 \times G(s) = s^2 \times \frac{K(s+z)}{s(s+P_1)(s+P_2)}$$

$$= s\, G(s)$$

$$\text{so} \lim_{s \to 0} s\cdot G(s) = 0.$$

$$= \infty$$

$$\boxed{\text{as} \lim_{s \to 0} s^2 G(s)\cdot H(s) = 0}$$

For type-2:-

$$G(s) = \frac{K(s+z_1)\ldots}{s^2(s+p_1)(s+p_2)\ldots}$$

For step i/p

$$e_{ss} = \lim_{s\to 0} \frac{R(s)\cdot s}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{1/s \cdot s}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{1}{1+G(s)\cdot H(s)}$$

$$= 0$$

$$\boxed{e_{ss} = 0}$$

For ramp i/p

$$e_{ss} = \lim_{s\to 0} \frac{1/s^2 \cdot s}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{1}{s+s\cdot G(s)\cdot H(s)}$$

$$= 1/\infty$$

$$= 0.$$

$$\boxed{e_{ss} = 0}$$

For unit parabolic i/p

$$e_{ss} = \lim_{s\to 0} \frac{1/s^3 \cdot s}{1+G(s)\cdot H(s)}$$

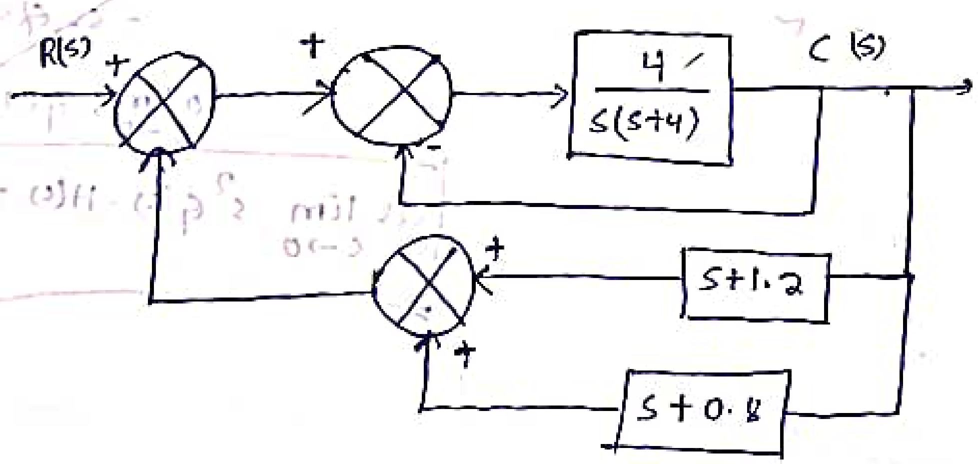
$$= \lim_{s\to 0} \frac{1}{s^2+s^2 G(s)\ldots}$$

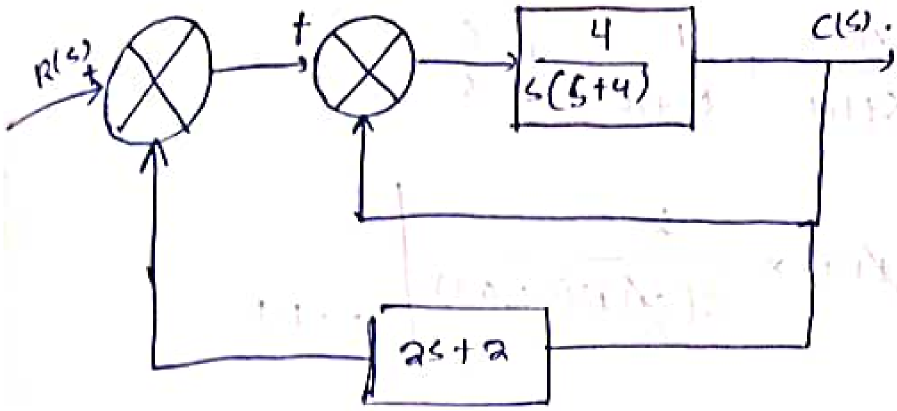
$$= \frac{1}{K_a}$$

$$\boxed{e_{ss} = 1/K_a}$$

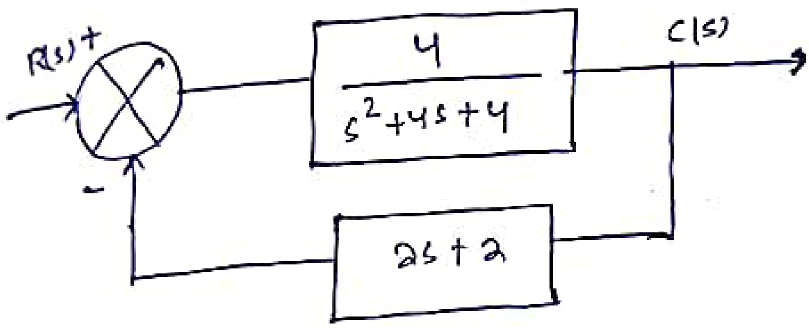
Date - 10 Feb, 2015 (20th class)

1. A control system shown in the figure determine the transfer function & derive an expression relating the o/p & time. If the i/p is a step having a magnitude of 2N





$$\frac{\frac{4}{s(s+4)}}{1+\frac{4}{s(s+4)}} = \frac{4}{s^2+4s+4}$$



$$T(s) = \frac{\frac{4}{s^2+4s+4}}{1+\frac{4}{s^2+4s+4}\times 2s+2}$$

$$= \frac{4}{s^2+12s+12}.$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2+12s+12}$$

$$\Rightarrow C(s) = T(s)\cdot R(s).$$

$$= \frac{4}{s^2+12s+12} \times \frac{2}{s}.$$

$$C(t) = \frac{A}{S+1.1} + \frac{B}{S+10.9} + \frac{C}{S}$$

$$A = (s+1.1) \times \frac{8}{s(s+1.1)(s+10.9)}\Bigg|_{s=-1.1}$$

$$= \frac{8}{(-1.1)(-1.1+10.9)}$$

$$=$$

$$C(t) = Ae^{-1.1t} + Be^{-10.9t} + C.$$

Q11 In an open loop system,

$$G(s) \cdot H(s) = \frac{2(s^2+3s+20)}{s(s+2)(s^2+4s+10)}$$

Determine the static error constant & steady state error for the i/p

(a) 5
(b) 4t
(c) $4t^2/2$

(a). $K_p = \lim_{s\to 0} G(s)\cdot H(s)$

$$= \lim_{s\to 0} \frac{2(s^2+3s+20)}{s(s+2)(s^2+4s+10)}$$

$$= \infty$$

$R(s) = 5/s$

$$e_{ss} = \frac{5}{1+K_p} = \frac{5}{1+\infty} = 5/\infty = 0.$$

b) <u>For Kv</u>

$$K_v = \lim_{s\to 0} s\cdot G(s)\cdot H(s)$$

$$= \lim_{s\to 0} \frac{\cancel{s}\cdot 2(s^2+3s+20)}{\cancel{s}\cdot(s+2)(s^2+4s+10)}$$

$$= \frac{40}{20}$$

$$= 2.$$

$R(s) = 4/s^2$

as $\lim_{s\to 0} \frac{s\cdot R(s)}{1+G(s)\cdot H(s)}$

$$= \lim_{s\to 0} \frac{s\cdot 4/s^2}{1+G(s)\cdot H(s)}.$$

$$e_{ss} = 4/K_v = 1/2 = 0.5.$$

c)

$$K_a = \lim_{s\to 0} s^2\, G(s)\cdot H(s).$$

$$= \lim_{s\to 0} \frac{\cancel{s^2}\times 2(s^2+3s+20)}{\cancel{s}(s+2)(s^2+4s+10)}$$

$R(s) = 4/s^3.$

$$= 0$$

$$e_{ss} = 4/K_a = 4/0 = \infty.$$

Q:- The open loop transfer function of a servo system with unity feedback is $G(s) = \frac{10}{s(0.1s+1)}$

Evaluate the static error coefficient for the system & obtain the steady state error of the system when subjected to an i/p given by the polynomial ~~eq~~

$$r(t) = a_0 + a_1 t + \frac{a_2}{2} t^2$$

Sol:-

Unity feedback:- $H(s) = 1$

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$= \lim_{s \to 0} \frac{10}{s(0.1s+1)}$$

$$= \infty.$$

$$e_{ss} = \frac{a_0}{1+K_p} = \frac{a_0}{\infty} = 0$$

$$K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s \frac{10}{s(0.1s+1)}$$

$$= \frac{10}{1} = 10.$$

$$e_{ss} = \frac{a_1}{K_v} = \frac{a_1}{10}.$$

$$K_a = \lim_{s \to 0} s^2 \cdot G(s) \cdot H(s)$$

$$= \lim_{s \to 0} \frac{s \, 10}{(0.1s+1)}$$

$$= \infty$$

$$e_{ss} = \frac{a_2}{K_a}$$

$$= 0.$$

$$(e_{ss})_{\text{Total error}} = e_{ss_1} + e_{ss_2} + e_{ss_3}$$

$$= \infty + a_1/10 + 0$$

$$= \infty. \quad \text{(Ans)//}$$

Q) The closed loop transfer function of a unity feedback control system is given as,

$$\frac{C(s)}{R(s)} = \frac{Ks+\beta}{s^2+\alpha s+\beta}.$$

Determine the steady state error for unit ramp i/p.

Soln:- Unity feed back system $H(s)=1$

$$\text{Error } E(s) = \frac{R(s)}{1+G(s)\cdot H(s)}.$$

$$T(s) = \frac{G(s)}{1+G(s)\cdot H(s)}.$$

$$\Rightarrow \frac{G(s)}{1+G(s)} = \frac{Ks+\beta}{s^2+\alpha s+\beta}.$$

$$\Rightarrow G(s)\left[s^2+\alpha s+\beta\right] = (ks+\beta)\left[1+G(s)\right]$$

$$\Rightarrow G(s)\left[s^2+\alpha s+\beta\right] = (ks+\beta) + G(s)(ks+\beta)$$

$$\Rightarrow G(s)\left[s^2+\alpha s+\beta-ks-\beta\right] = \left[ks+\beta\right]$$

$$\Rightarrow G(s) = \frac{ks+\beta}{s^2+s(\alpha-k)}$$

$$R(s) = 1/s^2 \text{ (unit ramp)}$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{s\times 1/s^2}{1+\frac{ks+\beta}{s^2+s(\alpha-k)}}$$

$$= \lim_{s\to 0} \frac{1/s}{\frac{s^2+s(\alpha-k)+ks+\beta}{s^2+s(\alpha-k)}}$$

$$= \lim_{s\to 0} \frac{1}{s}\times\frac{s^2+s(\alpha-k)}{s^2+s(\alpha-k)+ks+\beta}$$

$$= \lim_{s\to 0} \frac{s+(\alpha-k)}{s^2+s(\alpha-k)+ks+\beta}$$

$$= \frac{\alpha-k}{\beta} \quad \text{(Ans)}$$

ROOT LOCUS

- Root locus technique is used to determine the stability of closed loop control system from the knowledge of its openloop transfer function.

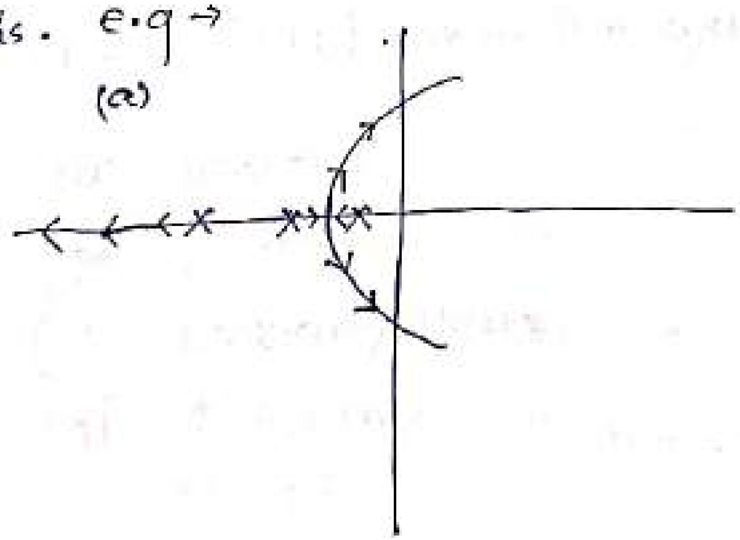
- Root locus is the path traversed by closed loop pole, when the system gain K is varied from zero to ∞.

- This concept is developed by W.R EVANS.

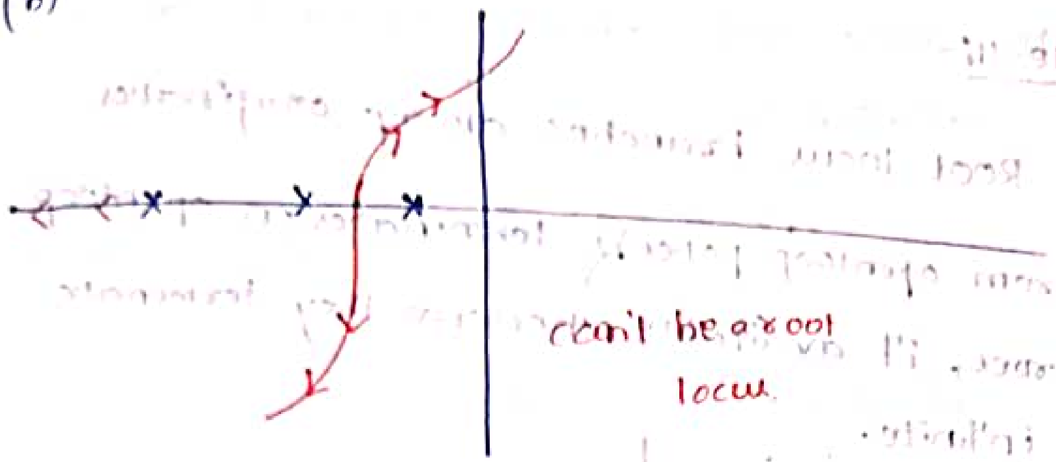
Rules for construction of Root locus:—

(1) Root locus is symmetrical with respect to real axis. e.g →

(a)



(b)



Rule - 2:-

The No. of Root locus branches is equal to the no. of openloop poles.

order of CE

$$e.g \quad G(s) = \frac{K(s+3)}{(s+1)(s+2)} \quad (\text{No branches } 2)$$

$$G(s) = \frac{K}{s(s+1)(s+3)} \quad (\text{No of branches } 3)$$

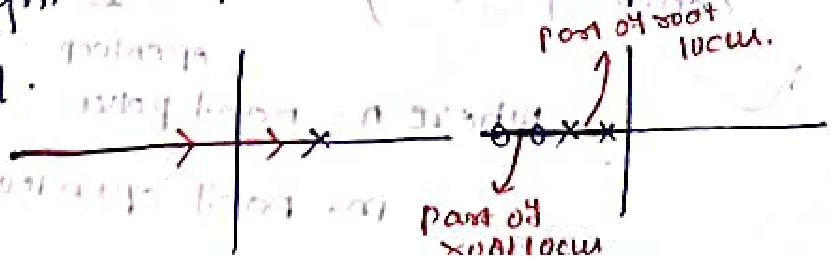
→ If zeroes are more than poles then no of root locus ~~branches~~ is equal to No of zeroes.

$$e.g - \frac{K(s+3)(s+4)}{s+1} \quad (\text{No of branches } 2)$$

[It is not practical system]

Rule-3:-

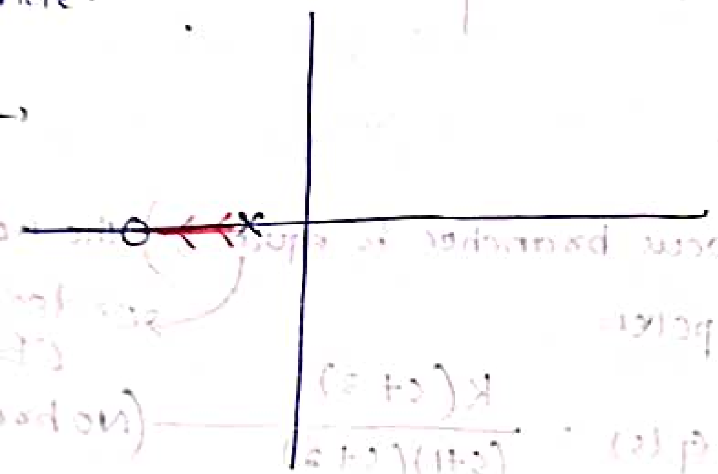
A point on real axis will exist on root locus if the total No of poles & zeroes to the right hand side of that point is odd.

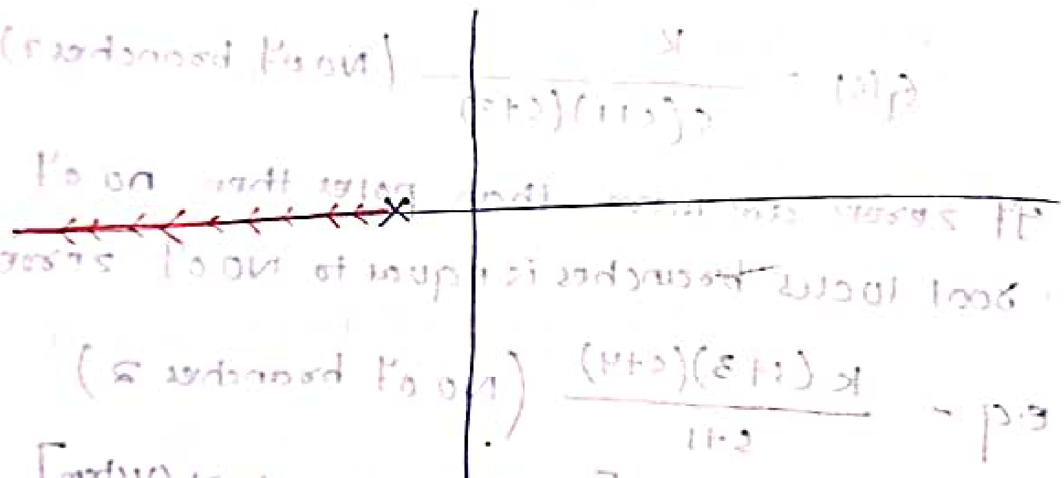


Rule-4:-

Root locus branches always originates from openloop poles & terminates on openloop zeroes, if available otherwise they terminate at infinite.

e.g ->





Rule-5:-

The branches terminating at infinite will follow the path shown by asymptotes.

$$\text{centroid of asymptotes} = \frac{\sum \text{Real part of poles} - \sum \text{Real parts of zeros}}{n-m}$$

where n - no of openloop poles

m - no of open loop zeroes.

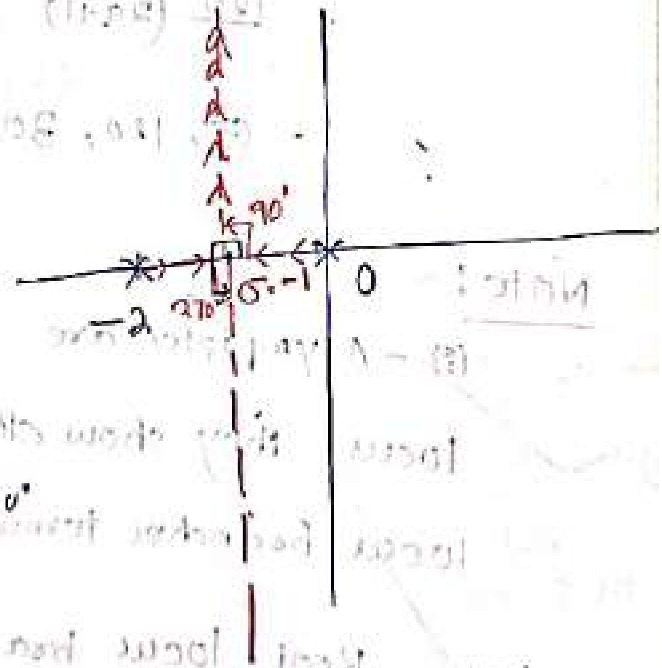
$n - m \rightarrow$ no of branches terminating at infinite.

Angle of Asymptotes $= \dfrac{180 \cdot (2q+1)}{n-m}$

$q = 0, 1, 2 \ldots (n-m-1)$

e.g. $G(s) = \dfrac{K}{s(s+2)}$

centroid $= \dfrac{[0+(-2)] - (0)}{2-0}$



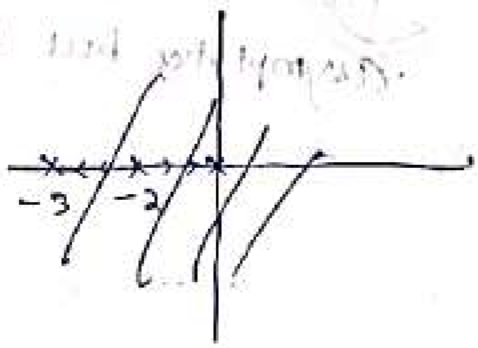
$A \cdot A = \dfrac{180(2q+1)}{n-m}$

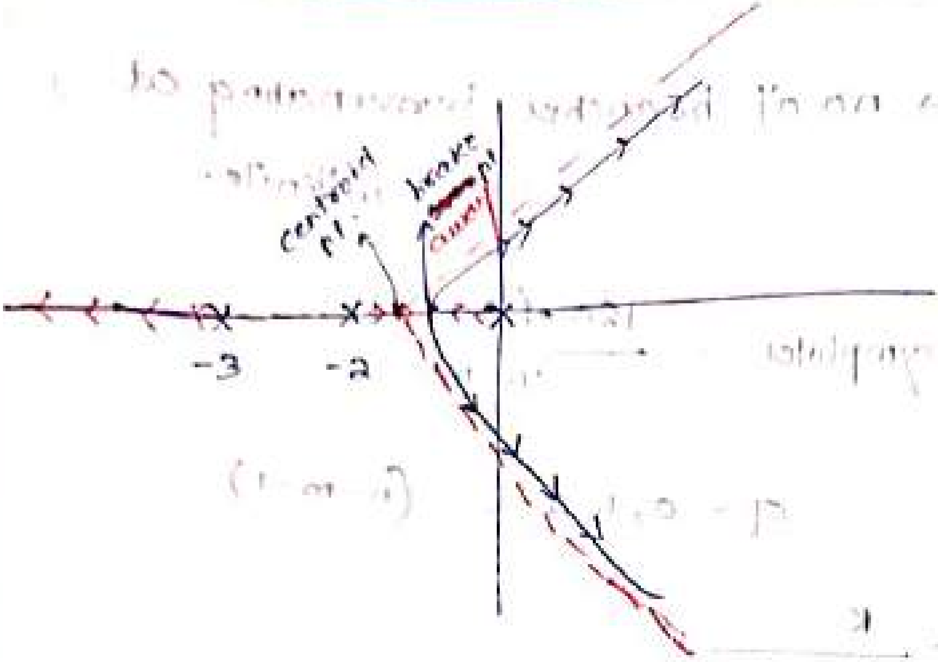
$q = 0, 1$

for $q=0$ $= \dfrac{180 \times (1)}{2} = 90°$

for $q=1$ $= \dfrac{180 \times (3)}{2} = 270°$

e.g:- $G(s) = \dfrac{K}{s(s+2)(s+3)}$





$$\text{Centroid} = \frac{(0-2-3)-(0)}{3} = \frac{5}{3}$$

$$A \cdot A = \frac{180(2q+1)}{n-3}$$

$$= \frac{180}{3}(2q+1) \quad (q = 0, 1, 2)$$

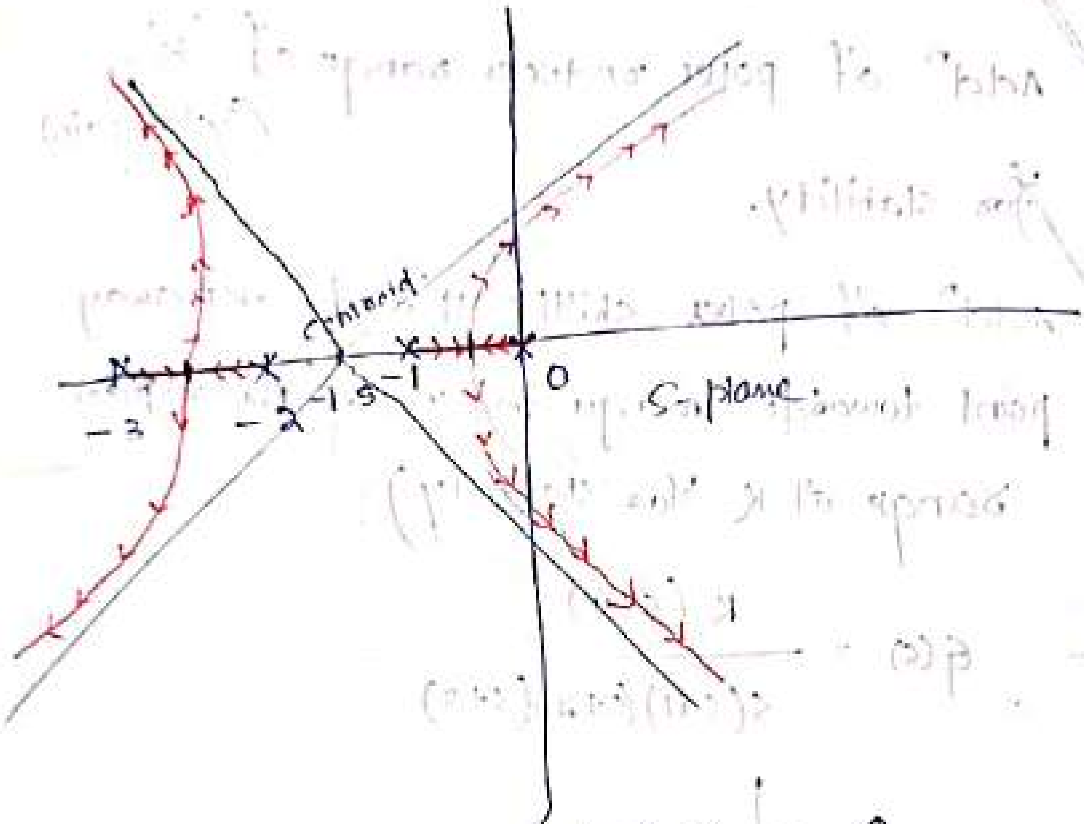
$$= 60, 180, 300.$$

Note :-

(i) - Asymptotes are not part of root locus, they show direction to the root locus branches terminating at infinite.

(ii) - Root locus branches can follow the asymptotes but they will never cross asymptotes

Eg → $G(s) = \dfrac{K}{s(s+1)(s+2)(s+3)}$.



$$\sigma = \frac{0+(-1)+(-2)+(-3) - 0}{4}$$

$$= \frac{-6}{4}$$

$$= -1.5$$

$$A \cdot A = \frac{180}{4}(2q+1)$$

$= 45, 135, 225, 315$

0, 1, 2, 3. V.V.V Imp.

1 Branches - 180

2 Branches - 90, 270

3 Branches - 60, 180, 300

4 Braches -

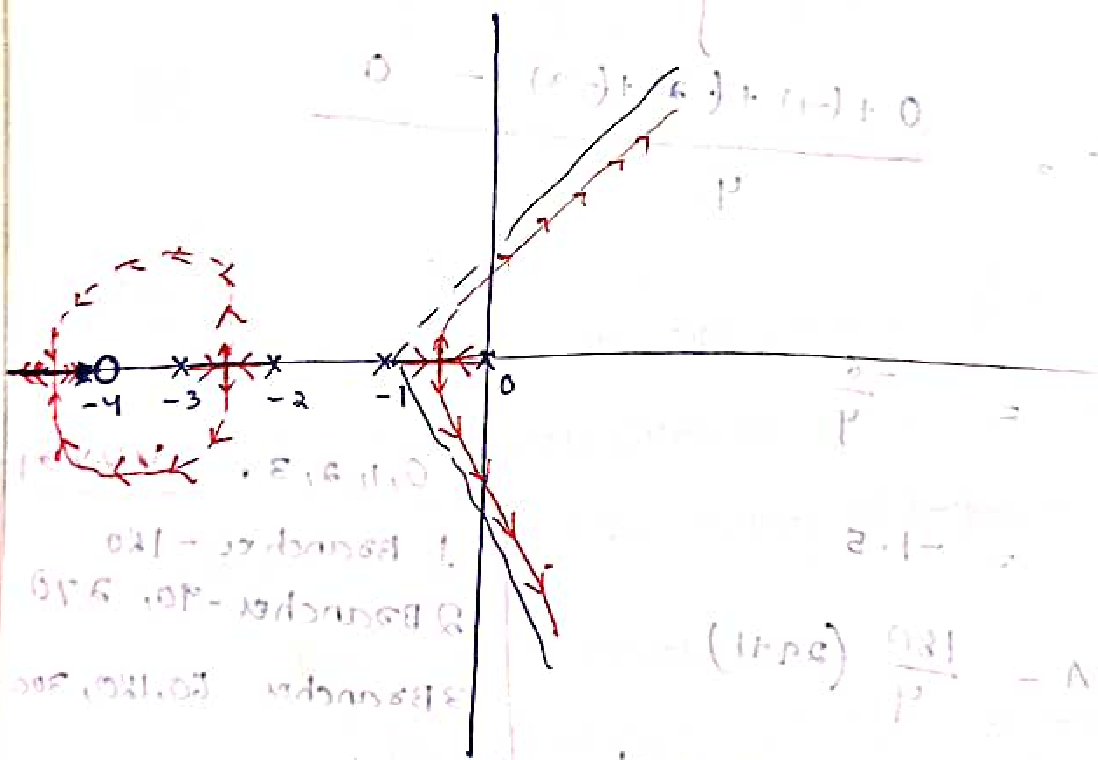
45°, 135, 225, 315

(i) (NOTE):- Add^n of poles reduces stability of the system.

(ii) Add^n of poles reduces range of K (System gain) for stability.

(iii) Add^n of poles shifts the brakeaway point towards imaginary axis. (This reduces range of K for stability).

e.g.:-
$$G(s) = \frac{K(s+4)}{s(s+1)(s+2)(s+3)}$$

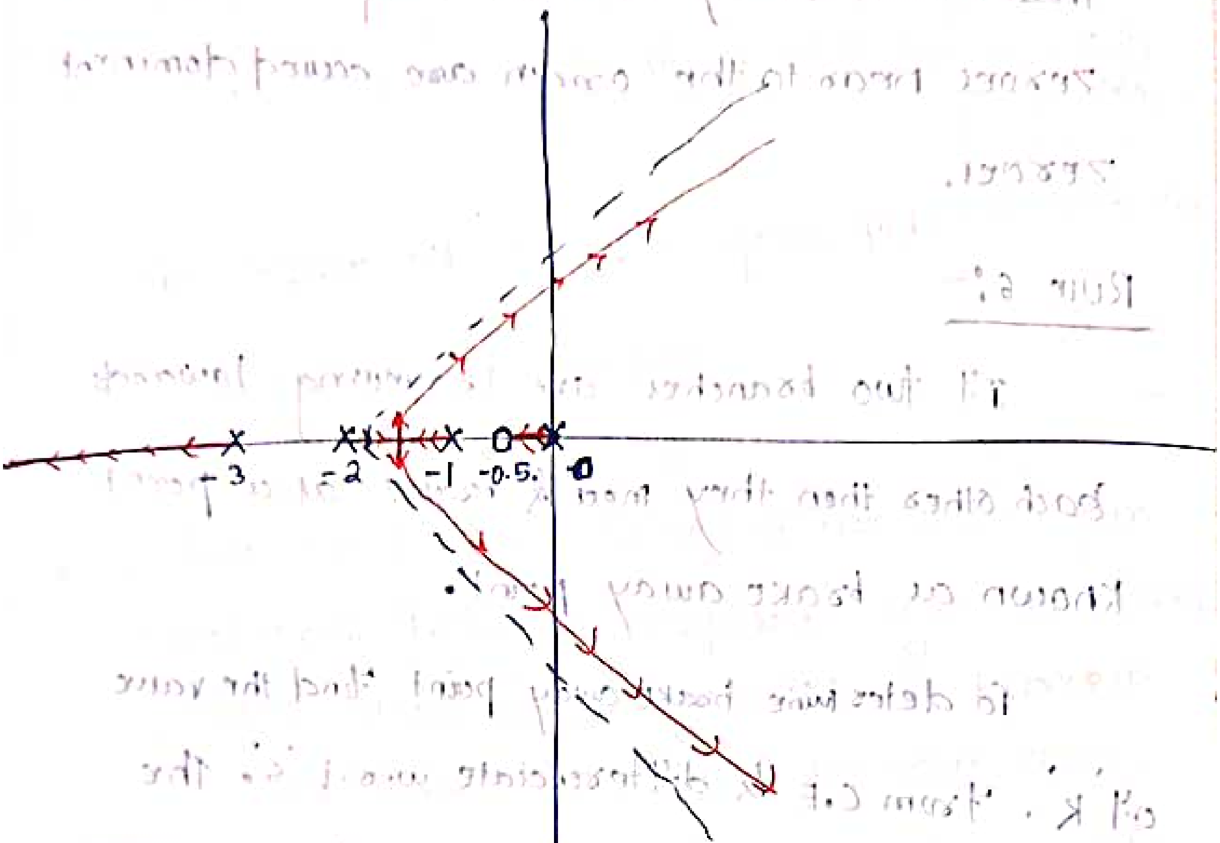


centroid = $\dfrac{(-3-2-1-0)-(-4)}{3}$

$= -2/3.$

eg:

$$G(s) = \frac{K(s+0.5)}{s(s+1)(s+2)(s+3)}$$



$$\text{Centroid} = \frac{[-1-2-3]-(-0.5)}{3}$$

$$\frac{-5.5}{3} = -1.83$$

NOTE:- → Addⁿ of zeros Improves stability of the system.

→ Addⁿ of zeroes Increases range of 'K' for stability.

→ Addition of zeroes shifts brakeaway point away from the origin.

4. zeroes near to the origin are more attractive than zeroes away from the origin. Therefore zeroes near to the origin are called dominent zeroes.

Rule - 6:-

If two branches are travelling towards each other then they meet & brake at a point known as brake away point.

To determine brake away point find the value of 'K' from C.E & differenciate w.r.t 's'. The value of 's' which satisfy $\frac{dK}{ds} = 0$ & gives positive value of K are valid brake away points.

If $G(s) = \frac{K\,N(s)}{D(s)}$.

C.E $1 + G(s)\,H(s) = 0$

$$= 1 + K\frac{N(s)}{D(s)} = 0$$

$$\boxed{K = -\frac{D(s)}{N(s)}}$$

- At brake away point $\frac{dK}{ds} = 0$

after solving this equation,

Substitute the value of 's' in $\boxed{K = -\frac{D(s)}{N(s)}.}$

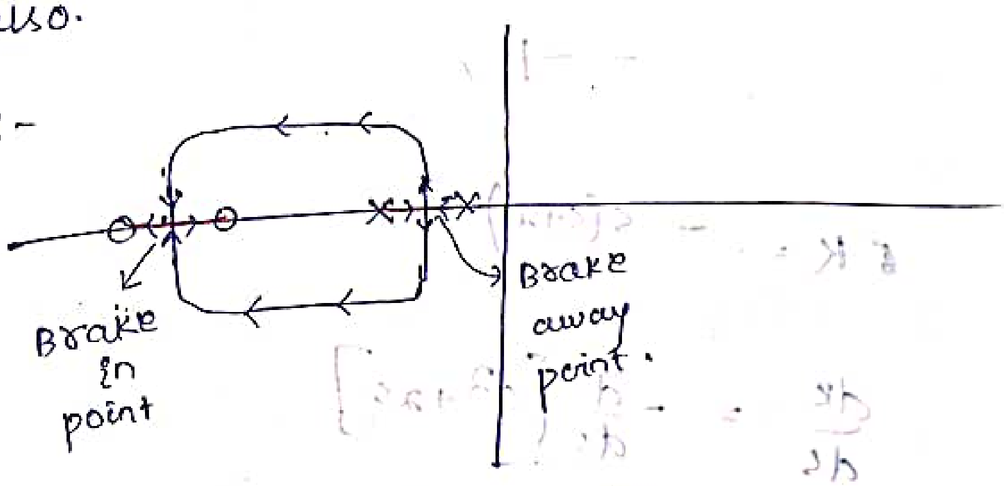
The value of 's' which gives positive K which is valid brake away point.

Note:-

- If two root locus branches are meeting on real axis from complex plane then the meeting point is called brake-in point. At brake in point the value of K is min^m on real axis.

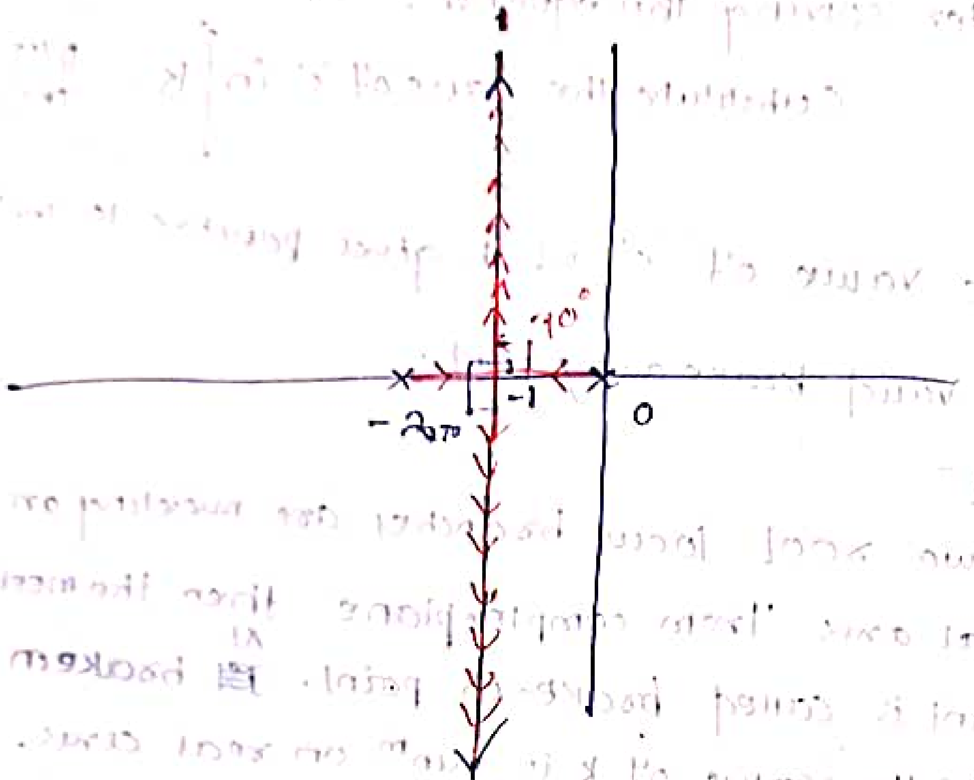
Then $\frac{dK}{ds} = 0$ is valid for brake in points also.

e.g.-



- (At Brake away point K is max^m on real axis. At Brake in point K is min^m on real axis.)

Q. $G(s) = \frac{K}{s(s+2)}$ sketch the exact root locus.



$$\text{centroid} = \frac{(-2)+0}{2} \quad A.A \ [90, 270°]$$

$$= -1,$$

$$K = -s(s+2)$$

$$\frac{dK}{ds} = -\frac{d}{ds}\left[s^2+2s\right]$$

$$\Rightarrow \left[-2s-2\right] = 0$$

$$\Rightarrow -2(s+1) = 0$$

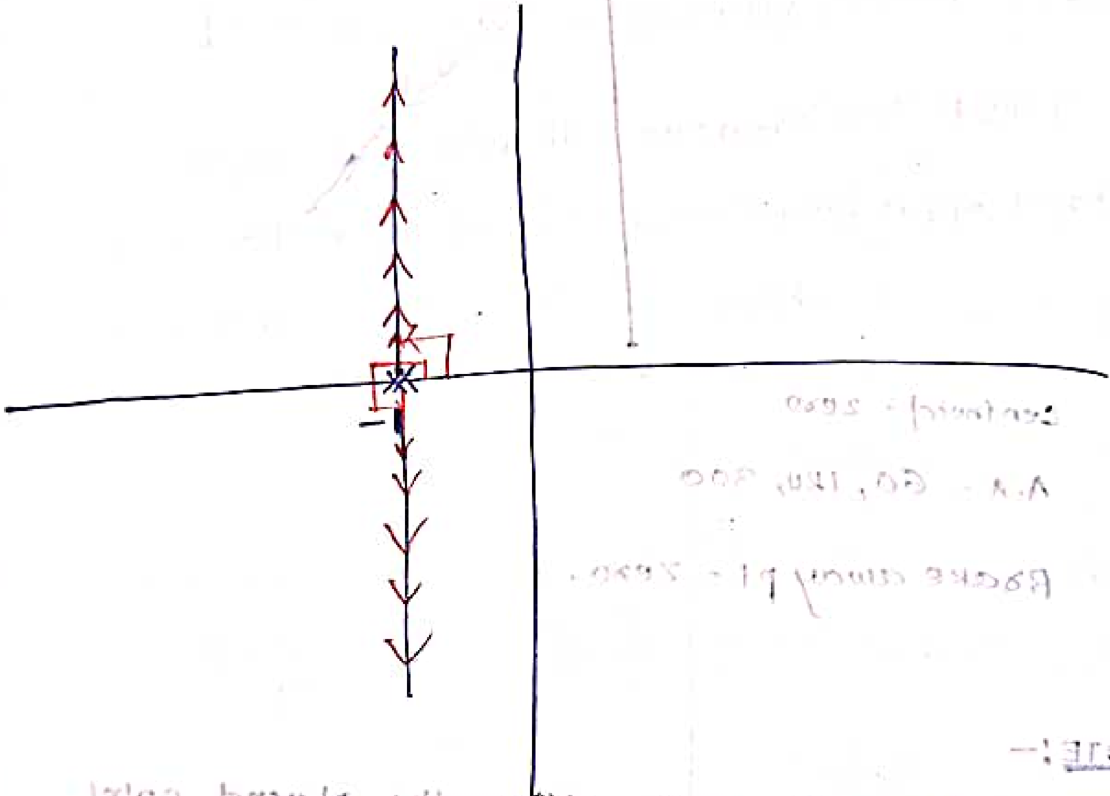
$$\Rightarrow s = -1$$

by putting $s = -1$

$K = 1$ which is positive

so it is brake away point.

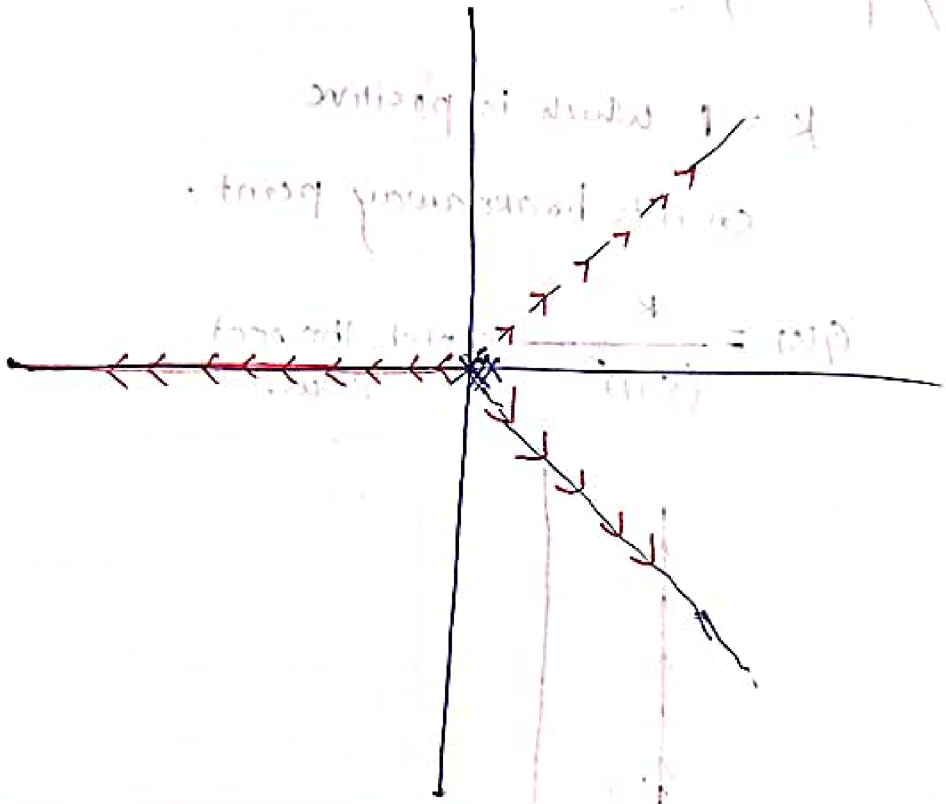
Q// $G(s) = \frac{K}{(s+1)^2}$ sketch the root locus.



centroid = -1 = brake away pt.

A A - 90, 270

Q^n // $G(s) = \frac{K}{s^3}$ draw the root locus.



centroid - zero

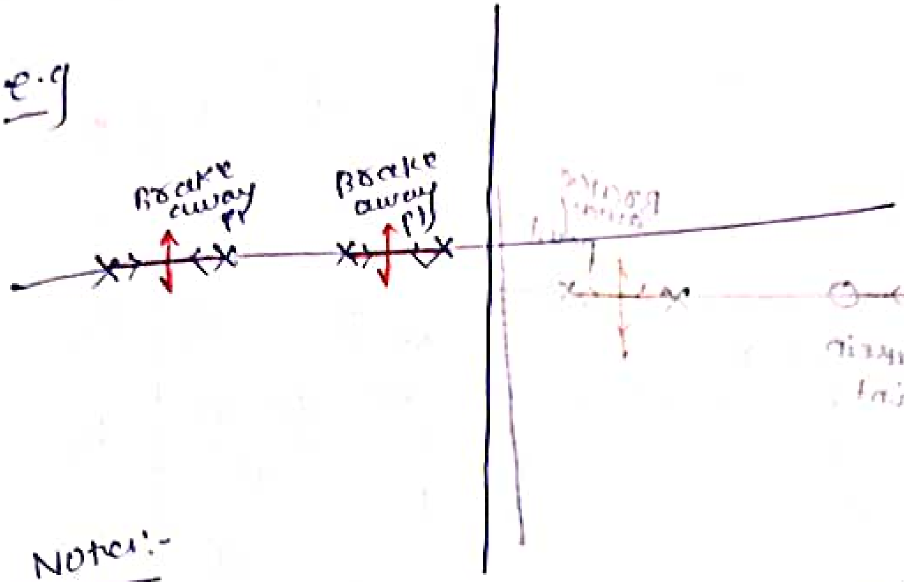
A.A - 60, 180, 300

Brake away pt - zero.

NOTE :-

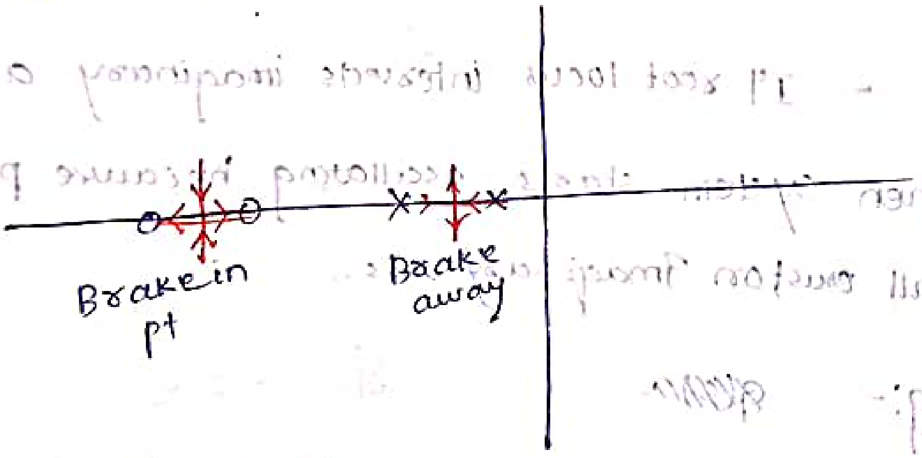
If there are two adjacently placed poles on real axis & the segment betn them is part of root locus then definitely there exists brake away pt betn the poles.

e.g

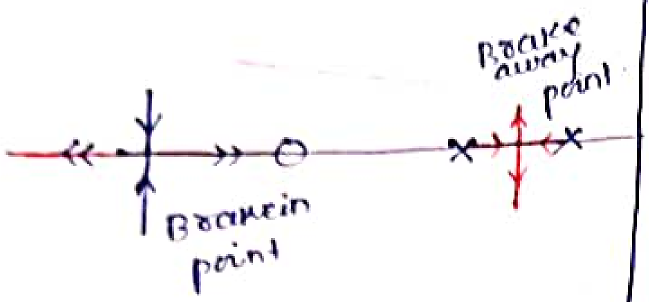


Note:-

- If there are two adjacently placed zero on real axis then the segment betn them is part of root locus then definitely there exists brake in point betn the zeroes.



- If there is a zero placed to the extreme left on real axis & the segment to the left of zero is part of root locus then definitely there exists brake in point to the left of zero.



Rule-7:-

If the root locus intersects imaginary axis, this intersection point can be determined by using Routh-Hurwitz criteria.

- If root locus intersects imaginary axis, then system starts oscillating because poles will exist on imaginary axis.

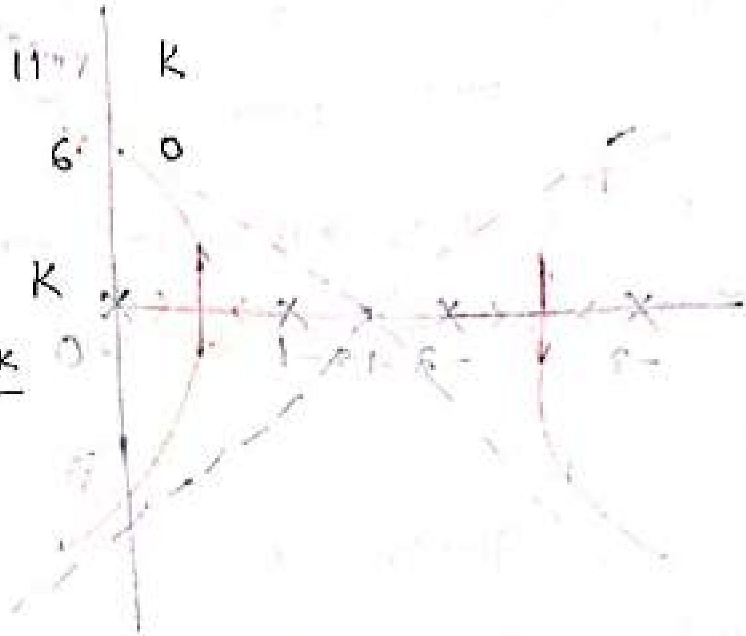
e.g.:-

Q. characteristic equation of a system is

$CE = s^4 + 6s^3 + 11s^2 + 6s + K = 0$

determine the intersection of root locus with imaginary axis.

s^4	1	11	K
s^3	6	6	0
s^2	10	K	
s^1	$\frac{60-6K}{10}$		
s^0	K		



$60 - 6K = 0$

$\Rightarrow 60 = 6K$

$\Rightarrow K = 10$ marginal

$10s^2 + K = 0$

$\Rightarrow 10s^2 + 10 = 0$

$\Rightarrow s^2 = -1$

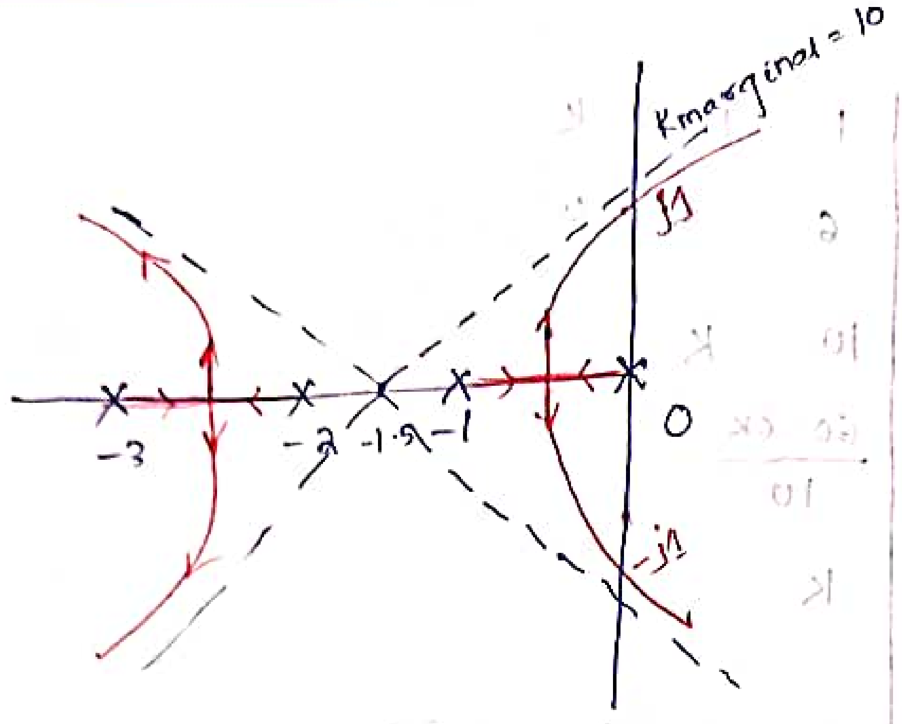
$\Rightarrow s = \pm j1$

$$CE = s^4 + 6s^3 + 11s^2 + 6s + K = 1 + G(s)\cdot H(s) = 0$$

$$\Rightarrow 1 + \frac{K\cdot 1}{(s^4+6s^3+11s^2+6s)} = 0$$

$$G(s) = \frac{K}{s(s^3+6s^2+11s+6)}$$

$$G(s) = \frac{K}{s(s+1)(s+2)(s+3)}.$$



Centroid = -1.5

A.A = 45, 135, 225, 315.

NOTE

For complex poles the root locus branches starts with an initial angle called as angle of departure. These branches starts with A.D angle of depr. & terminates either on zeros or at infinity.

Angle of departure (A.D)

$$180 - \phi$$

$$\text{where } \phi = \Sigma \phi_p - \Sigma \phi_z$$

$\Sigma \phi_p$ = Sum of angles contributed by remaining poles at the complex poles.

$\Sigma\phi_z$ = sum of the angles contributed by all the zeroes at the complex pole.

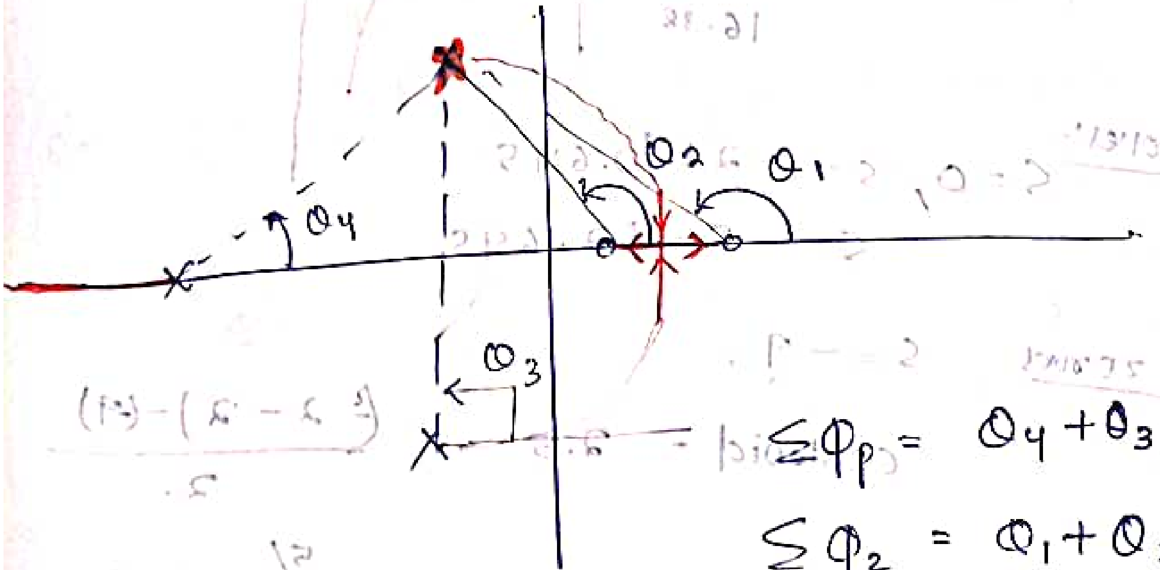
→ For complex zeroes the root locus branches arrives & terminates with the final angle called as angle of arrival.

- Angle of arrival, A·A = $180 - \phi$

where $\phi = \Sigma\phi_z - \Sigma\phi_p$

$\Sigma\phi_z$ - sum of angles contributed by remaining zeroes at the complex zero.

$\Sigma\phi_p$ - Sum of the angles contributed by all the poles at the complex zero.



$\Sigma\phi_p = \theta_4 + \theta_3$

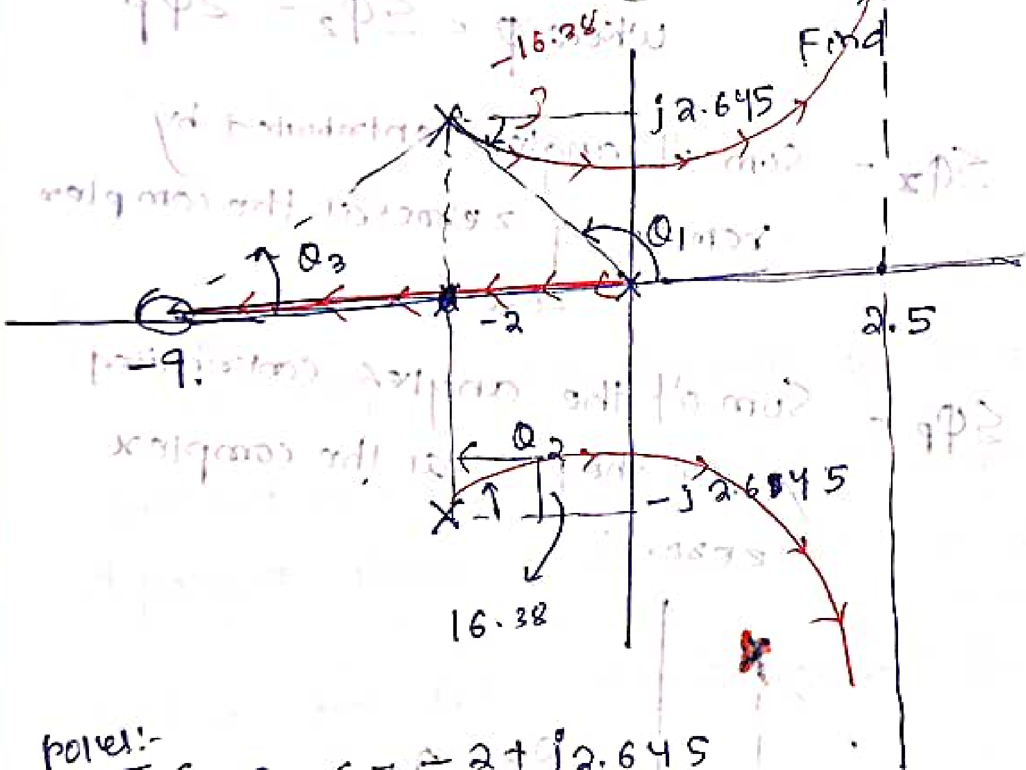
$\Sigma\phi_z = \theta_1 + \theta_2$

$$\phi = \Sigma\phi_p - \Sigma\phi_z$$
$$= (\theta_3 + \theta_4) - (\theta_1 + \theta_2)$$

$$A.D = 180 - \phi$$

Q"/ The open loop T.F. is given by

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$$



poles:- $s = 0,\ s = -2 + j2.645$
$s = -2 - j2.645.$

zeroes $s = -9.$

centroid - 2.5 $= \frac{(-2-2)-(-9)}{2}$

$= 5/2 = 2.5$

$$\varphi = \Sigma\varphi_p - \Sigma\varphi_z$$

$$= (\theta_1 + \theta_2) - \theta_3$$

$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{2.645}{2}\right) = 127.09.$$

$$\theta_2 = 90^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{2.645}{7}\right) = 20.69$$

$$\varphi = (127.09 + 90) - \cancel{52.90}\ 20.69.$$

$$= \cancel{164.38}\ 196.38.$$

$$A.D = 180 - \varphi$$

$$= -16.38$$

Frequency Domain Analysis

- Analysis of the system w.r.t frequency is called as frequency domain Analysis.
- In frequency domain Analysis we gain information about the system which can't be measured in time domain.
- The transfer function $T(s)$ when $s = j\omega$ is called as sinusoidal transfer function.

$$T(s)\Big|_{s=j\omega} \longrightarrow \text{Sinusoidal Transfer function.}$$

$s = j\omega$ converts the function from s-domain to frequency domain.

$$T(s)\Big|_{s=j\omega} = T(j\omega)$$

$$= |T(j\omega)| \angle T(j\omega)$$

$$\therefore M \angle \varphi$$

magnitude M - is the function of 'ω'.

The plot of M vs ω is called magnitude spectrum.

- phase 'ϕ' is function of ω. The plot of ϕ vs ω is called phase spectrum.

- magnitude & phase spectrum together is called frequency Response plot.

- The Basic frequency response plots are-

1. BODE plot:-

$$(20\log M) \text{ Vs } (\log\omega)$$

&

$$(\phi) \text{ Vs } (\log\omega)$$

2. Polar plot:-

(M) Vs (ϕ) [when ω is varied from 0 to ∞]

Frequency domain specification:-

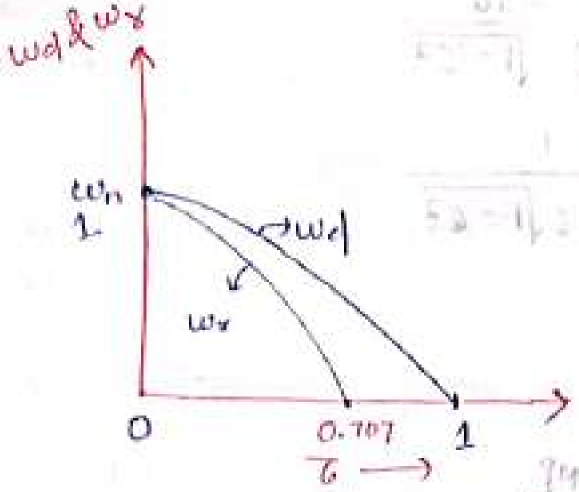
1. Resonant frequency:- (ω_r)

The frequency at which magnitude response reaches peak value is called resonant frequency.

$$M = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

at $\omega = \omega_r$, $\frac{dM}{d\omega} = 0$

on solving $\boxed{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$



$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\omega_r = \omega_n\sqrt{1-2\zeta^2}$

for $\zeta > 1/\sqrt{2}$, $\omega_r = 0$

for $\zeta > \frac{1}{\sqrt{2}}$, $M_r = 1$
(i.e at $\omega = 0$)

2. Resonant peak (M_r) :-

The ~~time~~ peak value of magnitude response is called resonant peak. it occurs at $\omega = \omega_r$.

at $\omega = \omega_r$, $M = M_r$

$$M_r = \frac{1}{\sqrt{\left(1-\left(\frac{\omega_r}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega_r}{\omega_n}\right)^2}}$$

on solving

$$\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

3. Band width :-

The range of frequencies over which a system performs within the designed specification is called Band width of the system.

mathematically - it is the range of frequencies where the signal power is more than 50%

at $\omega = \omega_b$, $M = 1/\sqrt{2}$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_b}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega_b}{\omega_n}\right)^2}}$$

on solving

v.v imp

$$\omega_b = \omega_n\sqrt{(1-2\zeta^2) + \sqrt{(1-2\zeta^2)^2+1}}$$

4. Cut-off Rate:-

The rate at which magnitude response decreases after $\omega = \omega_b$ is called cutoff rate.

5. Gain Margin:-

The additional gain which can be introduced in a system without affecting its stability is called gain margin.

$$\text{gain Margin} = \frac{K_{marginal}}{K}$$

Mathematically, It is defined as the reciprocal of magnitude of loop transfer function at phase cross over frequency.

$$G.M = \frac{1}{|G(j\omega)\,H(j\omega)|}\Bigg|_{\omega = \omega_{pc}}$$

ω_{pc} - phase cross over frequency.

at $\omega = \omega_{pc}$

$$\angle G(j\omega) \cdot H(j\omega) = -180^\circ$$

The frequency at which phase of loop transfer function is -180° is called phase cross over frequency.

6. Phase Margin :-

The additional phase which can be introduced in a system without affecting its stability is called phase margin.

mathematically,

$$P.M = 180 + \phi_{gc}$$

where $\phi_{gc} = \angle G(j\omega)\,H(j\omega)\Big|_{\omega = \omega_{gc}}$

ω_{gc} - gain cross over frequency at $\omega = \omega_{gc}$, $|G(j\omega)\, H(j\omega)| = 1$

Def^n of ω_{gc}:-

The frequency at which magnitude of loop transfer function is unity is called gain crossover frequency.

Q) Determine gain margin of the system with TF of the system $G(s) = \dfrac{K}{s(s+a)(s+b)}$

Sol^n:- $s = j\omega$. $G(j\omega) = \dfrac{K}{j\omega(j\omega+a)(b+j\omega)}$

$$|G(j\omega)| = \frac{K}{\omega\sqrt{a^2+\omega^2}\,\sqrt{b^2+\omega^2}}$$

$$\cancel{|G(j\omega)|}\ \angle G(j\omega) = \frac{0}{(90^\circ)\ \angle\tan^{-1}(\omega/a)\ \angle\tan^{-1}(\omega/b)}$$

$$= -90 - \tan^{-1}(\omega/a) - \tan^{-1}(\omega/b)$$

ω_{pc},

$$\angle G(j\omega)\, H(j\omega) = -180^\circ$$

$$\Rightarrow -90 - \tan^{-1}(\omega/a) - \tan^{-1}(\omega/b) = -180^\circ$$

$$\Rightarrow \tan^{-1}(\omega/a) + \tan^{-1}(\omega/b) = 90^\circ$$

$\Rightarrow$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{\omega}{a}+\frac{\omega}{b}}{1-\frac{\omega^2}{ab}}\right) = 90$$

$$1-\frac{\omega^2}{ab} = 0$$

$$\Rightarrow 1 = \frac{\omega^2}{ab}$$

$$\rightarrow \omega_{pc} = \sqrt{ab} \text{ rads/sec}$$

$$\left| G(j\omega) \right|_{\omega=\omega_{pc}} = \frac{K}{\omega_{pc}\sqrt{a^2+\omega_{pc}^2}\sqrt{b^2+\omega_{pc}^2}}$$

$$= \frac{K}{\sqrt{ab}\sqrt{a^2+ab}\sqrt{b^2+ab}}$$

$$= \frac{K}{ab(a+b)}$$

$$G.M = \frac{1}{|G(j\omega)|}$$

$$= \frac{ab(a+b)}{K}$$

<u>Case-1</u> $K < ab(a+b)$

$G.M > 1$; system is stable.

<u>Case-2</u> $K = ab(a+b)$

$G.M = 1$; system is marginally stable.

Case 3

$K > ab(a+b)$

$G_m < 1$, system is unstable.

NOTE: very very important

1. Gain margin as Number

$$G.M = \frac{K_{marginal}}{K}$$

(ii) Stable system, $G.m > 1$

(iii) Marginal stable system $G.m = 1$

(iv) Unstable system $G.m < 1$

2. Gain margin in dB →

$$G.M_{dB} = 20 \log (G.m_{(number)})$$

(i) Stable system → $G.M_{dB}$ is positive

(ii) marginally stable system - $G.m_{dB}$ is ~~stable~~ zero.

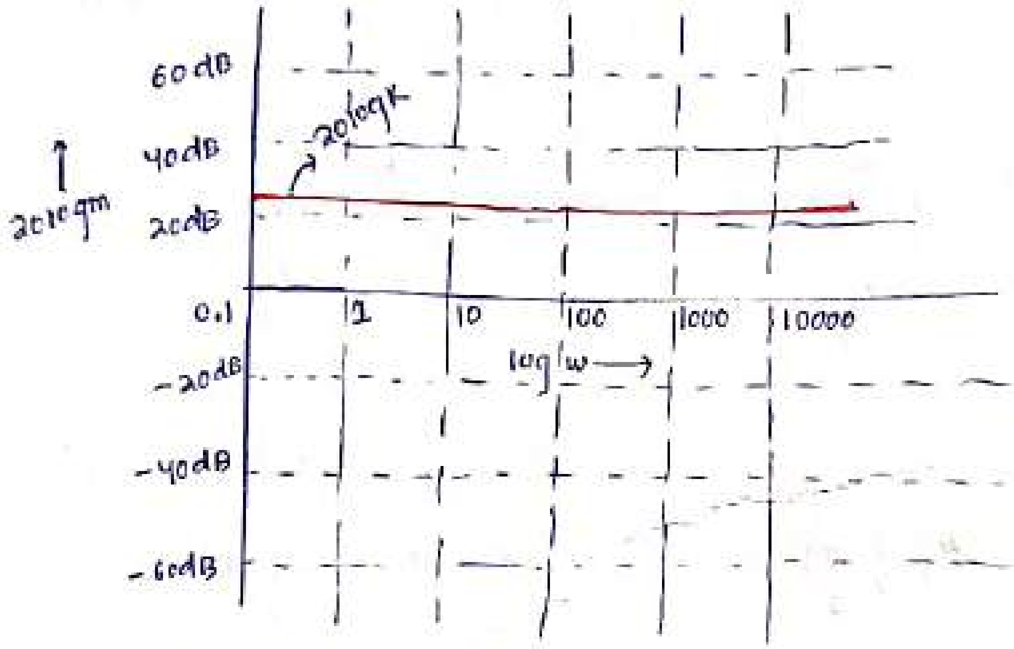
(iii) unstable system - $G.M_{dB}$ in negative.

Bode Plot

The plot of $(20\log m)$ vs $(\log\omega)$ & (ϕ) vs $(\log\omega)$ is called Bode plot.

→ $20\log M$ converts multiplications & divisions into simple additions.

→ $\log\omega$ axis gives the advantage of representing lowest frequency to highest frequency in the order of "GHz."



In Bode plot the system will be in Time constant form.

For Bode plot, transfer function must be in time constant form. eg →

$$TF = \frac{K(1+sT_1)(1+sT_2)^n \dots}{s^n(1+sT_1')(1+sT_2')\dots}$$

General factors in transfer function :-

1. constant K
2. Factor K/s (or) K/s^n
3. Factor Ks or $K.s^n$
4. Factor $(1+sT)$ or $(1+sT)^n$
5. Factor $\frac{1}{1+sT}$ or $\frac{1}{(1+sT)^n}$
6. Factor $s^2+2\zeta\omega_n s+\omega_n^2$

in Numerator or denominator.

Bode plot of factors :-

1. constant 'K'

$G(s) = K$

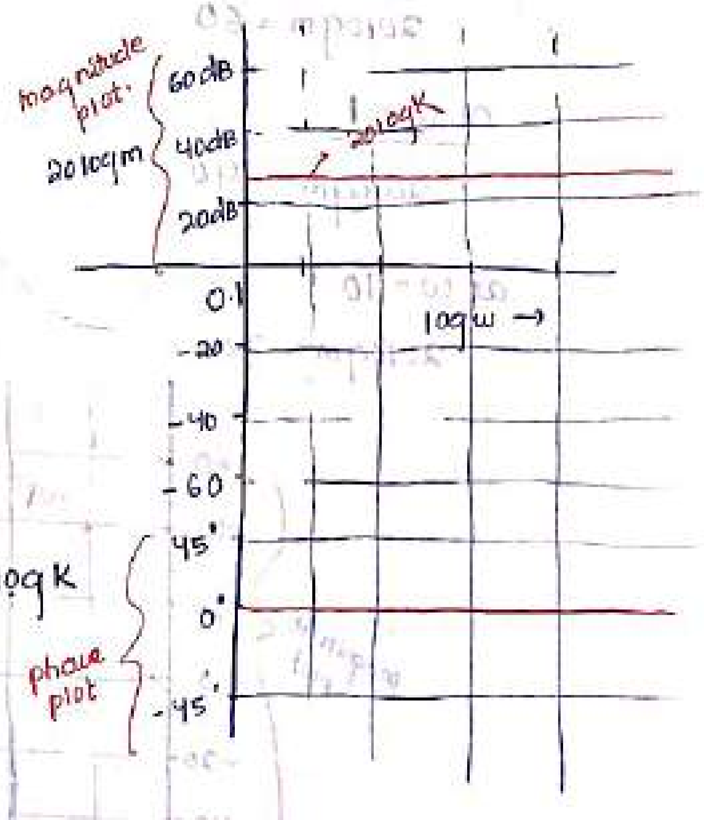
$s = j\omega$

$G(j\omega) = K$

$M = K$

$\phi = 0$

$20\log m = 20\log K$



2. Factor $G(s) = K/s$

$$G(j\omega) = K/j\omega$$

$$M = \frac{K}{\omega}$$

$$\phi = -90^\circ$$

$$20\log m = 20\log\left(\frac{K}{\omega}\right)$$

$$= 20\log K - 20\log\omega$$

Let $K = 100$

$$20\log m = 40 - 20\log\omega$$

at $\omega = 0.1$

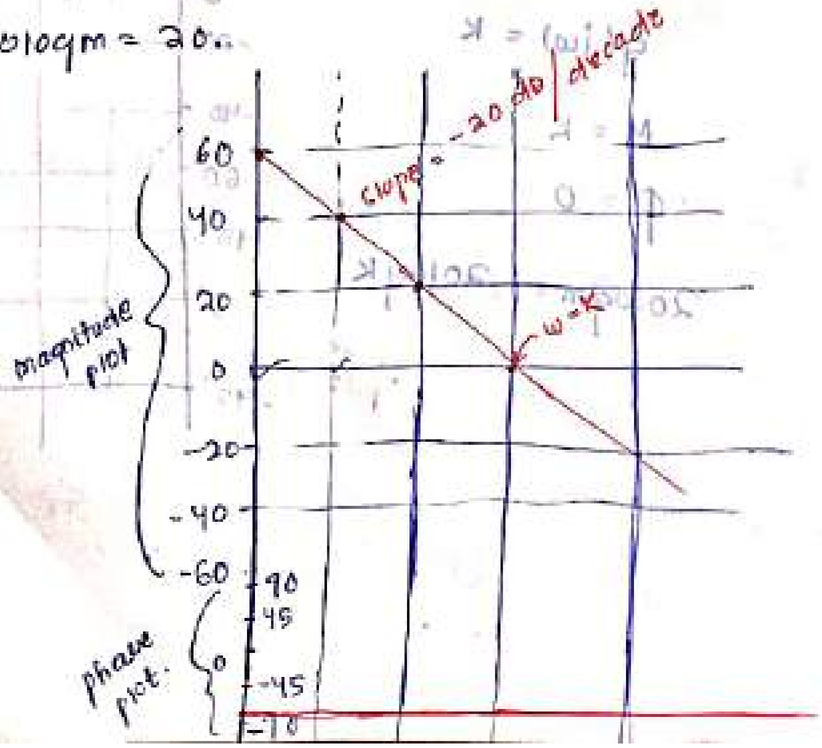
$$20\log m = 60$$

at $\omega = 1$

$$20\log m = 40$$

at $\omega = 10$

$$20\log m = 20$$



Bode plot of the factor K/s is a straight line with slope of -20dB/decade & passing through 0dB line with $\omega = K$.

$$0\text{dB} \Rightarrow 20\log m = 0$$

$$M = 1$$

$$\frac{K}{\omega} = 1$$

$$\Rightarrow \underline{\omega = K}.$$

3. Factor K/s^n

$$G(s) = K/s^n$$

$$G(j\omega) = \frac{K}{(j\omega)^n}$$

$$m = K/\omega^n$$

$$\phi = -n \times 90^\circ$$

$$20\log M = 20\log\left(K/\omega^n\right)$$

$$= 20\log K - n \times 20\log\omega$$

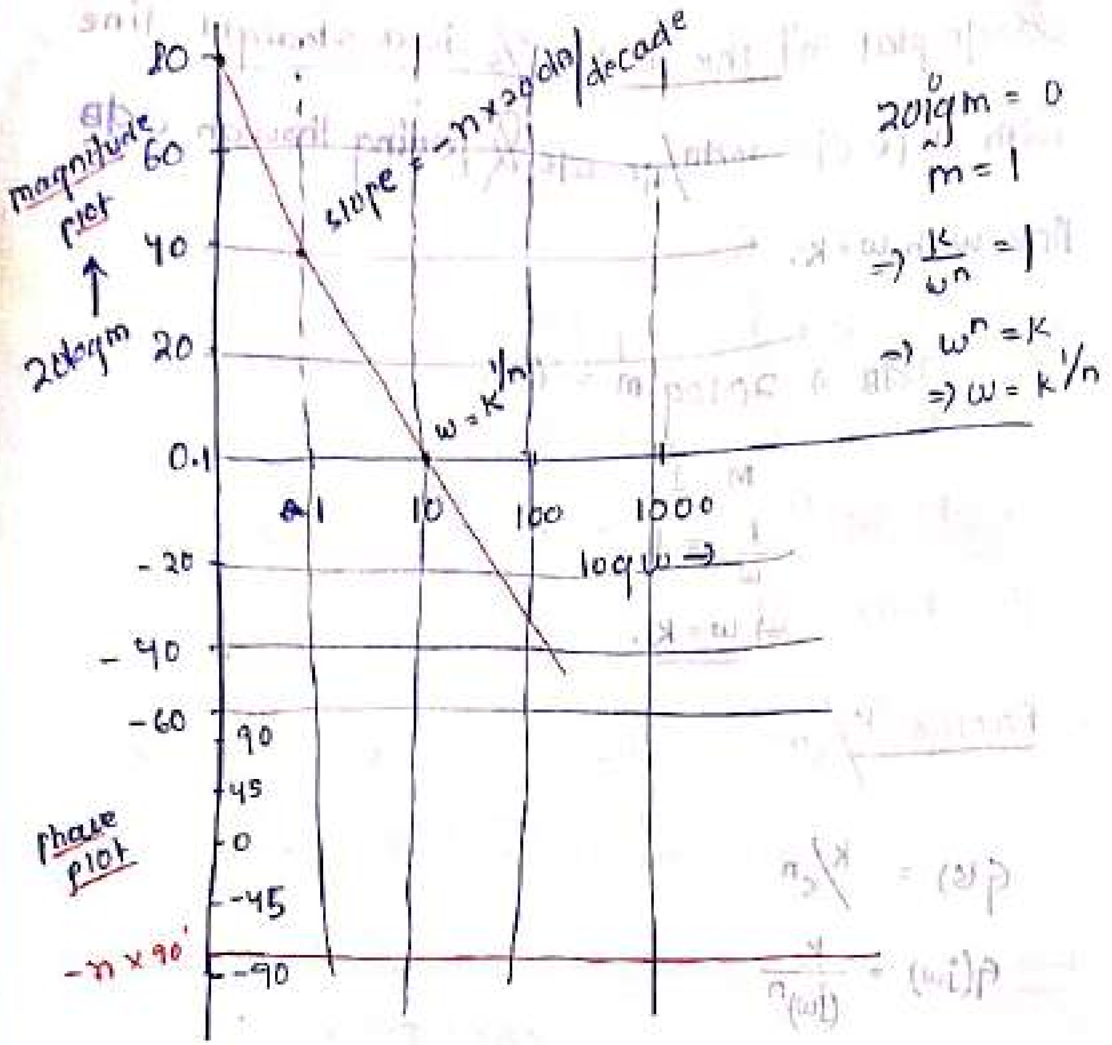
Let $K = 100$, $n = 2$

$$20\log M = 40 - 20n\log\omega = 40 - 40\log\omega$$

$\underline{\text{at } \omega = 0.1}$, $20\log m = 80$

$\underline{\text{at } \omega = 10}$ $20\log m = -40$

$\underline{\text{at } \omega = 10}$ $20\log m = 0$



3. Factors Ks or Ks^n :-

$G(s) = Ks$

$s = j\omega$

$G(j\omega) = K(j\omega)$

$M = K\omega$

$\phi = 90°$

$20\log M = 20\log(K\omega)$

$= 20\log K + 20\log\omega$

Let $K=100$,

$20\log M = 40 + 20\log w$

<u>$w = 0.1$</u>

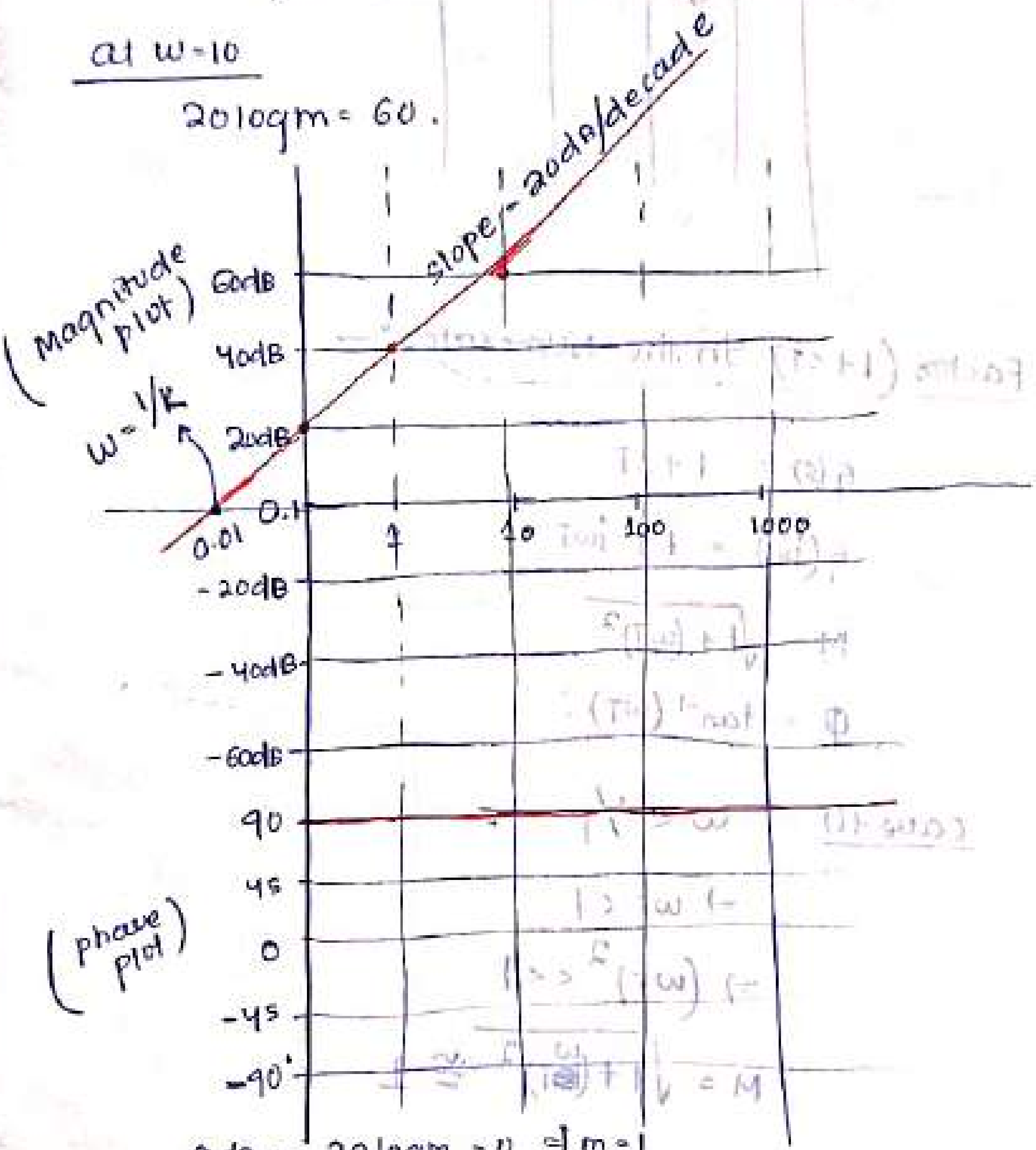
$20\log m = 20$

<u>at $w=1$</u>

$20\log m = 40$

<u>at $w=10$</u>

$20\log m = 60.$



$0dB = 20\log m = 0 \Rightarrow m = 1$

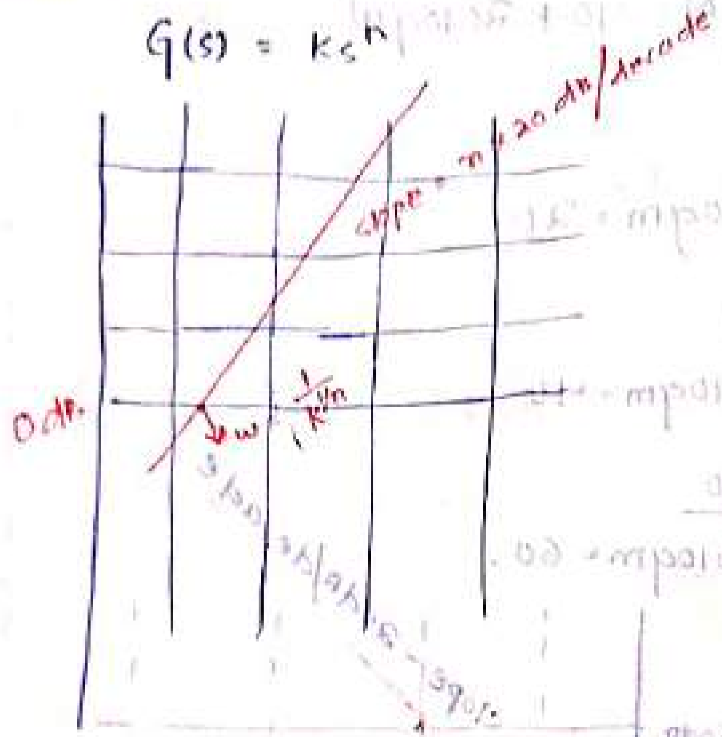
$m = 1$

$Kw = 1$

$\Rightarrow w = 1/K$

Factor ks^n

$$G(s) = ks^n$$



Factor $(1+sT)$ In the Numerator:—

$$G(s) = 1 + sT$$

$$G(j\omega) = 1 + j\omega T$$

$$M = \sqrt{1 + (\omega T)^2}$$

$$\phi = \tan^{-1}(\omega T).$$

case (i) $\quad \omega < 1/T$

$$\Rightarrow \omega T < 1$$

$$\Rightarrow (\omega T)^2 << 1$$

$$M = \sqrt{1 + (\omega T)^2} \simeq 1$$

$$20\log M = 0$$

case-2

$\omega > 1/T$

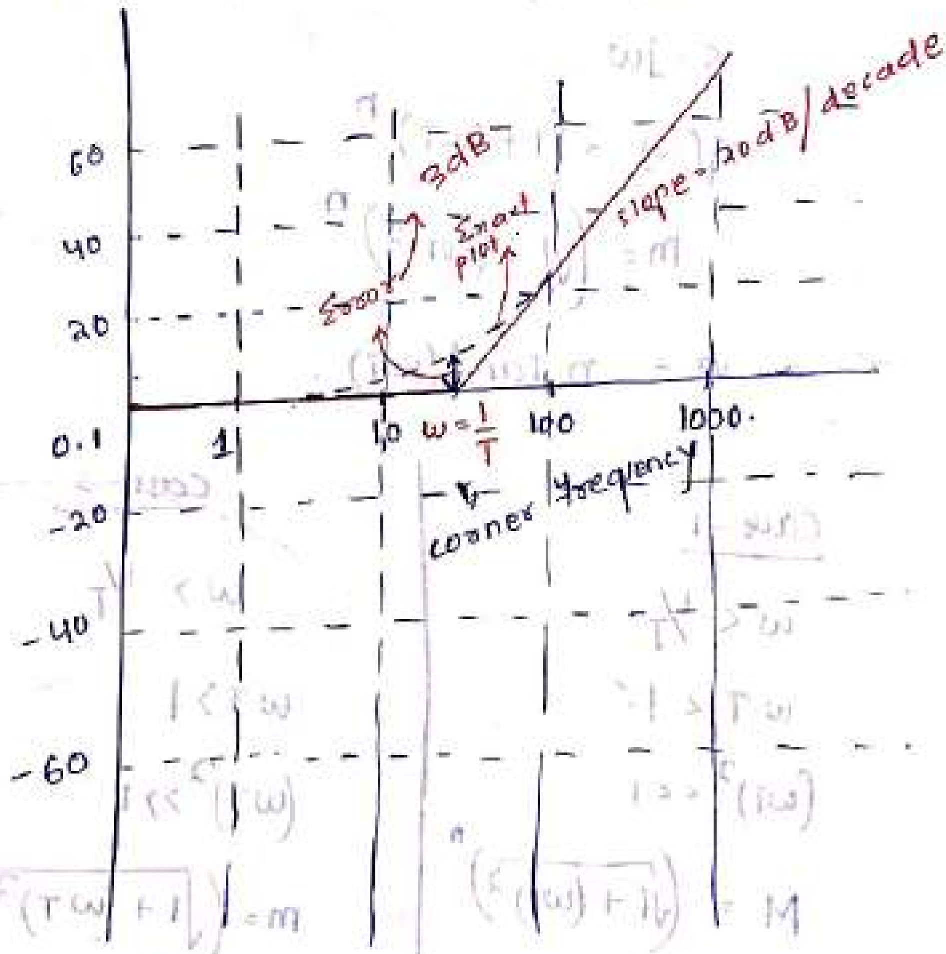
$\rightarrow \omega T > 1$

$\rightarrow (\omega T)^2 >> 1$

$M = \sqrt{1 + (\omega T)^2}$

$\simeq \omega T$

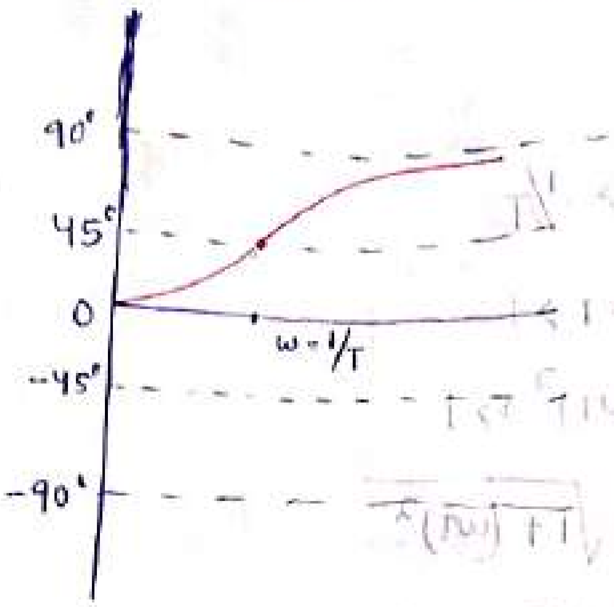
$20 \log m = 20 \log T + 20 \log \omega$. (It is similar to the factor ks)



$\omega = 1/T$

$m = \sqrt{2}$

$20 \log M = 20 \log \sqrt{2} = 3dB$ (error)



Similarly for the factor

$$G(s) = (1 + sT)^{n}$$

$$s = j\omega$$

$$G(j\omega) = (1 + j\omega T)^{n}$$

$$m = \left(\sqrt{1 + (\omega T)^2}\right)^{n}$$

$$\phi = n \tan^{-1}(\omega T).$$

Case - 1

$\omega < 1/T$

$\omega T < 1$

$(\omega T)^2 << 1$

$M = \left(\sqrt{1 + (\omega T)^2}\right)^{n}$

$\simeq 1$

$20 \log m = 0\,dB$

Case - 2

$\omega > 1/T$

$\omega T > 1$

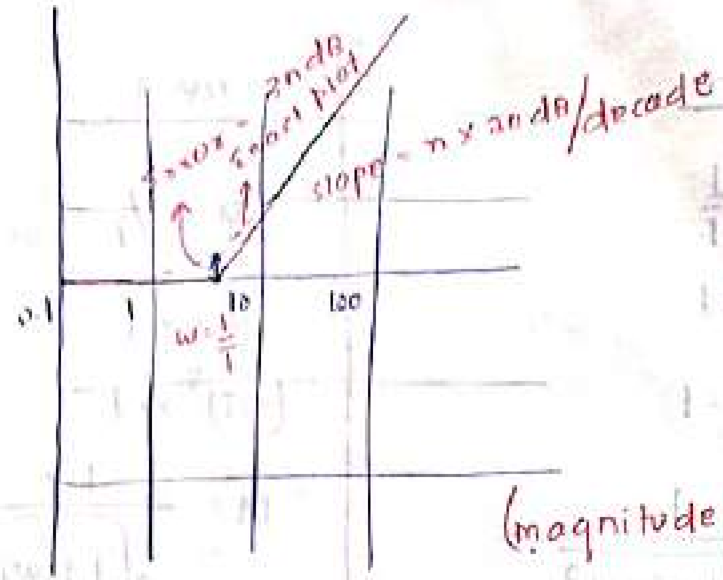
$(\omega T)^2 >> 1$

$m = \left(\sqrt{1 + (\omega T)^2}\right)^{2}$

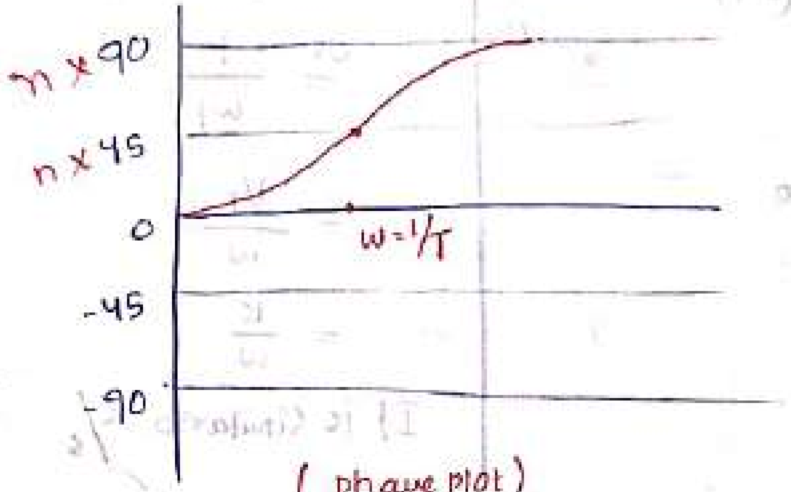
$\simeq (\omega T)^{n}$

$\simeq T^{n} \cdot \omega^{n}$

It is similar to the factor Ks^{n}.



(magnitude plot)



(phase plot)

Factor $G(s) = \dfrac{1}{1+sT}$

$s = j\omega$

$$G(j\omega) = \frac{1}{1+j\omega T}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+(\omega T)^2}}$$

$$\phi = -\tan^{-1}(\omega T).$$

case-1

$\omega < \frac{1}{T}$

$\omega T < 1$

$(\omega T)^2 << 1$

$M = \frac{1}{\sqrt{1+(\omega T)^2}}$

$\simeq 1$

$20 \log M = 0$

case-2

$\omega > 1/T$

$\omega T > 1$

$(\omega T)^2 >> 1$

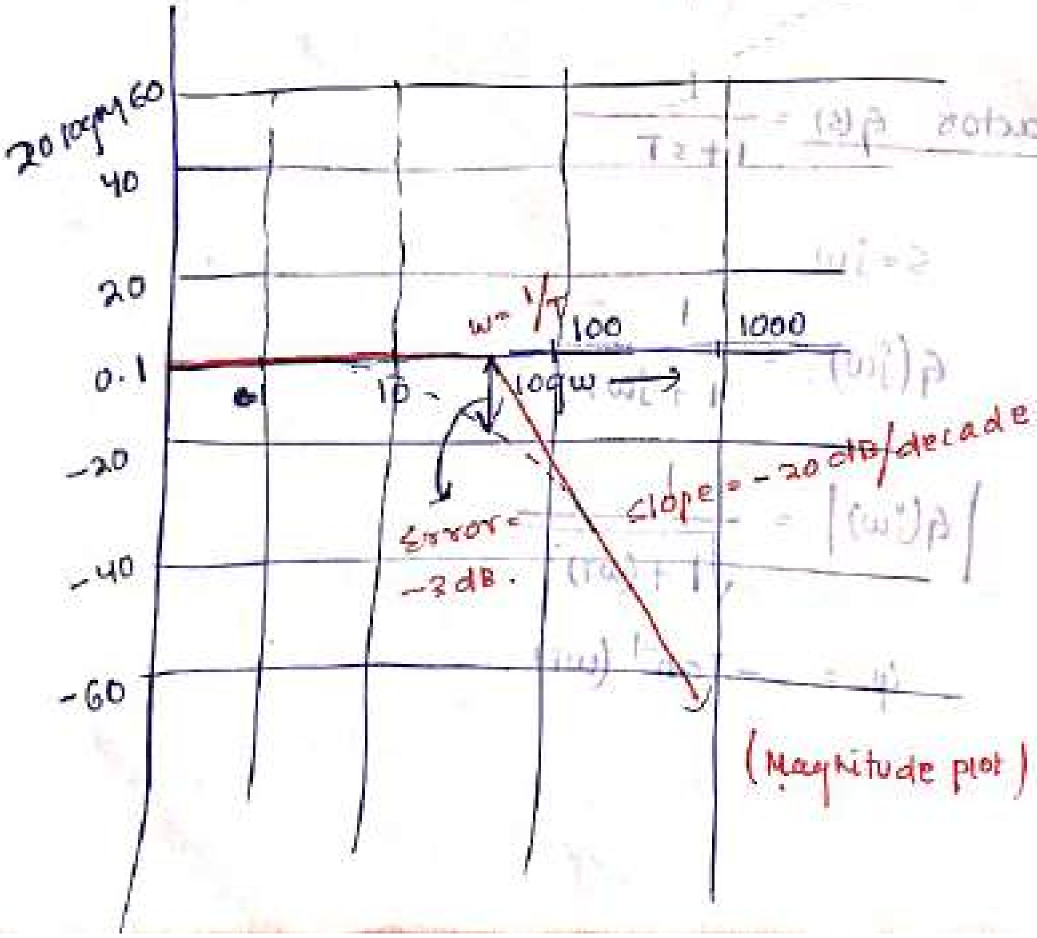
$M = \frac{1}{\sqrt{1+(\omega T)^2}}$

$\simeq \frac{1}{\omega T}$

$= \frac{1/T}{\omega}$

$= \frac{K}{\omega}$

It is similar to K/s



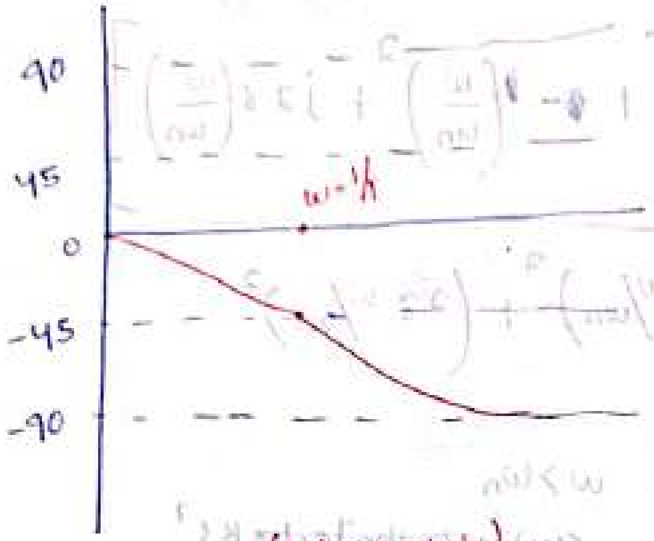
(Magnitude plot)

$\omega = \frac{1}{T}$

$m = \frac{1}{\sqrt{2}}$

$20\log m = 20\log 1/\sqrt{2}$

$= -3\,dB$ (error)



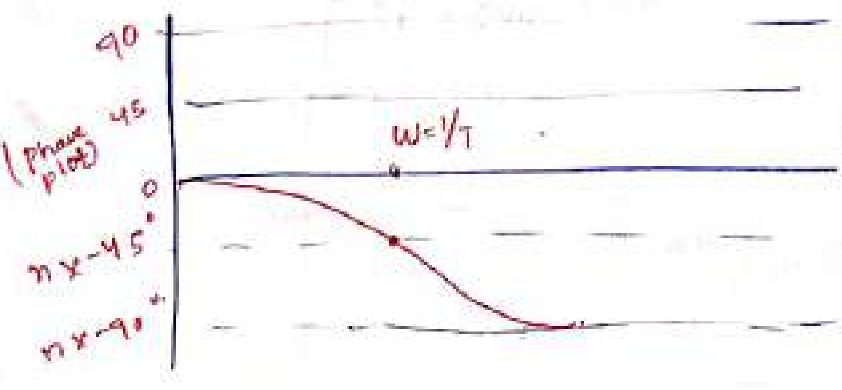
(phase plot)

Similarly for the factor $G(s) = \frac{1}{(1+sT)^n}$

The same Bode plot will be there as $(1+sT)$

but slope will be $-n \times 20\,dB/decade$

& error will be $-3n\,dB$.



(phase plot)

Quadratic factor :-

$$G(s) = s^2 + 2\zeta\omega_n + \omega_n^2$$

$$G(s) = \omega_n^2\left(1 + \left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(s/\omega_n\right)\right)$$

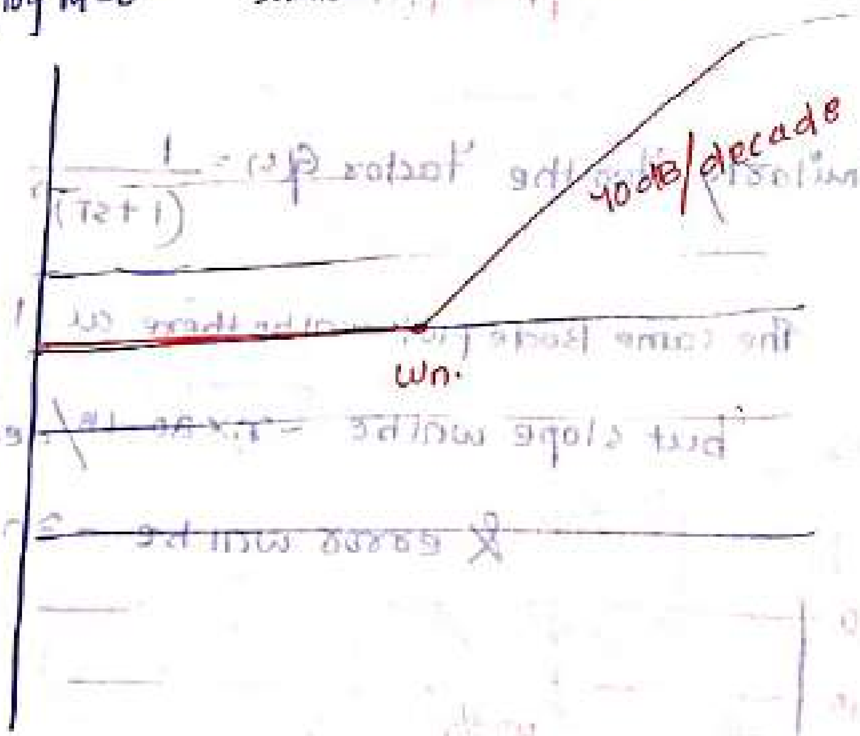
$s = j\omega$

$$G(j\omega) = \omega_n^2\left[1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\left(\frac{\omega}{\omega_n}\right)\right]$$

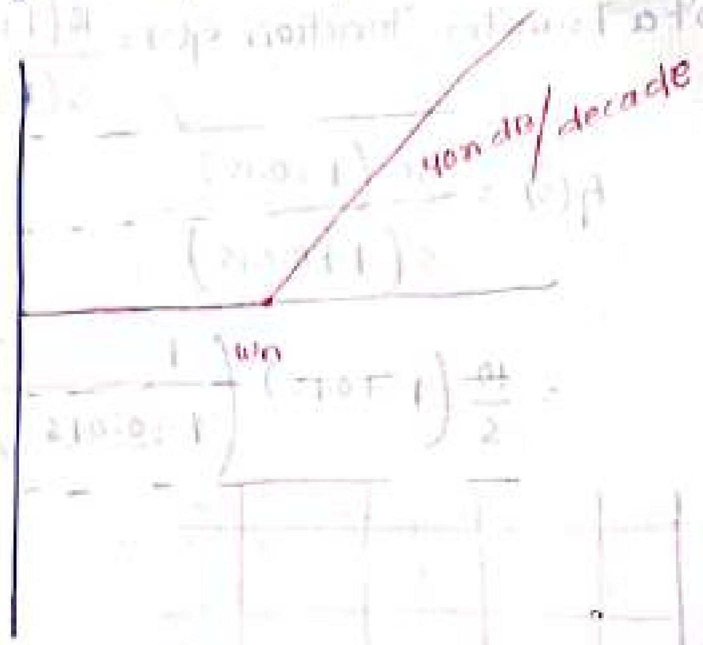
$$M = \sqrt{\left[1 - \left(\omega/\omega_n\right)^2\right]^2 + \left(2\zeta\,\omega/\omega_n\right)^2}$$

$\omega < \omega_n$; $\omega > \omega_n$

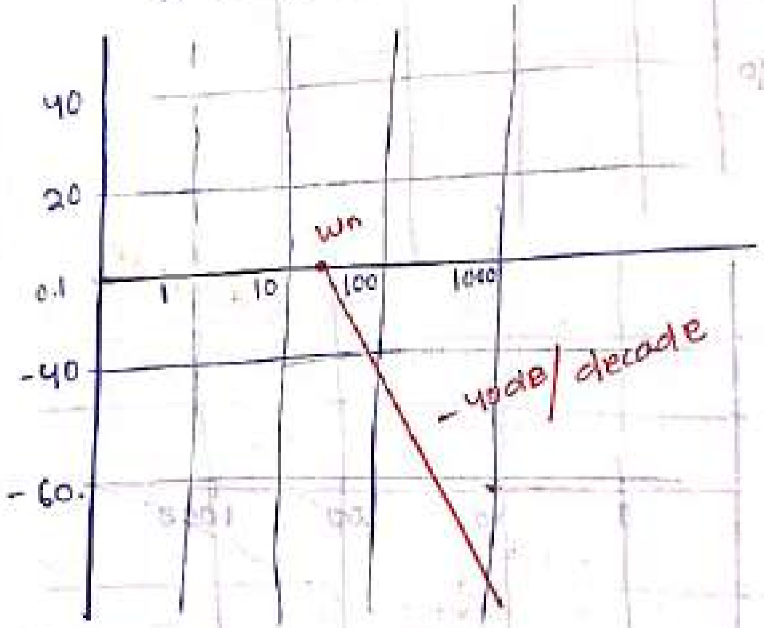
$20\log M = 0$ similar to the factor ks^2



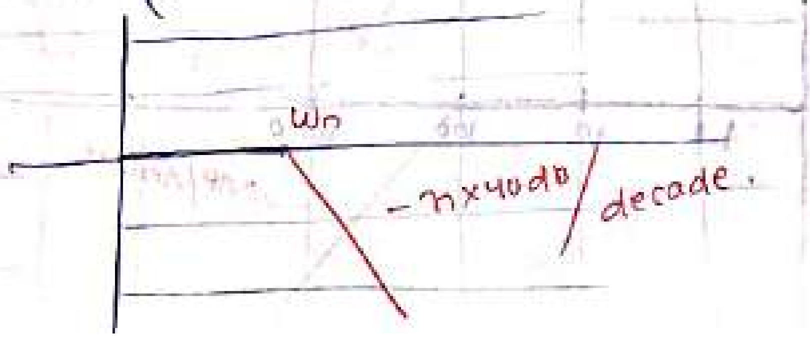
similarly for $G(s) = \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)^n$



$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



$$G(s) = \frac{1}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)^n}$$

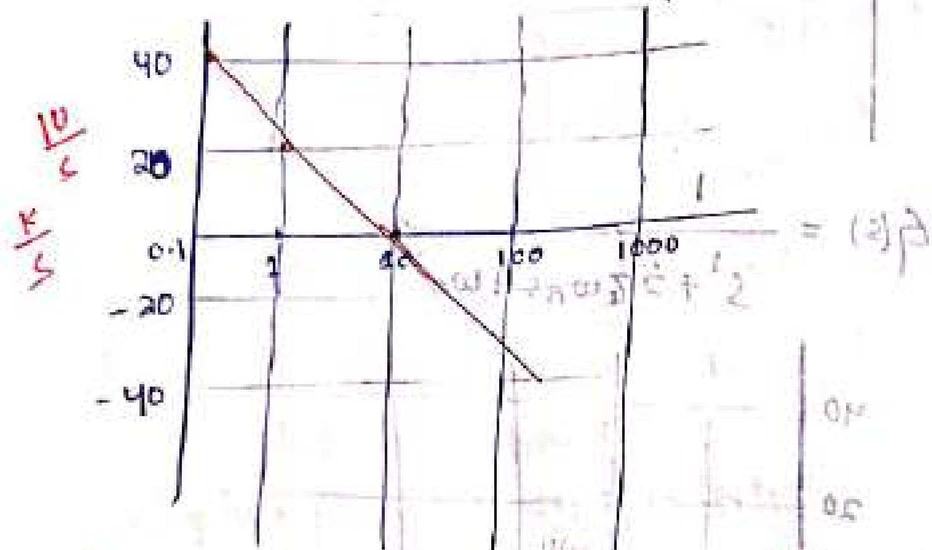


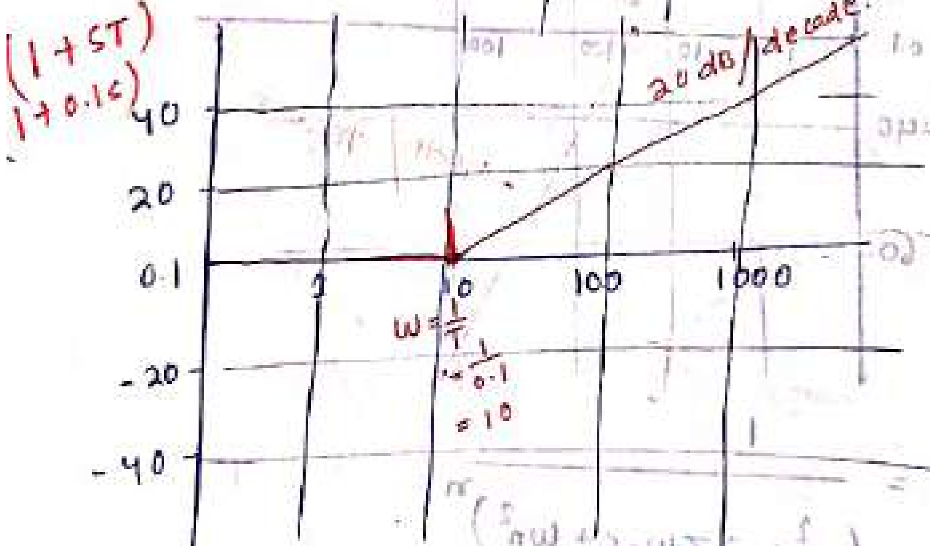
Q. Sketch the Bode plot of a control system of a transfer function $G(s) = \frac{10(1+0.1s)}{s(1+0.01s)}$

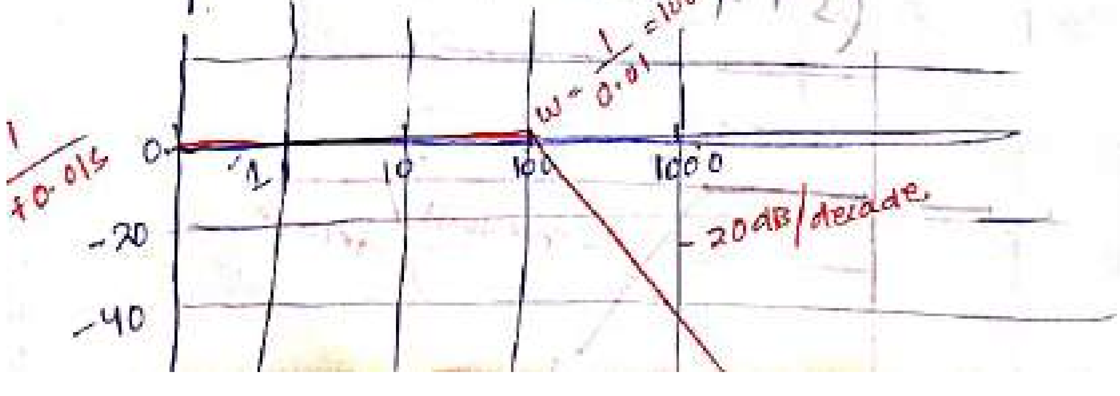
Sol:-

$$G(s) = \frac{10(1+0.1s)}{s(1+0.01s)}$$

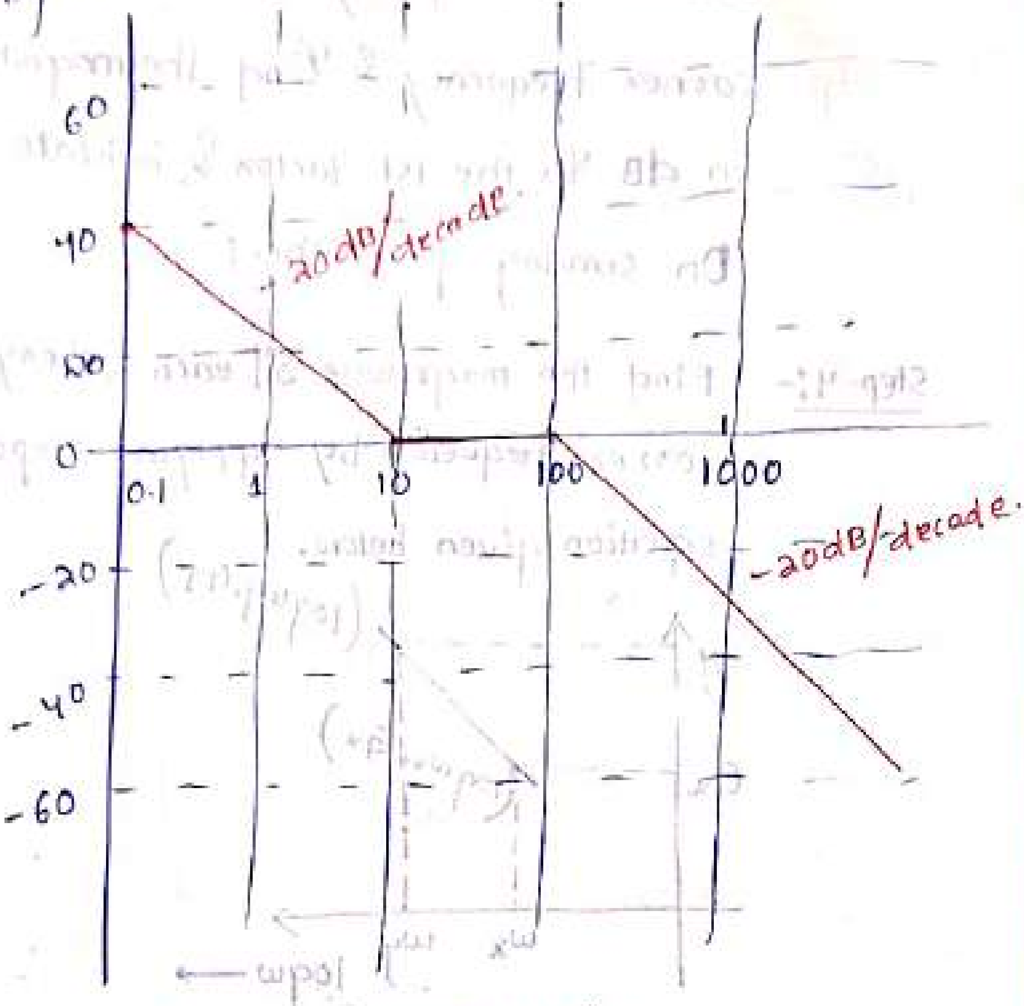
$$= \frac{10}{s}(1+0.1s)\left(\frac{1}{1+0.01s}\right)$$







Resultant Bode plot by combining the three.



procedure for sketching Bode plot :-

Step-1 :- convert the transfer function into time constant form.

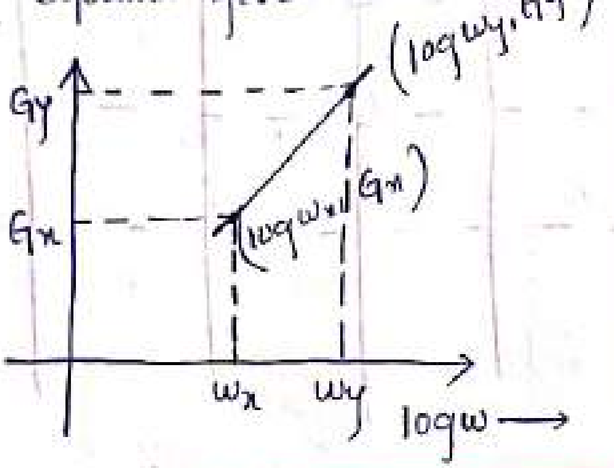
Step-2 :- Identify the factors in the increasing order of corner frequency & fill the table as given below.

Sl NO	Factor	corner freq^n	slope	change in slope

Step-3:-

choose a frequency lower than 1st corner frequency & find the magnitude in dB for the 1st factor & indicate On semilog graph sheet.

Step-4:- Find the magnitude of each & every corner frequency by applying slope equation given below.



gain at wy, $\boxed{G_y = \text{slope} \log\left(\frac{\omega_y}{\omega_x}\right) + G_x}$

$$\text{slope} = \frac{G_y - G_x}{\log \omega_y - \log \omega_x}$$

$$G_y = \text{slope} \log\left(\frac{\omega_y}{\omega_x}\right) + G_x$$

ω_y → present freq, G_x → prev gain, ω_x → prev freq., slope → bringing to that freq.

Step-5:-

choose a frequency higher than last corner frequency & find the magnitude by using slope equation given in step 4.

Step-1

Indicate all the magnitudes on semilog graph sheets & join them by straight lines in the increasing order of corner frequencies.

Q11 $G(s) = \dfrac{10(1+\underline{0.1}s)}{s(1+\underline{0.01}s)}$

Step-2

SI NO	factor	C.F	slope	change in slope
1	$10/s$	–	-20	-20
2.	$(1+0.1s)$	10	$+20$	0
3.	$\dfrac{1}{(1+0.1s)}$	100	-20	-20
4.		1000	-	

Step-3

$\omega = 1$

$G(s) = 10/s$

$M = 10/\omega$ (maginute)

$20 \log M = 20\log\left(\frac{10}{\omega}\right)$

$= 20\log\left(\frac{10}{1}\right)$

$G(1) = 20.$

$\omega = 1$

Step-4

$$G_{10} = \text{slope} \times \log\left(\frac{10}{1}\right) + G_1$$

$$= -20 \log(10) + 20$$

$$= 0$$

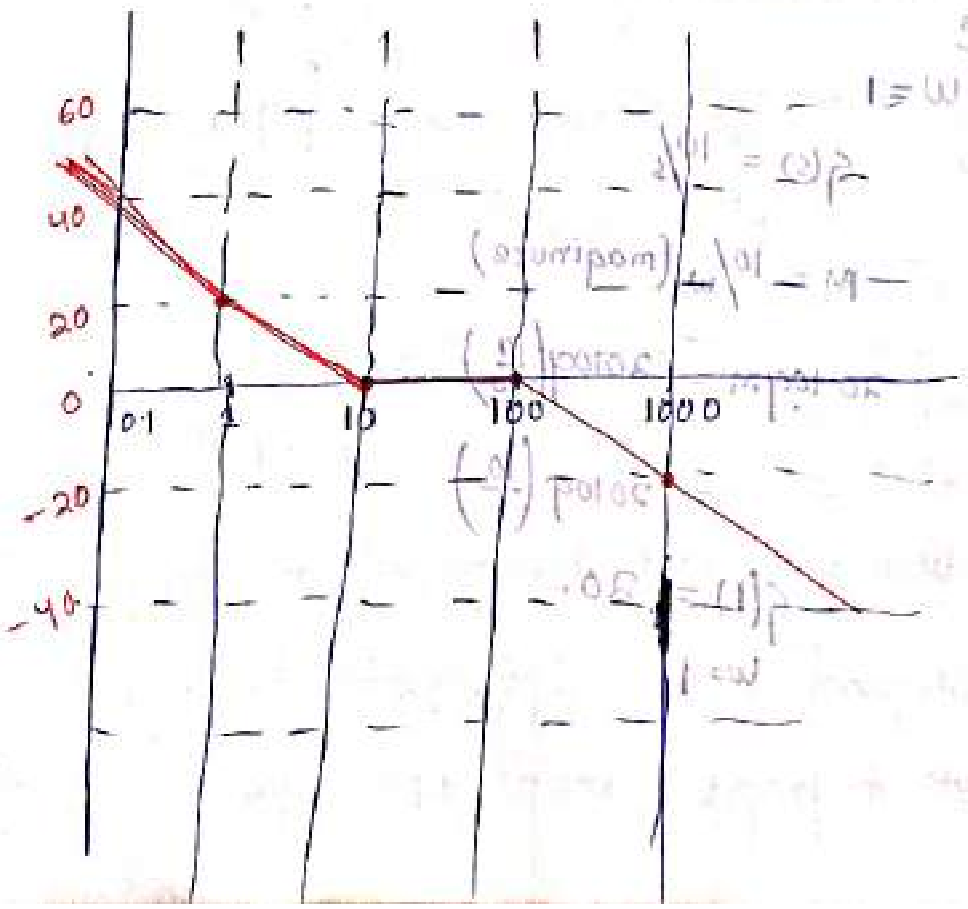
$$G_{100} = 0 \times \log\left(\frac{100}{10}\right) + G_{10}$$

$$= 0$$

Step-5

$$G_{1000} = -20 \log\left(\frac{1000}{100}\right) + G_{100}$$

$$= -20.$$



Q^n $\quad G(s) = \frac{5(s+1)(s+100)}{s^2(s+10)}$

Step-2

Sl NO	Factor	C.F	slope	change in slope
1	$50/s^2$	-	-40	-40
2.	$(1+\frac{1}{1}s)$	1	20dB	-20
3.	$(1+\frac{1}{10}s)$	10	-20dB	-40
4.	$(1+\frac{1}{100}s)$	100	$+20$dB	$-\cancel{6}0$ 2
5.				

Step-1 :- converting into time constant form we get

$$\frac{5(1+s)\,100\left(1+\frac{s}{100}\right)}{10s^2\left(1+\frac{s}{10}\right)}$$

$$= \frac{50\left(1+\frac{1}{1}s\right)\left(1+\frac{1}{100}s\right)}{s^2\left(1+\frac{1}{10}s\right)}$$

CF=1 CF=100 CF=10

Step-3 for $\omega = 0.1$

$G(s) = \frac{50}{s^2}$

$M = \frac{50}{\omega^2} \quad 20\log\left(\frac{50}{\omega^2}\right) = 20\log\frac{50}{(0.1)^2}$

$= 74$dB.

step-4

$$G_1 = -40 \times \log\left(\frac{1}{0.1}\right) + G_{0.1}$$

$$= -40 \times \log\left(\frac{1}{0.1}\right) + 74$$

$$= 34\,dB.$$

$$G_{10} = -20 \times \log\left(\frac{10}{1}\right) + G_1$$

$$= -20 + 34$$

$$= 14\,dB$$

slope that is coming to the [illegible]

present ω

$$G_{100} = -40 \times \log\left(\frac{100}{10}\right) + G_{10}$$

previous ω

previous gain

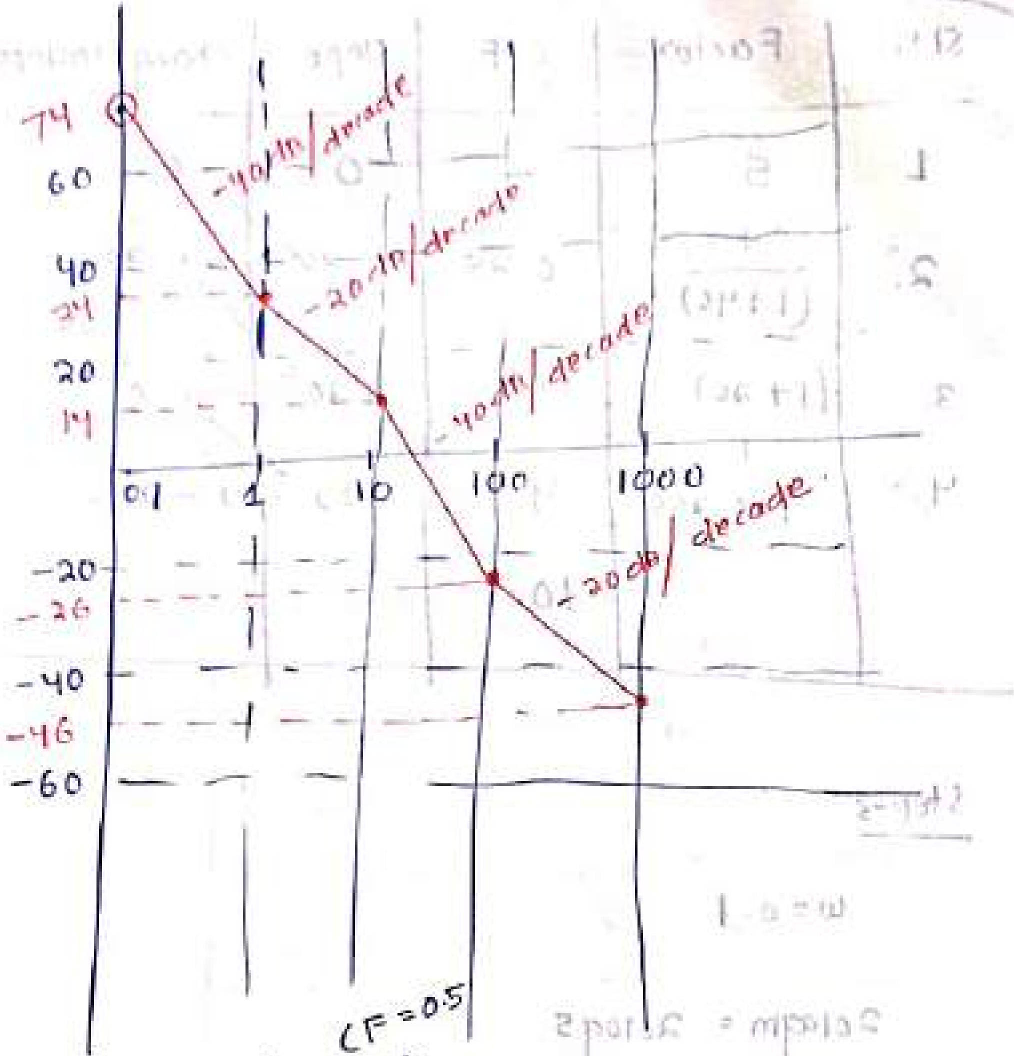
$$= -40 \quad 14$$

$$= -26$$

$$G_{1000} = -20 \times \log\left(\frac{1000}{100}\right) + G_{100}$$

$$= -20 \times \log(10) - 26$$

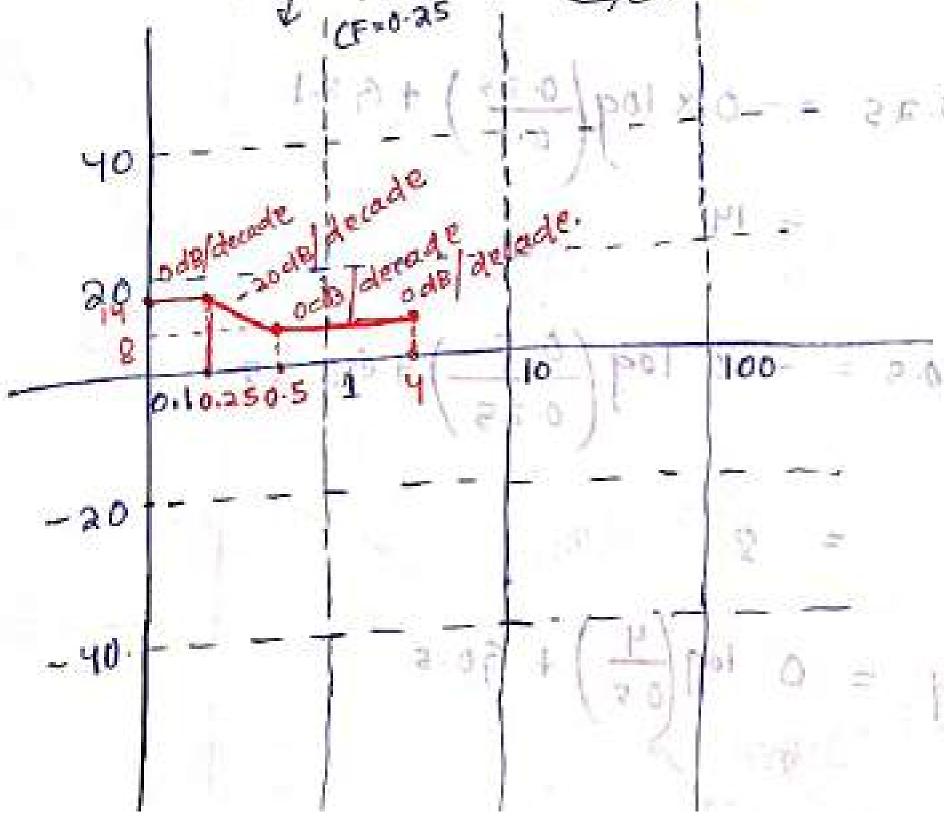
$$= -46\,dB.$$



CF = 0.5

Q. $G(s) = \dfrac{5(1+2s)}{(1+4s)(1+0.25s)}$

CF = 0.25   C.F = 4



step-2

Sl NO	Factor	C.F	slope	change in slope
1.	5	--	0	0
2.	$\frac{1}{(1+4s)}$	0.25	-20	-20
3.	$(1+2s)$	0.5	+20	0
4.	$\frac{1}{1+0.25s}$	4	-20	-20.
		10		

Step-3

$\omega = 0.1$

$20\log m = 20\log 5$

$= 14$

Step-4

$G_{0.25} = -0 \times \log\left(\frac{0.25}{0.1}\right) + G_{0.1}$

$= 14$

$G_{0.5} = -20 \log\left(\frac{0.5}{0.25}\right) + G_{0.25}$

$= 8$

$G_4 = 0 \log\left(\frac{4}{0.5}\right) + G_{0.5}$

$= 8.$

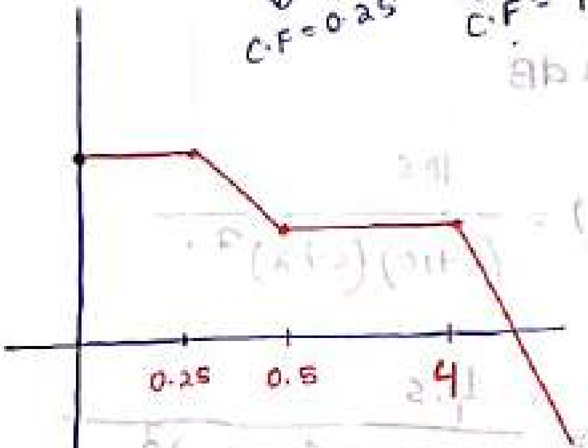
$G_{10} = -20\log\left(\frac{10}{4}\right) + G_4$

$= 0.02$

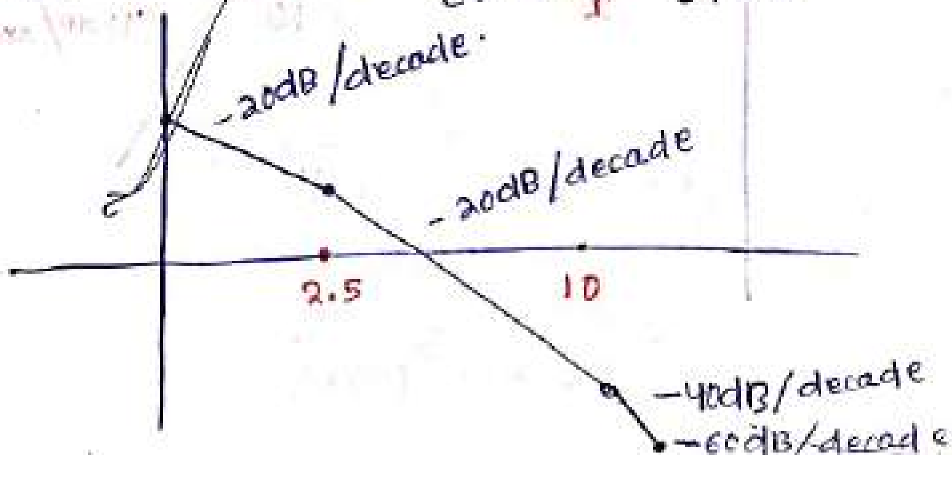
$= 0\,dB.$

Gate Method :-

$$G(s).\ H(s) = \frac{5(1+\underbrace{2s}_{C.F=0.5})}{\underbrace{(1+4s)}_{C.F=0.25}\underbrace{(1+0.25s)}_{C.F=4}}$$



Q^n_{11} $$G(s).\ H(s) = \frac{10}{s\underbrace{(1+0.4s)}_{C.F=2.5}\underbrace{(1+0.1s)}_{C.F=10}}$$



$$m = \frac{10}{\omega}$$

$$G_{25} = (-20)\log\left(\frac{2.5}{1}\right) + 20$$

$$20\log m = 20\log\left(\frac{10}{1}\right)$$

$$= 20$$

$$= 12.04$$

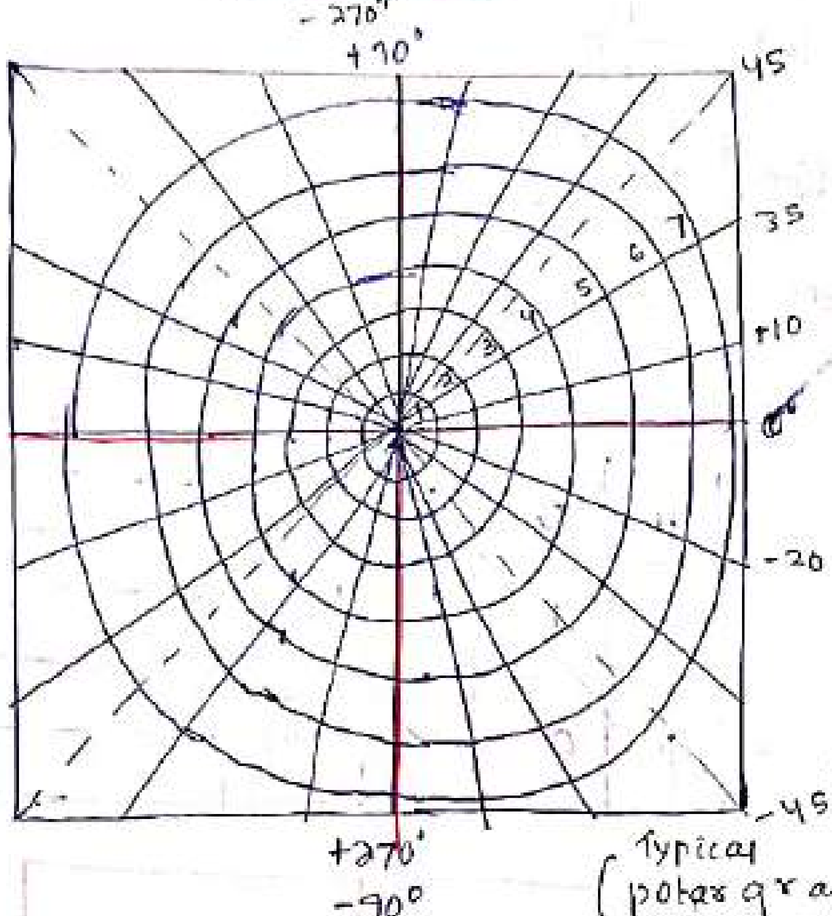
$$\simeq 12\,dB.$$

$$G_{10} = -40\log\left(\frac{10}{2.5}\right) + 12$$

$$= -12.08$$

$$\simeq -12\,dB$$

POLAR PLOT



Typical (polar graph sheet)

For loop T.F = $G(s)\,H(s)$

Sinusoidal T.F is $G(j\omega)\,H(j\omega)$

$$G(j\omega)\,H(j\omega) = \left|G(j\omega)\,H(j\omega)\right| \angle G(j\omega)\,H(j\omega)$$

$$= r \angle \theta$$

$r = f(\omega)$ (function of ω)

$\theta = f(\omega)$ (").

polar plots are plotted on polar graph sheets)

- The plot of magnitude (r) vs phase (θ) when ω is varied from 0 to ∞ is called polar plot.

- Concentric circle on polar graph sheet represents magnitudes & radial lines represents phase angle.

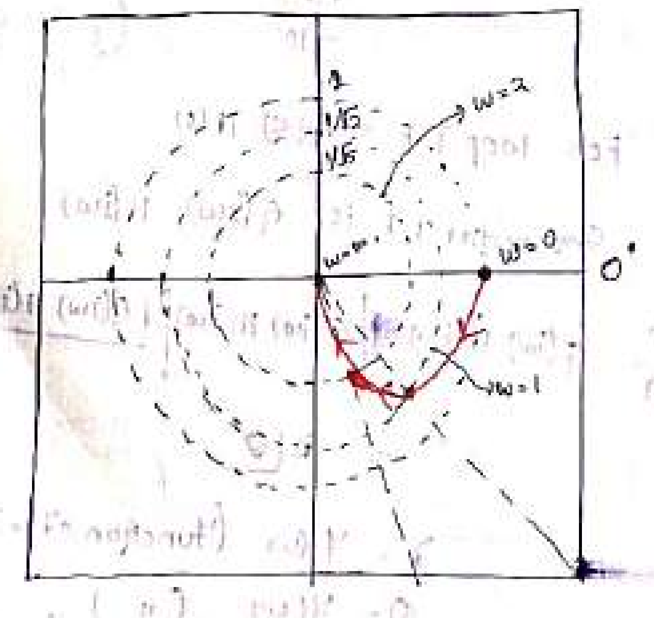
e.g.:- $G(s) = \frac{1}{1+s}$

$G(j\omega) = \frac{1}{1+j\omega}$

$M = |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$

$\phi = \angle G(j\omega) = -\tan^{-1}(\omega)$.

ω	0	1	2	- - - -	∞
M	1	$\frac{1}{\sqrt{2}}$	$1/\sqrt{5}$	- - - -	0
ϕ	0	-45°	-63.4°	- - - -	(-90°)



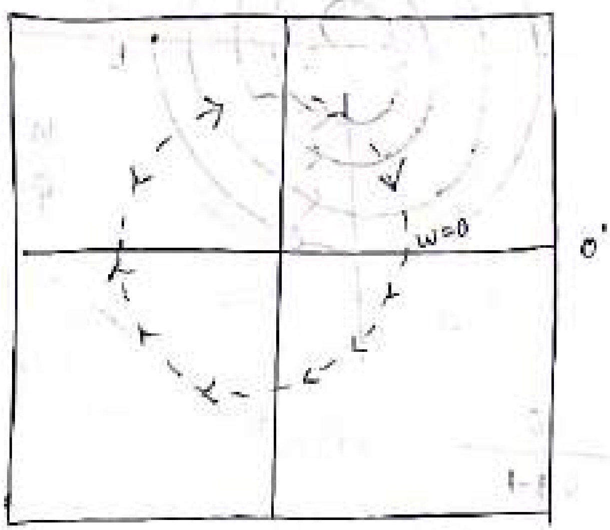
Q // $G(s) = e^{-3s}$ (transportation lag)

$G(j\omega) = e^{-3j\omega}$

$|G(j\omega)| = 1$

$\angle G(j\omega) = -(3\omega)$

ω	0	1	- - -	∞
M	1	1	— —	1
ϕ	0	-3ω	—	$-\infty$



Q // $G(s) = \dfrac{e^{-3s}}{1+2s}$

$G(j\omega) = \dfrac{e^{-3j\omega}}{1+2j\omega}$

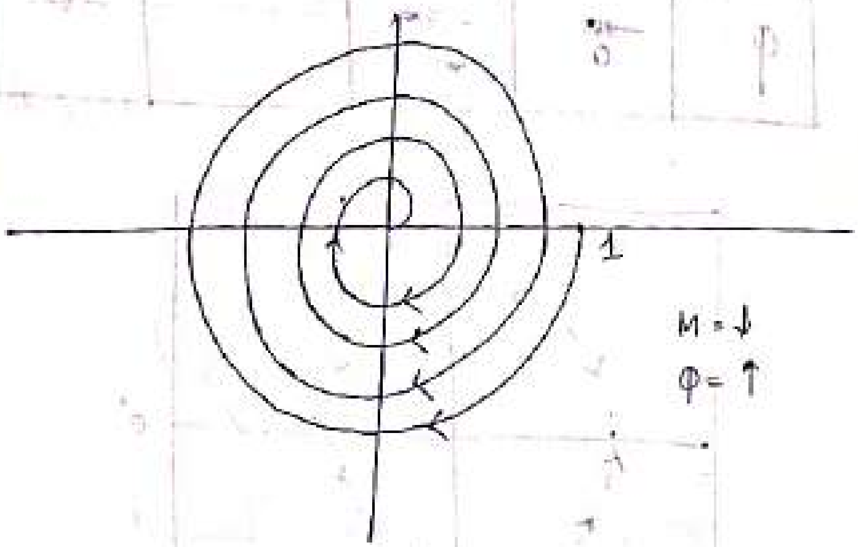
$|G(j\omega)| = \dfrac{1}{\sqrt{1+2^2\omega^2}} = \dfrac{1}{\sqrt{1+4\omega^2}}$

$\angle G(j\omega) = -3\omega - \tan^{-1}(2\omega)$.

ω	0	1	2	...	∞
M	1	$1/\sqrt{5}$	$1/\sqrt{17}$		0
ϕ	0°	$-3-45$ $=-48^\circ$	-81°	...	...

$$M = \frac{1}{\sqrt{1+4\omega^2}}$$

$$\phi = -3\omega - \tan^{-1}(2\omega).$$



$M = \downarrow$
$\phi = \uparrow$

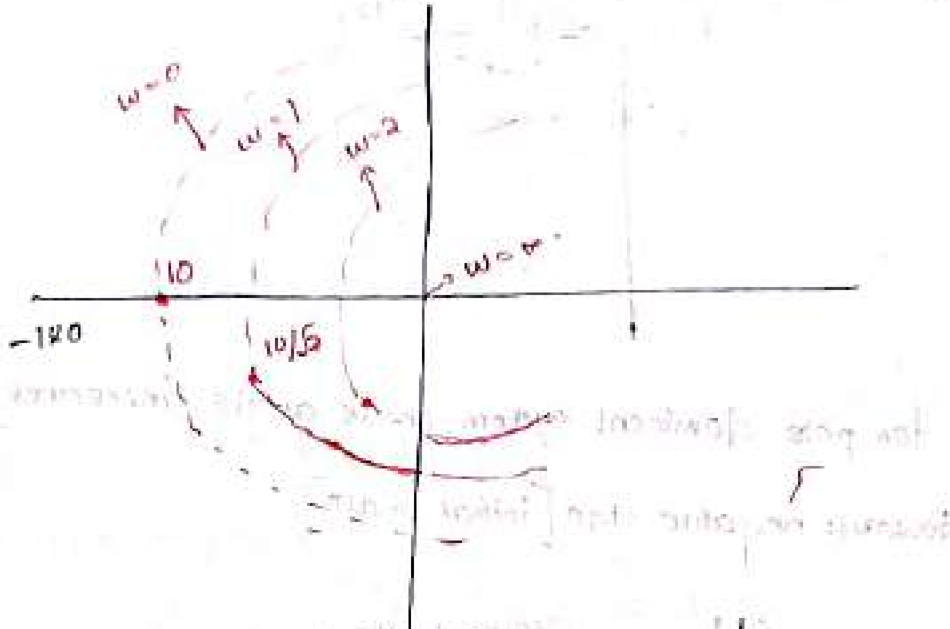
Q^n // $G(s) = \dfrac{10}{s-1}$

$$G(j\omega) = \frac{10}{j\omega - 1}$$

$$|G(j\omega)| = \frac{10}{\sqrt{1+\omega^2}} \qquad -\left(\pi - \tan^{-1}(\omega)\right)$$

$$\angle G(j\omega) = -\pi + \tan^{-1}(\omega).$$

ω	0	1	2	----	∞
M	10	$10/\sqrt{2}$	----		0
ϕ	$-180°$	$-135°$	----		$-90°$

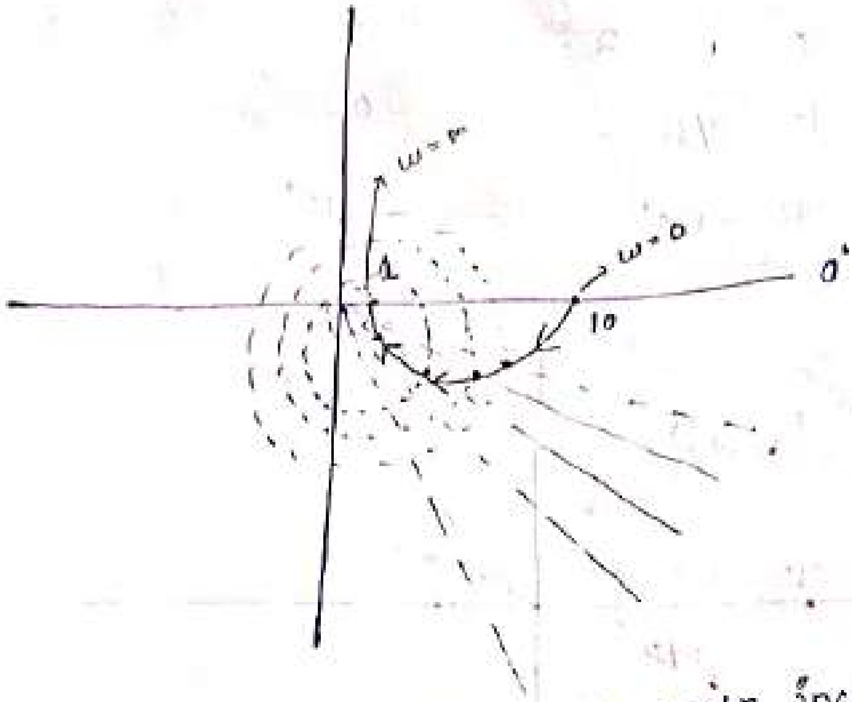


Q11 $G(s) = \dfrac{s+10}{s+1}$

$$M = \frac{\sqrt{100+\omega^2}}{\sqrt{1+\omega^2}}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{1}\right)$$

ω	0	1	2	10	100	----	∞
M	10	7.10	4.56	1.4		----	1
ϕ	0	-39.2	-52.1	-39.2	-5	----	0



- For pole dominent system phase angle increases towards negative side [initial phase].

Q) - $G(s) = \frac{s+1}{s+10}$, zero dominate.

$$G(j\omega) = \frac{j\omega+1}{j\omega+10}$$

$$|G(j\omega)| = \frac{\sqrt{\omega^2+1}}{\sqrt{100+\omega^2}}$$

$$\angle G(j\omega) = \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{10}\right)$$

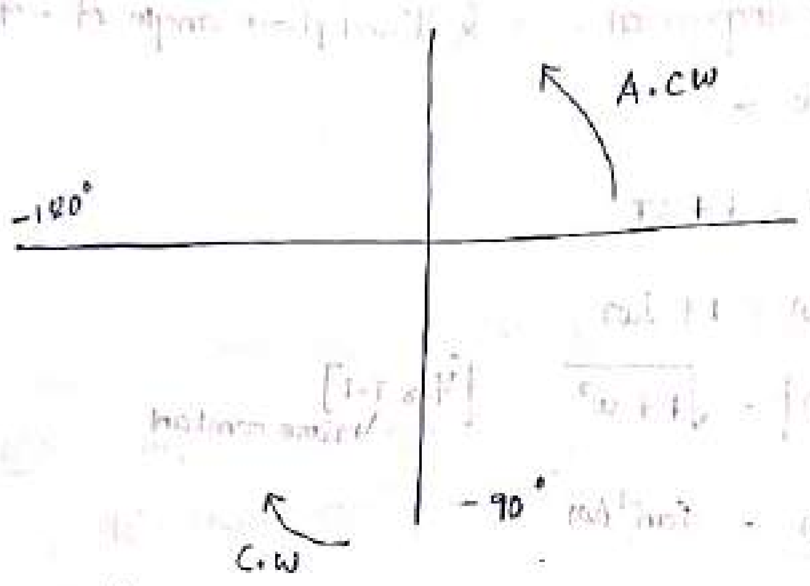
ω	0	1	20	100	100	∞
M	0.01	0.14	0.219	0.710	.01	1
φ	0	39.28	52.12	32.28	5.13	0

NOTE - Finite zero always contributes initial phase angle of zero degrees at $\omega = 0$ & final phase angle of $+90^\circ$ at $\omega = \infty$.

Q. $$G(s) = \frac{10(s+10)}{s(s+1)(s+6)}$$

$\omega = 0$; $\phi = -90^\circ$, $M = \infty$ $\left[0^\circ - j90^\circ\right]$

$\omega = \infty$; $\phi = -180^\circ$, $M = 0$



For GATE

procedure for polar plots :-

step-1
$\Rightarrow$ Find the initial magnitude, phase. Final magnitude, phase

i.e $\omega = 0$, $M_1 \lfloor \phi_1$

$\omega = \infty$, $M_2 \lfloor \phi_2$

step-2
$\Rightarrow$ Observe the transfer function for pole dominence or zero dominence.

Step-3:-

starting direction-

pole dominent $\rightarrow$ it is clockwise.

zero dominent $\rightarrow$ it is anticlockwise.

Step-4:-

Ending direction:-

$$\phi = \phi_2 - \phi_1$$

= Final phase - Initial phase.

If ϕ is positive, it is anticlockwise direction.

ϕ is negative, it is clockwise direction.

Step-5:-

Join starting & Ending directions to get polar plots.

NOTE:- (i) when zeroes are present, always use minimum path for joining starting & ending directions.

(ii) The above method is applicable to Min^m phase transfer functions only.

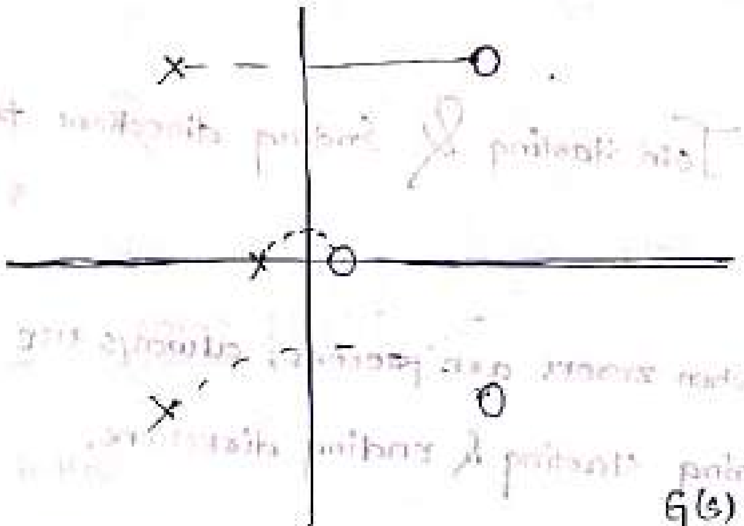
(iii) when all the poles and zero are lieing on the left half of s-plane, then the transfer function is called minimum phase transfer function.

e.g:- $G(s) = \frac{10(s+1)}{s(s+3)(s+4)}$ - stable Minimum phase
(as poles and zeroes are lying on left half)

$G(s) = \frac{10(s-1)}{s(s+3)(s+4)}$ - stable Non-Minm phase

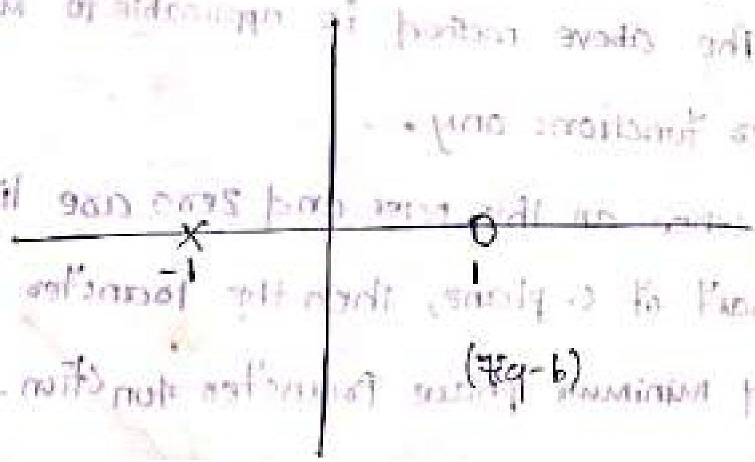
$G(s) = \frac{10(s+1)}{s(s-3)(s+4)}$ - unstable & Non-minm phase.

- when each & every pole on the left half of s-plane has mirror image of zero in the right half of s-plane then the transfer function is called <u>All pass Transfer function</u>



$G(s) = \frac{s-1}{s+1}$

$M = 1$



(79-b)

e.g

$$G(s) = \frac{s-1}{s+1}$$

its pole zero plot is shown in fig (b)

M = 1 , Independent of frequency.

i.e the system allows all the frequencies.

Therefore it is called Allpass T.F.

Q 11 $$G(s) = \frac{1}{s(s+1)(s+2)}$$

step-1 Initial - $\infty \angle -90^\circ$

final - $0 \angle -270^\circ$

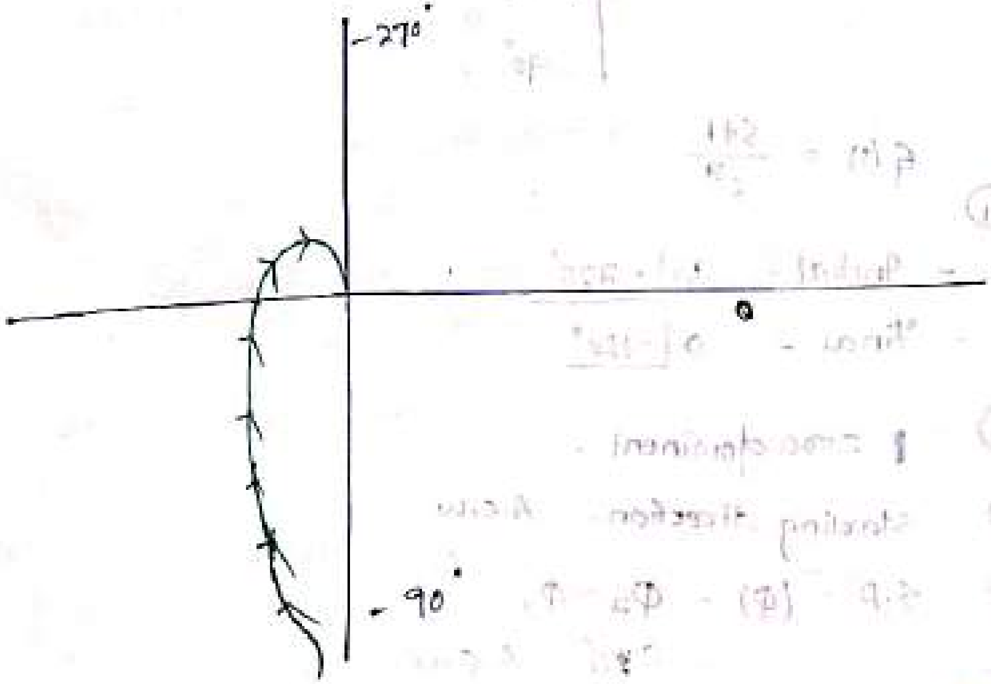
step-2 pole dominence

step-3 starting direction - C.W

step-4 Ending direction - $(\phi) = \phi_2 - \phi_1$

$= -270 + 90$

$= -180^\circ$



Q^n $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$

(1) Initial - $\frac{1}{6}\angle 0^\circ$

Final - $0 \angle -270^\circ$

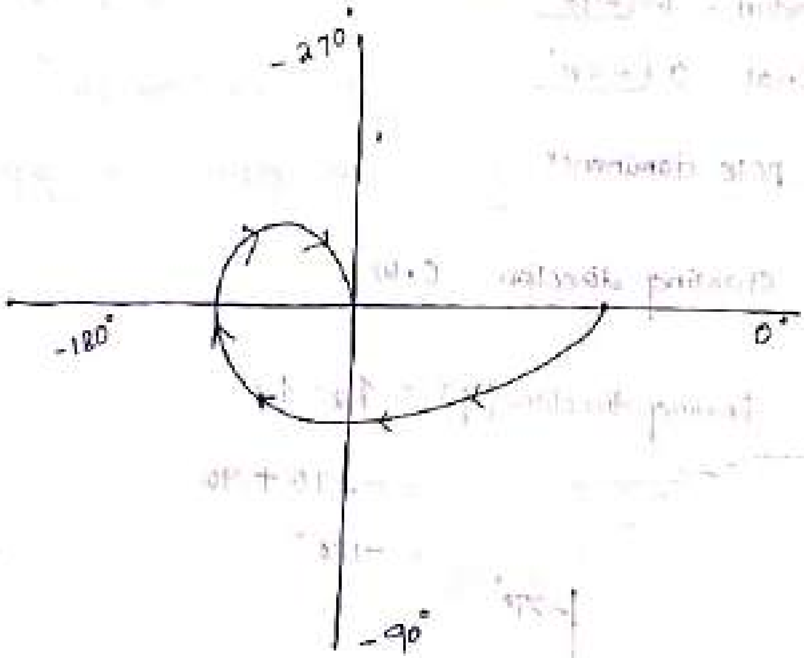
(2) - pole dominent.

(3) starting direction - C.W

(4) S.D - $(\phi) = \phi_2 - \phi_1$

$= -270 - 0$

$= -270$ C.W.



Q^n $G(s) = \dfrac{s+1}{s^3}$

(1)

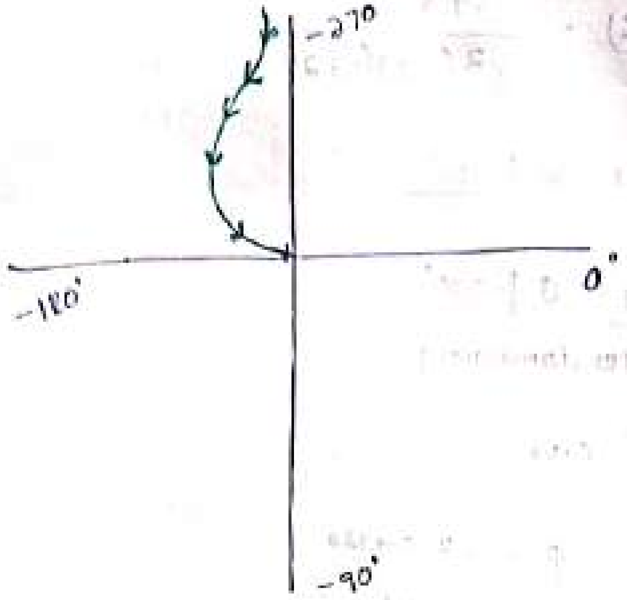
- Initial - $\infty \angle -270^\circ$
- final - $0 \angle -180^\circ$

(2) zero dominent.

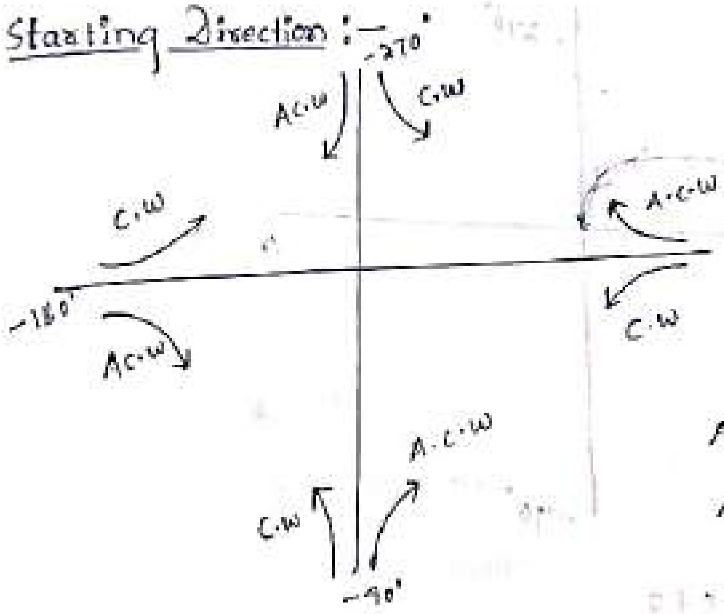
(3) starting direction - A.C.W

(4) S.D - $(\phi) = \phi_2 - \phi_1$

$= 90^\circ$ A.C.W.



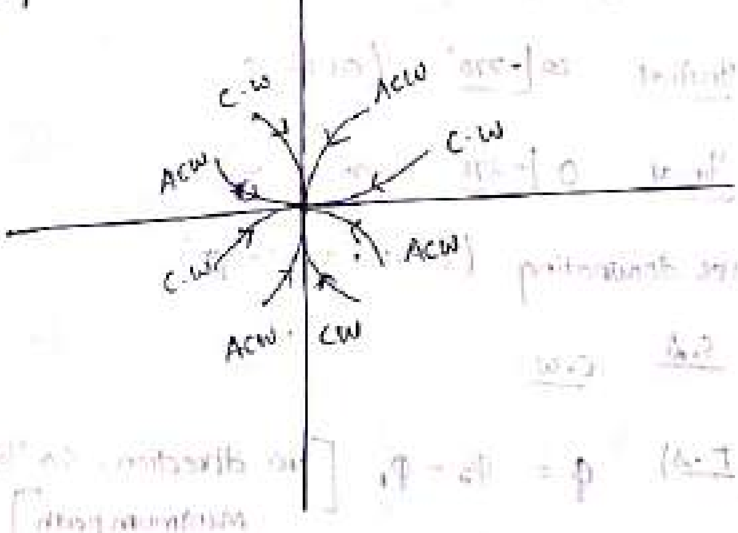
Starting Direction :-



Angle is decreasing - means $-270 \to -180° \to -90°$ A.C.W.

Angle in increasing means $-90 \to -180 \to -270°$ then clock wise.

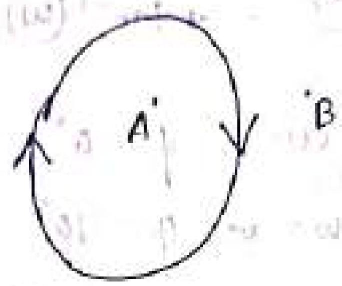
Ending direction



Nyquist plot

Encircled:- If a pt lies inside the path then it will said to be encircled by the path.

Enclosed:- If a point lies to the right hand side of the path then it will said to be enclosed by the path.



A - is enclosed in anti clock wise

B - ~~Not enclosed~~. Enclosed
Not encircled.

A → Encircled in c.w
→ enclosed

B → Not encircled
→ Not enclosed.

Analytic function:-

If a function & its derivatives exists at a point then function is said to be analytic at that point.

e.g:- $F(s) = \dfrac{10(s+1)(s-2)}{(s+3)(s+4)(s+5)}$

F(s) is not analytic at its poles & zeros.

i.e $s=-1, s=2, s=-3, s=-4,$
$s=-5.$

principle of Arguement :-

If n no. of poles & m no of zeros of the function F(s) are enclosed by s-plane contour & if this contour is mapped upto F(s) it will encircle the origine N times in anticlock wise direction.

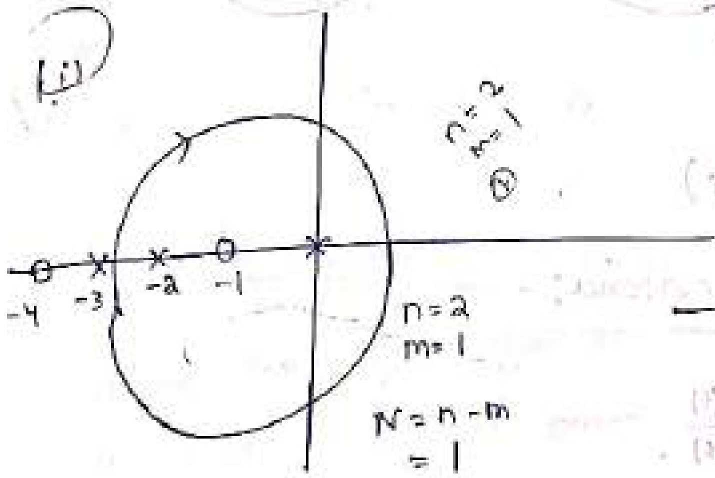
Where N = ~~ ~~ n - m

if N → +ve (A.C.W encirclement)

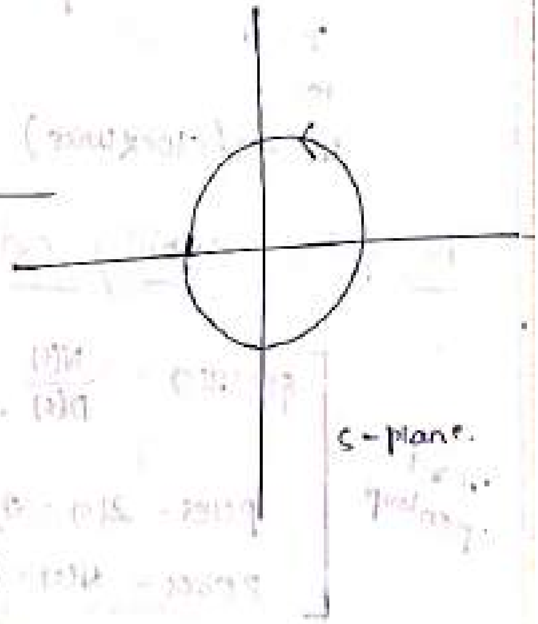
N → -ve (C.W encirclement)

N → 0 (No encirclement)

(i)

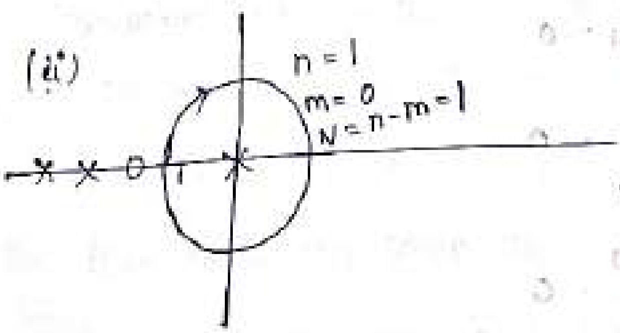


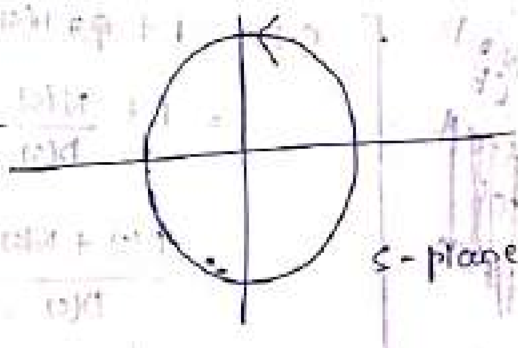
(s-plane)



s-plane.

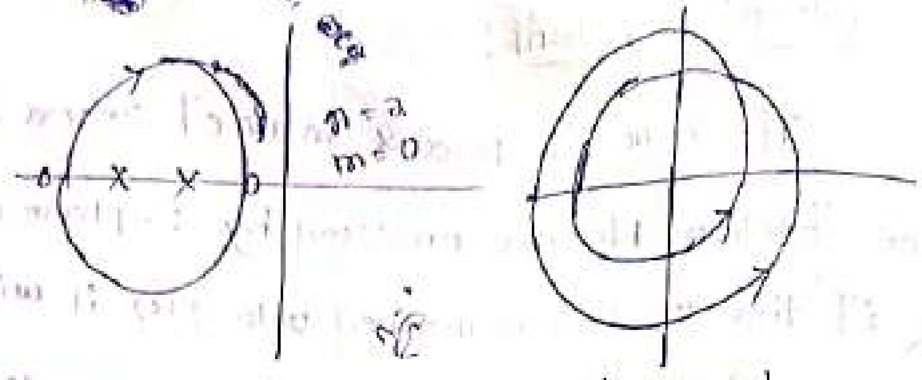
(ii)



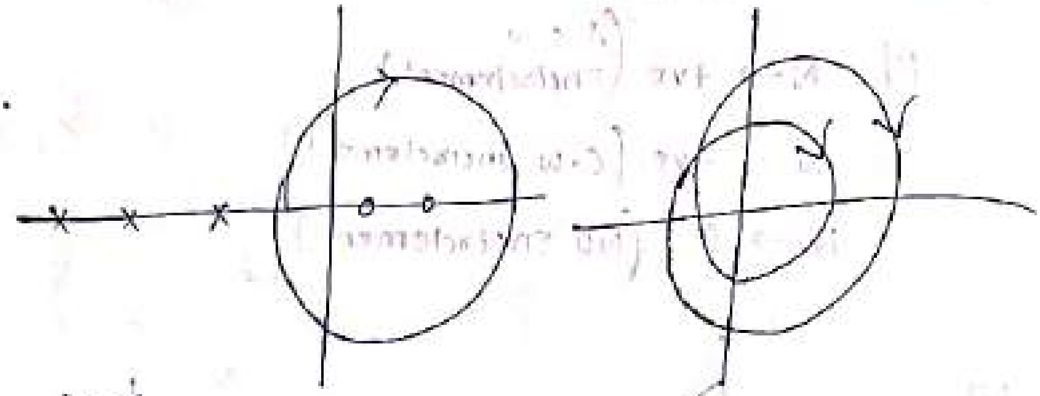


s-plane

3.



4.



$n = 0$

$m = 2$

$N = -2$ (clockwise)

Nyquist Stability criteria:-

w.r.t openloop

$$G(s)H(s) = \frac{N(s)}{D(s)}.$$

poles - $D(s) = 0$

zeros - $N(s) = 0$

w.r.t CE

closed loop system.

$$C.E = 1 + G(s)H(s) = 0$$

$$= 1 + \frac{N(s)}{D(s)} = 0$$

$$\Rightarrow \frac{D(s) + N(s)}{D(s)} = 0$$

$$\Rightarrow \frac{N'(s)}{D(s)} = 0$$

w.r.t CE

poles $D(s) = 0$

zeros $N'(s) = 0$

w.r.t control system:

$$TF = \frac{G(s)}{1 + G(s).H(s)}.$$

poles of closed loop – $1 + G(s).H(s) = 0$

$N'(s) = 0$

- therefore we can conclude that poles of openloop control system are poles of C.E.
- poles of closed loop control system are zeros of CE.

- $CE = 1 + G(s).H(s) = 0$

or

$G(s).H(s) = -1$

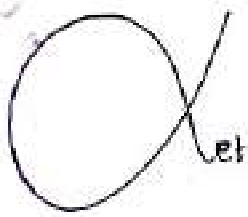
If we choose, an arbitrary contour in s-plane such that, it encloses entire right half of s-plane & if this contour is mapped onto open loop transfer function $G(s).H(s)$. we can predict the stability of of closed loop control system by observing the no of encirclement of the point $(-1, 0)$. If the mapping of contour encircles the point $(-1, 0)$ in clockwise direction, then definitely some closed loop poles will be present in the right half of s-plane & closed loop system is unstable.

2) If the mapping of contour encircles the point (−1, 0) in anticlockwise direction then,

(a) closed loop system is stable if No of encirclements is equal to no. of openloop poles in the right half of 's' plane.

(b) the ~~open~~ closed loop system is unstable if no. of encirclements is not equal to No of Open loop poles in the right half of s-plane.

Mapping of contour :−



Let,

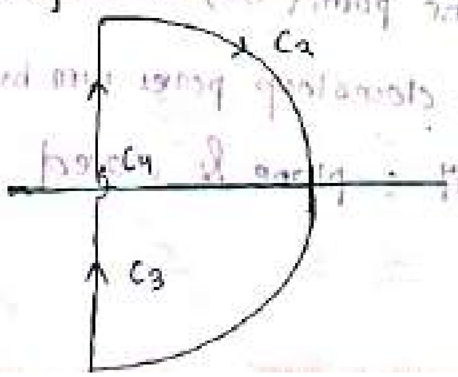
$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots(1+sT_m)}{s^x(1+sT_1')(1+sT_2')\cdots(1+sT_y')}$$

m = No of ~~poles~~ zeroes

n = x + y = No of poles

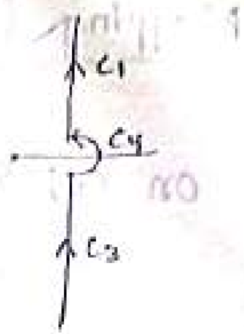
x = No of poles at origin.

choosing s− plane contour as shown in figure.



Section C_1

$s = j\omega, \; \omega \to 0 \text{ to } \infty$



Section - C_2

$s = \lim R \cdot e^{j\theta}$

$R \to \infty$

$\theta \to \frac{\pi}{2}$ to $-\frac{\pi}{2}$

Section - C_3

$s = j\omega, \left(\omega \to -\infty \text{ to } 0\right)$

$s = -j\omega \; \left(\omega \to \infty \text{ to } 0\right)$

at section C_4

$s = \lim R e^{j\theta}$

$R \to 0$

$\theta \to -\pi/2$ to $\pi/2$.

C_1 Mapping of C_1:-

$s = j\omega$

$\omega = 0 \text{ to } \infty$

$G(j\omega)\, H(j\omega) = \left|G(j\omega)\, H(j\omega)\right| \angle G(j\omega)\, H(j\omega)$

$= M \angle \phi \; (\omega \to 0 \text{ to } \infty)$

mapping of C_1 is polar plot of $G(s) \cdot H(s)$.

Mapping Of C_2 :-

on C_2; $s = \lim Re^{j\theta}$

$R \to \infty$

$\theta \to \frac{\pi}{2}$ to $-\frac{\pi}{2}$

ignoring 1, compare to the magnitude of s in $(1+sT)$

$(1+sT) \approx sT$

$$G(s) \cdot H(s) = \frac{k\, sT_1 \cdot sT_2 \cdots sT_m}{s^{x} sT_1' \cdot sT_2' \cdots sT_y'}$$

$$= \frac{K' s^m}{s^x \cdot s^y}$$

$$= \frac{K s^m}{s^n}$$

$$= \frac{K}{s^{n-m}}$$

$$G(s) \cdot H(s) = \frac{K}{s^{n-m}}$$

$$= \frac{K}{(Re^{j\theta})^{n-m}}$$

$$= \underbrace{0 \cdot e^{-j\theta(n-m)}}$$

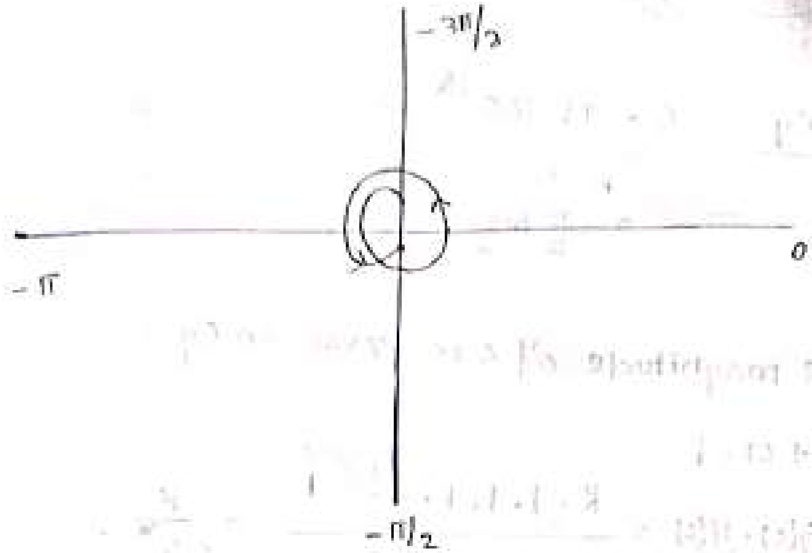
(It represents an arc of magnitude zero 0 varying from $-(n-m)\frac{\pi}{2}$ to $+(n-m)\frac{\pi}{2}$

e.g. $G(s) \cdot H(s) = \frac{K}{s(s+2)(s+3)}$.

$n = 3$

$m = 0$

$-(n-m)\frac{\pi}{2}$ to $(n-m)\frac{\pi}{2}$



Mapping OF C_2 :-

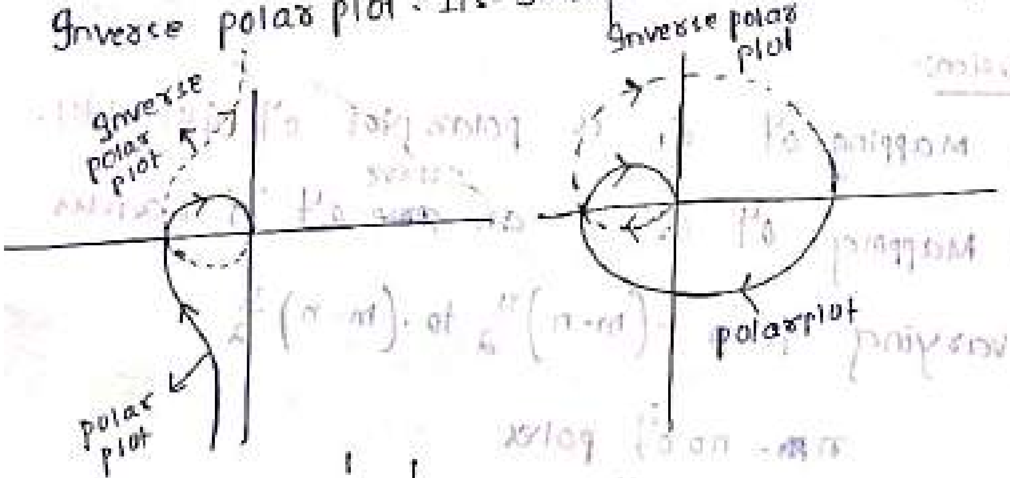
C_2, $s = j\omega$

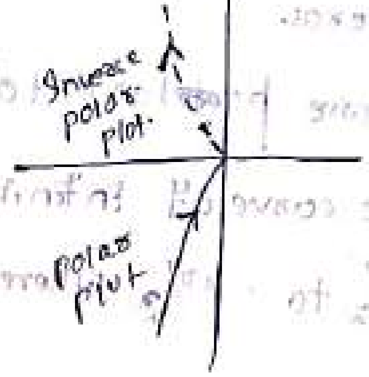
ω varies from $-\infty$ to 0.

$$G(s) \cdot H(s) \Big|_{s=j\omega} = G(j\omega)\,H(j\omega).$$

polar plot when ω is varied from $-\omega$ to 0 is called

Inverse polar plot. Its Image is opposite to polar plot.





Mapping of C_4:-

On C_4 $\quad s = \lim Re^{j\theta}$

$R \to 0$

$\theta \; -\frac{\pi}{2}$ to $\frac{\pi}{2}$

as the magnitude of s is zero on C_4

$1 + sT = 1$

$$G(s) \cdot H(s) = \frac{K \cdot 1 \cdot 1 \cdot 1 \cdots 1}{s^n \; 1 \cdot 1 \cdots} = \frac{K}{s^n}.$$

$$= \frac{K}{(Re^{j\theta})^n}$$

It is an arc of infinite radius varying from

$$= \infty \; e^{-j\theta n}$$

$\boxed{n\frac{\pi}{2} \text{ to } -n\frac{\pi}{2}}$ where n is type no of the system.

$$-\theta n \left(\theta \to \left(-\frac{\pi}{2} \text{ to } \frac{\pi}{2}\right) \right)$$

$n\frac{\pi}{2}$ to $-n\frac{\pi}{2}$

Conclusion:-

(1) Mapping of C_1 is polar plot of $G(s) \cdot H(s)$.

(2) Mapping of C_2 is an curve of 0 radius varying from $-(m-n)\frac{\pi}{2}$ to $(m-n)\frac{\pi}{2}$

n - no of poles

m - no of zeros.

(3) Mapping of C_3 is inverse polar plot of $G(s) \cdot H(s)$.

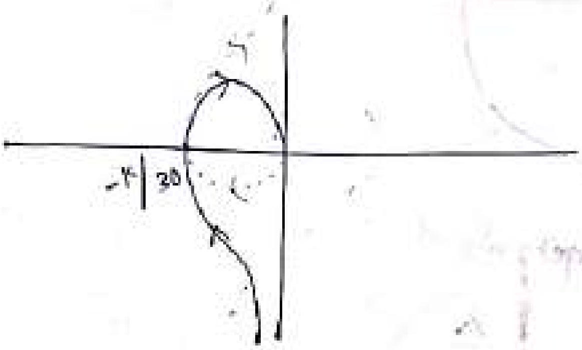
(4) Mapping C_4 is the curve of infinite radius varying from $n\pi/2$ to $-n\pi/2$ where n = type number

e.g.:- $G(s) \cdot H(s) = \dfrac{K}{s(s+2)(s+3)}$.

Sketch the Nyquist plot.

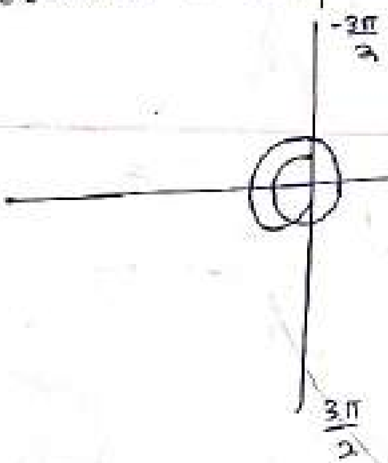
C1 Polar plot

No zero
Type-1
order-3

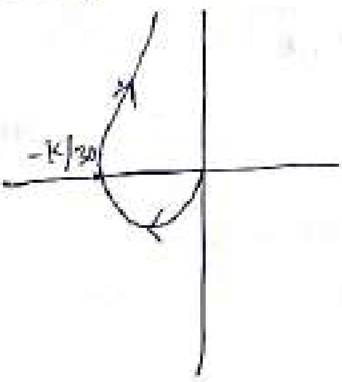


C2 $n = 3$
$m = 0$

Curve of '0' magnitude from $-\frac{3\pi}{2}$ to $\frac{3\pi}{2}$.



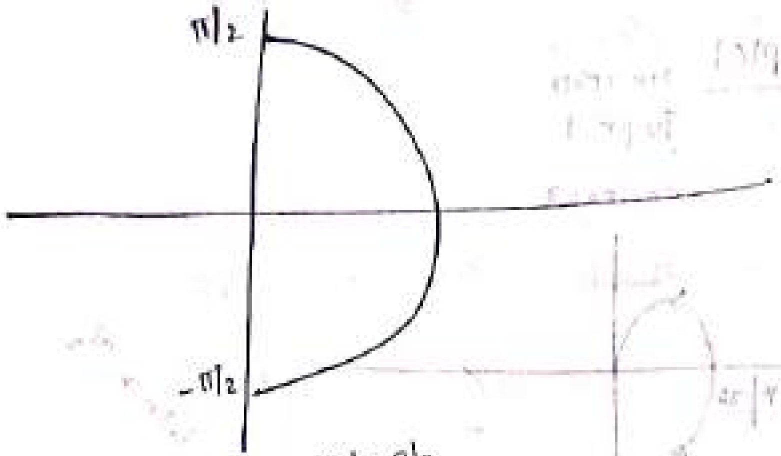
C3 Inverse polar plot:-



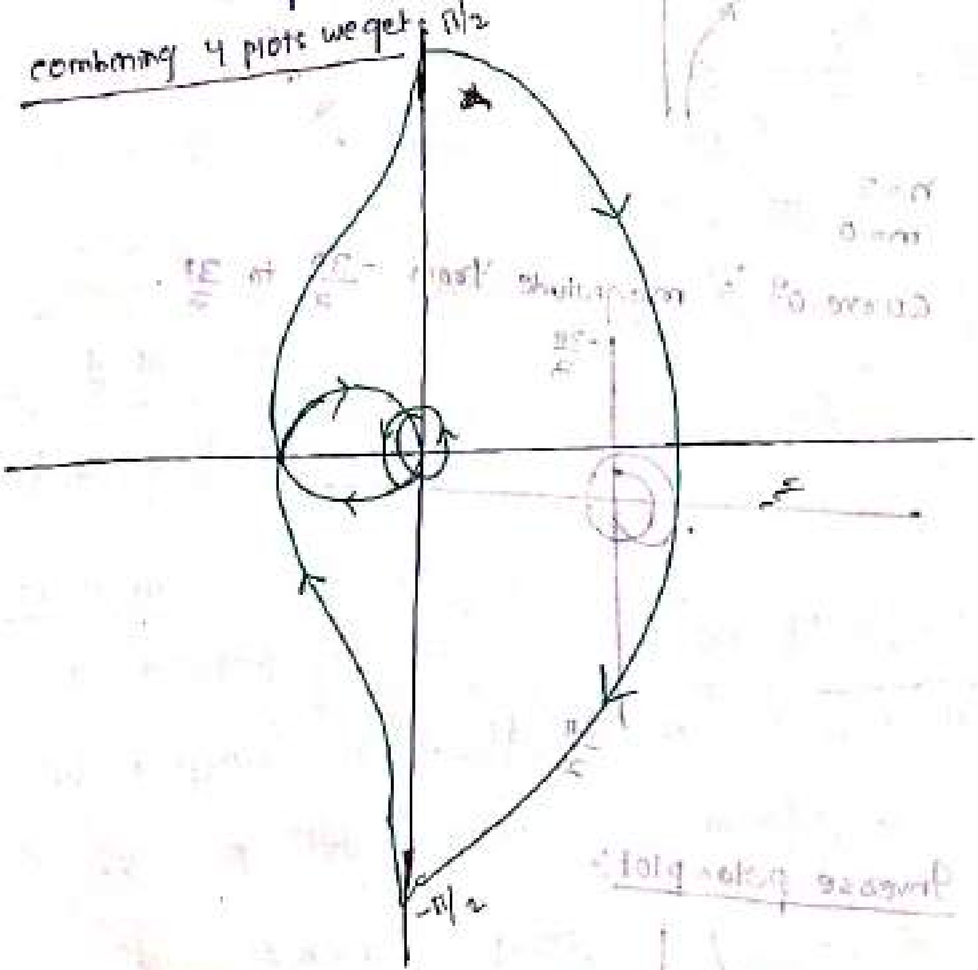
<u>C4</u>

<u>$x=1$</u> Curve $x\frac{\pi}{2}$ to $-x\,\pi/2$

$x^{|Q|}=1$ 'ca type. 1



<u>combining 4 plots we get:</u>



After putting nyquist plot observe the ~~fn~~ encirclements of the point (−1,0)

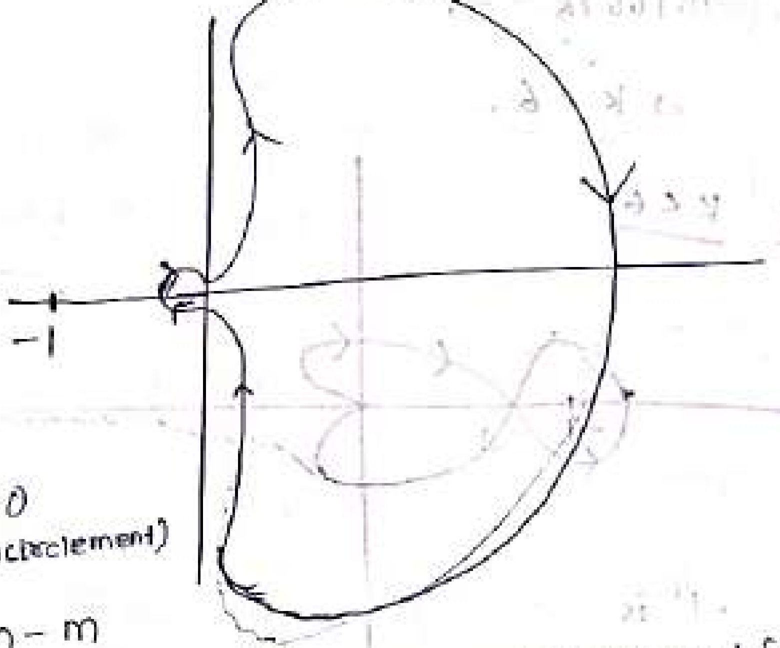
$$N = \underbrace{n}_{\text{No of openloop poles in the right half of s plane}} - \underbrace{m}_{\text{No of closed loop poles in the right half of s-plane}}$$

N → no. of encirclements of (−1,0) by Nyquist plot

$N = n - m$

Q^n// Comment on the stability of a c.s −

$$G(s).\,H(s) = \frac{s}{s(1-s)}$$ The Nyquist plot of the system is shown in the figure.



$N = 0$
(no encirclement)

$N = n - m$

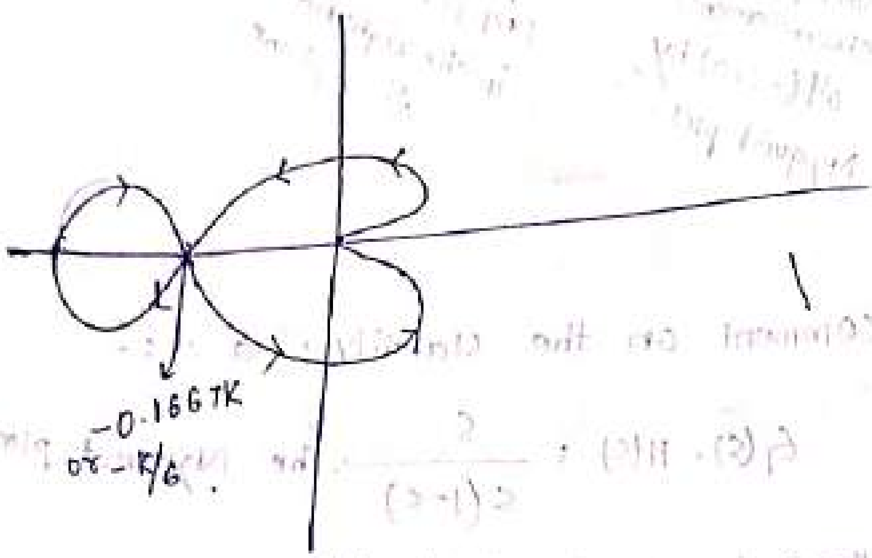
$0 = 1 - m$ ($n = 1$ one pole is present in right half)

$\Rightarrow m = 1$

↳ one pole is present in the right half of closed loop T.F so System is unstable.

Q. what is the range of K for stability of a system with loop T.F

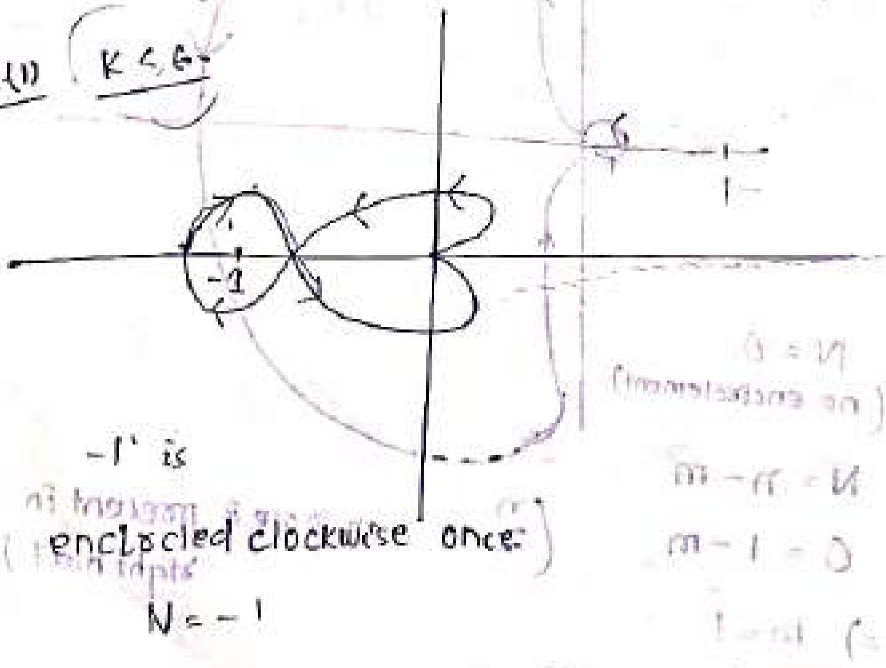
$$G(s) \cdot H(s) = \frac{K(1+0.5s)(1+s)}{(1+10s)(s-1)}$$



$-0.1667K = -1$

$\Rightarrow K = 6$

Case (1) $K < 6$



-1 is encircled clockwise once.

$N = -1$

$N = n - m$

$-1 = 1 - m$

$\Rightarrow m = 2$

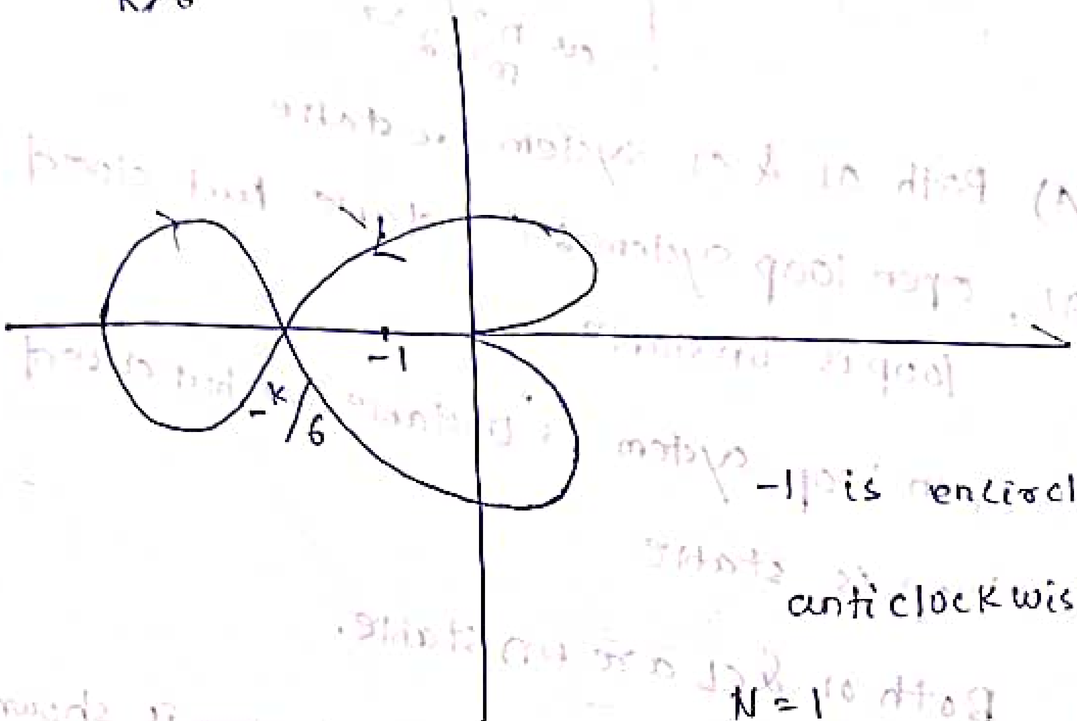
two closed loop poles are

present are present at right half of s - plane

so closed loop pole is unstable.

case-2

K > 6



-1 is encircled

anticlockwise once.

N = 1

$N = n - m$

$\Rightarrow 1 = 1 - m$

$\Rightarrow m = 0$ ← stable