C.V. RAMAN POLYTECHNIC, BHUBANESWAR

LECTURE NOTE

STRUCTURAL DESIGN - I, (Th.1)

SEM-4TH

BRANCH- CIVIL ENGINEERING

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Introduction

Reinforced concrete, as a composite material, has occupied a special place in the modern construction of different types of structures due to its several advantages. Due to its flexibility in form and superiority in performance, it has replaced, to a large extent, the earlier materials like stone, timber and steel. Further, architect's scope and imaginations have widened to a great extent due to its mouldability and monolithicity. Thus, it has helped the architects and engineers to build several attractive shell forms and other curved structures. However, its role in several straight line structural forms like multistoried frames, bridges, foundations etc. is enormous.

Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions.

Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones.

Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

Properties of Concrete

The important properties of concrete, which govern the design of concrete mix are as follows

(i) Weight

The unit weights of plain concrete and reinforced concrete made with sand, gravel of crushed natural stone aggregate may be taken as 24 KN/m3 and 25 KN/m3 respectively.

(ii) Compressive Strength

With given properties of aggregate the compressive strength of concrete depends primarily on age, cement content and the water cement ratio are given Table 2 of IS 456:2000. Characteristic strength are based on the strength at 28 days. The strength at 7 days is about two-thirds of that at 28 days with ordinary portland cement and generally good indicator of strength likely to be obtained.

(iii) Increase in strength with age

There is normally gain of strength beyond 28 days. The quantum of increase depends upon the grade and type of cement curing and environmental conditions etc.

(iv) Tensile strength of concrete

The flexure and split tensile strengths of various concrete are given in IS 516:1959 and IS 5816:1970 respectively when the designer wishes to use an estimate of the tensile strength from compressive strength, the following formula can be used

Flexural strength, $f_{cr}=0.7\sqrt{f_{ck}}$ N/mm2

(v) Elastic Deformation

The modulus of elasticity is primarily influenced by the elastic properties of the aggregate and to lesser extent on the conditions of curing and age of the concrete, the mix proportions and the type of cement. The modulus of elasticity is normally related to the compressive characteristic strength of concrete

$E_c=5000\sqrt{f_{ck}}$ N/mm2

Where E_c= the short-term static modulus of elasticity in N/mm2

f_{ck}=characteristic cube strength of concrete in N/mm2

(vi) Shrinkage of concrete

Shrinkage is the time dependent deformation, generally compressive in nature. The constituents of concrete, size of the member and environmental conditions are the factors on which the total shrinkage of concrete depends. However, the total shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing for a given humidity and temperature. The cement content, however, influences the total shrinkage of concrete to a lesser extent. The approximate value of the total shrinkage strain for design is taken as 0.0003 in the absence of test data (cl. 6.2.4.1).

(vii) Creep of concrete

Figure1.1: Stress-strain curve of concrete

Creep is another time dependent deformation of concrete by which it continues to deform, usually under compressive stress. The creep strains recover partly when the stresses are released. Figure 1.2.2 shows the creep recovery in two parts. The elastic recovery is immediate and the creep recovery is slow in nature.

Thus, the long term deflection will be added to the short term deflection to get the total deflection of the structure. Accordingly, the long term modulus *Ece* or the effective modulus of concrete will be needed to include the effect of creep due to permanent loads. The relationship between *Ece* and *Ec* is obtained as follows:

$$c_c = \frac{f}{E_c}$$

Where, c_c=short term strain at the age of loading at a stress value of f_c

θ=creep co-efficient = f_c/E_c

s_{cr}=ultimate creep strain

The values of θ on 7th, 28th and 365th day of loading are 2.2, 1.6 and 1.1 respectively.

Then the total strain= $\varepsilon_c + \varepsilon_{cr} = \frac{f}{E_{ce}}$

Where, *Ece* = effective modulus of concrete.

From the above Equation, we have

$$E_{ce} = \frac{E_c}{1+\theta}$$

The effective modulus of *Ece* of concrete is used only in the calculation of creep deflection.

It is seen that the value of creep coefficient θ is reducing with the age of concrete at loading.

It may also be noted that the ultimate creep strain ε_{cr} does not include short term strainε_c.

The creep of concrete is influenced by

• Properties of concrete

• Water/cement ratio

• Humidity and temperature of curing

• Humidity during the period of use

• Age of concrete at first loading

• Magnitude of stress and its duration

• Surface-volume ratio of the member

(f) Thermal expansion of concrete

The knowledge of thermal expansion of concrete is very important as it is prepared and remains in service at a wide range of temperature in different countries having very hot or cold climates. Moreover, concrete will be having its effect of high temperature during fire. The coefficient of thermal expansion depends on the nature of cement, aggregate, cement content, relative humidity and size of the section. IS 456 stipulates (cl. 6.2.6) the values of coefficient of thermal expansion for concrete / for different types of aggregate.

Workability and Durability of Concrete

Workability and durability of concrete are important properties to be considered. The relevant issues are discussed in the following:

The workability of a concrete mix gives a measure of the ease with which fresh concrete can be placed and compacted. The concrete should flow readily into the form and go around and cover the reinforcement, the mix should retain its consistency and the aggregates should not segregate. A mix with high workability is needed where sections are thin and/or reinforcement is complicated and congested. The main factor affecting workability is the water content of the mix. Admixtures will increase workability but may reduce strength. The size of aggregate, its grading and shape, the ratio of coarse to fine aggregate and the aggregate-to-cement ratio also affect workability to some degree.

Measurement of workability

(a) Slump test

The fresh concrete is tamped into a standard cone which is lifted off after filling and the slump is measured. The slump is 25–50 mm for low workability, 50–100 mm for medium workability and 100–175 mm for high workability. Normal reinforced concrete requires fresh concrete of medium workability. The slump test is the usual workability test specified.

(b) Compacting factor test

The degree of compaction achieved by a standard amount of work is measured. The apparatus consists of two conical hoppers placed over one another and over a cylinder. The upper hopper is filled with fresh concrete which is then dropped into the second hopper and into the cylinder which is struck off flush. The compacting factor is the ratio of the weight of concrete in the cylinder to the weight of an equal volume of fully compacted concrete. The compacting factor for concrete of medium workability is about 0.9.

Durability of concrete

A durable concrete performs satisfactorily in the working environment during its anticipated exposure conditions during service. The durable concrete should have low permeability with adequate cement content, sufficient low free water/cement ratio and ensured complete compaction of concrete by adequate curing. For more information, please refer to cl. 8 of IS 456.

Design mix and nominal mix concrete

In design mix, the proportions of cement, aggregates (sand and gravel), water and mineral admixtures, if any, are actually designed, while in nominal mix, the proportions are nominally adopted. The design mix concrete is preferred to the nominal mix as the former results in the grade of concrete having the specified workability and characteristic strength (vide cl. 9 of IS 456).

Batching

Mass and volume are the two types of batching for measuring cement, sand, coarse aggregates, admixtures and water. Coarse aggregates may be gravel, grade stone chips or other man made aggregates. The quantities of cement, sand, coarse aggregates and solid admixtures shall be measured by mass. Liquid admixtures and water are measured either by volume or by mass (cl. 10 of IS 456).

Properties of reinforcing steel

Steel reinforcement used in reinforced concrete may be of the following types

(a) 1. Mild steel bars conforming to IS 432 (part-I)
2. Hot rolled mild steel conforming to IS 1139
(b) 1. Medium tensile steel conforming to IS 432 (part-I)
2. Hot rolled medium tensile steel.
(c) 1. Hot rolled High Yield Strength Deformed (HYSD) steel conforming to IS 1139.
2. Cold-worked steel HYSD bars steel conforming to IS 1786.
(d) 1. Hard drawn steel wire fabric conforming to IS 1566.
2. Rolled steel made from structural steel conforming to Is 226.

1. the most important characteristic of a reinforcing bar is its stress strain curve and the important property yield stress or 0.2% proof stress, as the case may be.
2. The modules of elasticity E for these steel is 2x105 N/mn2.

3. Mild steel bars have yield strength of 250 N/mm2 and hence it is known as Fe 250.

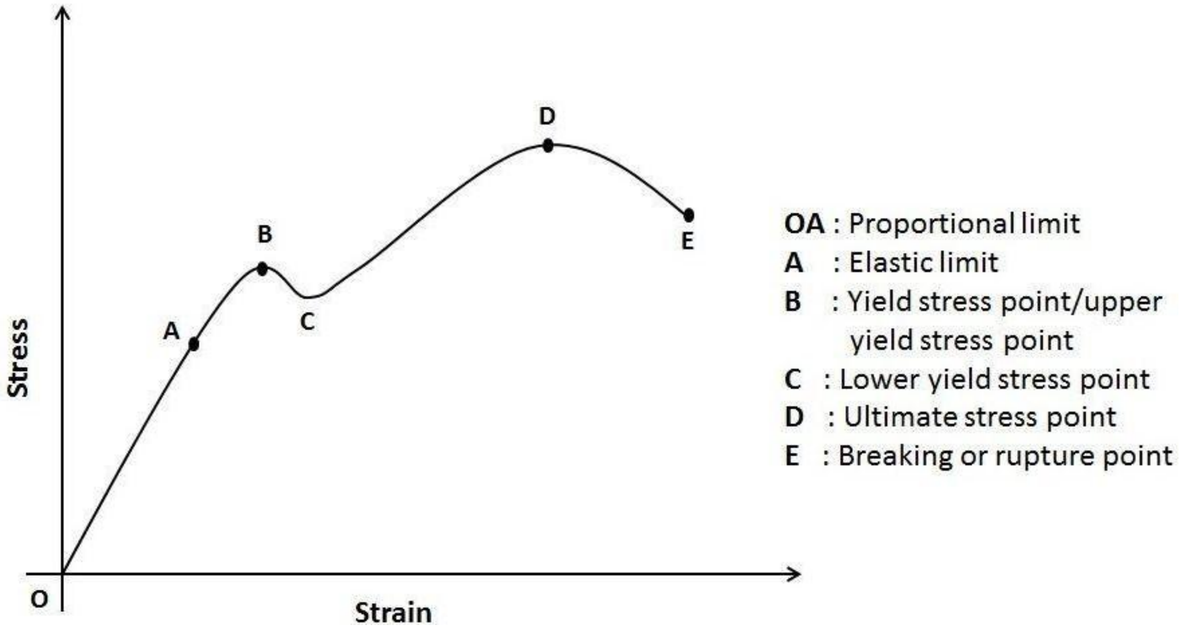
4. HYSD bars may be hot rolled high yield strength bars or cold rooked steel high strength deformed bars. The latter are also known as cold twisted deformed bars or Tor steel and are available in different grades

i) Fe 415 ii) 500 iii) Fe 550 having 0.2% proof stress as 415N/mm2, 500N/mm2 and 550 N/mm2

5. The reinforcing bars should have sufficient % of elongation.

6. Its coefficients of thermal expansion should be more or less equal to the cement concrete.

Stress-strain curves for reinforcement



Stress-strain curves for reinforcement

Above figures show the representative stress-strain curves for steel having definite yield point and not having definite yield point, respectively. The characteristic yield strength *fy* of steel is assumed as the minimum yield stress or 0.2 per cent of proof stress for steel having no definite yield point. The modulus of elasticity of steel is taken to be 200000 N/mm2.

For mild steel, the stress is proportional to the strain up to the yield point. Thereafter, post yield strain increases faster while the stress is assumed to remain at constant value of *fy*.

For cold-worked bars (Fig. 1.3), the stress is proportional to the strain up to a stress of 0.8 *fy*. Thereafter, the inelastic curve is defined as given below:

Stress	Inelastic strain
0.80 fy	Nil
0.85 fy	0.0001
0.90 fy	0.0003
0.95 fy	0.0007
0.975 fy	0.0010
1.00 fy	0.0020

Linear interpolation is to be done for intermediate values. The two grades of cold-worked bars used as steel reinforcement are Fe 415 and Fe 500 with the values of fy as 415 N/mm2 and 500 N/mm2, respectively.

Method of RCC design

A reinforced concrete structure should be designed to satisfy the following criteria-

i) Adequate safety, in items stiffness and durability

iii) Reasonable economy.

The following design methods are used for the design of RCC Structures.

a) The working stress method (WSM)

b) The ultimate load method (ULM)

c) The limit state method (LSM)

Working Stress Method (WSM)

This method is based on linear elastic theory or the classical elastic theory. This method ensured adequate safety by suitably restricting the stress in the materials (i.e. concrete and steel) induced by the expected working leads on the structures. The assumption of linear elastic behaviour considered justifiable since the specified permissible stresses are kept well below the ultimate strength of the material. The ratio of yield stress of the steel reinforcement or the cube strength of the concrete to the corresponding permissible or working stress is usually called factor of safety.

The WSM uses a factor of safety of about 3 with respect to the cube strength of concrete and a factor of safety of about 1.8 with respect to the yield strength of steel.

Ultimate load method (ULM)

The method is based on the ultimate strength of reinforced concrete at ultimate load is obtained by enhancing the service load by some factor called as load factor for giving a desired margin of safety .Hence the method is also referred to as the load factor method or the ultimate strength method.

In the ULM, stress condition at the state of in pending collapse of the structure is analysed, thus using, the non-linear stress – strain curves of concrete and steel. The safely measure in the design is obtained by the use of proper load factor. The satisfactory strength performance at ultimate loads does not guarantee satisfactory strength performance at ultimate loads does not guarantee satisfactory serviceability performance at normal service loads.

Limit state method (LSM)

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended. It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded.

Limit state are two types

i) Limit state of collapse

ii) Limit state of serviceability.

Limit states of collapse

The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from bucking due to elastic bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavourable combination of loads on the structure using the appropriate factor of safely.

Limit state of serviceability

Limit state of serviceability deals with deflection and crocking of structures under service loads, durability under working environment during their anticipated exposure conditions during service, stability of structures as a whole, fire resistance etc.

Characteristic and design values and partial safety factor

1. Characteristic strength of materials.

The term _characteristic strength' means that value of the strength of material below which not more than minimum acceptable percentage of test results are expected to fall. IS 456:2000 have accepted the minimum acceptable percentage as 5% for reinforced concrete structures. This means that there is 5% for probability or chance of the actual strength being less than the characteristic strength.

The design strength should be lower than the mean strength (fm)

Characteristic strength = Mean strength –K x standard deviation or

$fk=fm-K\delta$

Where, fk=characteristic strength of the material

fm=mean strength

K=constant =1.65

δ=standard deviation for a set of test results.

Characteristic strength of concrete

Characteristic strength of concrete is denoted by fck (N/mm2) and its value is different for different grades of concrete e.g. M 15, M25 etc. In the symbol _M' used for designation of concrete mix, refers to the mix and the number refers to the specified characteristic compressive strength of 150 mm size cube at 28 days expressed in N/mm2

Characteristic strength of steel

Until the relevant Indian Standard specification for reinforcing steel are modified to include the concept of characteristic strength, the characteristic value shall be assumed as the minimum yield stress or 0.2% proof stress specified in the relevant Indian Standard specification. The characteristic strength of steel designated by symbol fy (N/mm2)

Characteristic loads

The term _Characteristic load' means that values of load which has a 95% probability of not being exceeded during that life of the structure.

The design load should be more than average load obtained from statistic, we have

Fk=Fm+Kδ

Where, Fk=characteristic load;

Fm= mean load

K=constant=2.65;

δ=standard deviation for the load.

Design strength of materials

The design strength of materials (fd) is given by

$$f_d = \frac{f_k}{\gamma_m}$$

Where, fk=characteristic strength of material.

γ_m= partial safety factor appropriate to the material and the limit state being considered m

Design loads

The design load (Fd) is given by.

Fd=Fk.γ_f

γ_f=partial safety factor appropriate to the nature of loading and the limit state being considered.

The design load obtained by multi plying the characteristic load by the partial safety factor for load is also known as factored load.

Partial safety factor (γ_m) for materials

When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor, γ_m should be taken as 1.15 for steel.

Thus, in the limit state method , the design stress for steel reinforcement is given by f_y/1.15=0.87f_y

According to IS 456:2000 for design purpose the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength of concrete in cube and partial safety factor γ_{mc} =1.5 shall be applied in addition to this. Thus, the design stress in concrete is given by

Limit state of collapse in flexure

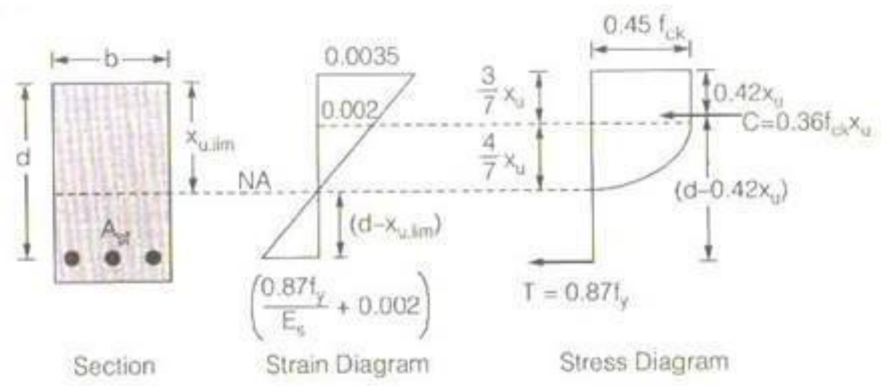
Assumptions

a) Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.

b) The maximum compressive strain in concrete (at the outermost fibre) s_{cu}shall be taken as 0.0035 in bending.
c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given below in figure 1.6. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y, = 1.5 shall be applied in addition to this.

d) The tensile strength of the concrete is ignored.
e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor $\gamma_m$equal to 1.15 shall be applied.
f) The maximum strain in the tension reinforcement in the section at failure shall not be less $\frac{f_y}{1.15E_s} + 0.002$

Limiting Depth of Neutral Axis



Based on the assumption given above, an expression for the depth of the neutral axis x_u at the ultimate limit state, can be easily obtained from the strain diagram in Fig. 1.8. Considering similar triangles,

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \frac{0.87f_y}{E_s} + 0.002}$$

According to IS 456:2000 cl no 38.1 (f) ,when the maximum strain in tension reinforcement is equal to , then the value of neutral axis will be . $\frac{0.87f_y}{E_s} + 0.002$,$x_{u,max}$

The values of $x_{u,ax}/d$ for different grades of steel, obtained by applying Eq. (2), are given by

Steel Grade	Fe 250	Fe 415	Fe 500
$x_{u,max}/d$	0.53	0.48	0.46

Depth of Neutral Axis

For any given section, the depth of the neutral axis is given by

$$x_{u,} = \frac{0.87f_y A_{st}}{0.361f_{ck} b}$$

valid only if resulting , $x_{u,}$is less than$x_{u,max}$

Ultimate Moment of Resistance

The *ultimate moment of resistance MR* of a given beam section is obtainable from Eq. (3). The lever arm *z*, for the case of the singly reinforced rectangular section [Fig. 1.8, Fig. 1.9] is given by

Z=d-0.416$x_{u,}$

Accordingly, in terms of the concrete compressive strength

$M_r = 0.361f_{ck}bx_u$ (d-0.416$x_{u,}$) for all x_u

Alternatively, in terms of the steel tensile stress,

$M_r = f_{st}A_{st}$ (d-0.416$x_{u,}$) for all x_u

With *fst=0.87fy* for *xu* ≤ *xu*,max

Limiting Moment of Resistance

The *limiting moment of resistance* of a given (singly reinforced, rectangular) section, according to the Code (Cl. G–1.1), corresponds to the condition, defined by Eq. (2). From Eq. (9), it follows that:

$M_r = 0.361f_{ck}bx_{u,ax}$ (d-0.416$x_{umax,}$)

$=_r 0.361f_{ck} b \frac{x_{u,ax}}{d}$ (1-d-$\frac{0.416x_{umax,}}{d}$)

Limiting Percentage Tensile Steel

Corresponding to the limiting moment of resistance$M_{u,im}$, there is a limiting percentage tensile steel ,

$$P_{t,\mathrm{im}} = \frac{100 \times A_{st}}{bd}$$

An expression for is obtainable from Eq. (7) with: . ,lim tp,max u uxx

$$\frac{X_{u,ax}}{d} = \frac{0.87f_y}{0.361f_{ck}} \times \frac{P_{t,\mathrm{im}}}{100}$$

Modes of failure: Types of section

Balanced section

In balanced section, the strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is X_u=$X_{u,ax}$

Under-reinforced section

An under-reinforced section is the one in which steel percentage (pt) is less than critical or
limiting percentage (*pt*,lim). Due to this the actual NA is above the balanced NA and
$xu \le xu$,max .

Over-reinforced section

In the over reinforced section the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and . Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value

$Xu > Xu$,max .

Computation of *Mu*

The corresponding expressions of *Mu* are given below for the three cases:

(i) When *xu* < *xu*, *max*
In this case the concrete reaches 0.0035, steel has started flowing showing ductility (Strain >$\frac{0.87f_y}{E_s} + 0.002$). So, the computation of *Mu* is to be done using the tensile force of steel in this case. $\frac{0.87f_y}{E_s} + 0.002$In this case the concrete reaches 0.0035, steel has started

flowing showing ductility (Strain > $\frac{0.87f_y}{E_s} + 0.002$). So, the computation of *Mu* is to be done using the tensile force of steel in this case.

Therefore, *Mu* = *T* (lever arm) = 0.87 *fy Ast* (*d* - 0.42 *xu*)

When *xu* = *xu, max*

In this case steel just reaches the value of $\frac{0.87f_y}{E_s} + 0.002$ and concrete also reaches its maximum value. The strain of steel can further increase but the reaching of limiting strain of concrete should be taken into consideration to determine the limiting *M u* as *Mu,lim* here. So, we have

$$Mu,lim = 0.36\frac{K_{u,max}}{d}\left(1 - 0.42\frac{K_{u,max}}{d}\right)f_{ck}\,bd^2$$

When $x_u > x_{u,max}$

In this case, concrete reaches the strain of 0.0035, tensile strain of steel is much less than $\frac{0.87f_y}{E_s} + 0.002$ and any further increase of strain of steel will mean failure of concrete, which is to be avoided.

Numerical Problems

(5) The value thus obtained shall be rounded upto nearest 25 mm.

(6) Now $d = D_{overall} - 40$ mm for case 1

$= D_{overall} - 60$ mm for case 2.

(7) Determine $\frac{M_u}{b\,d^2}$, p_t and A_{st}, using equation (6-10) as per the case of under-reinforced section. Also determine $A_{st,lim}$.

(8) Select the bar size and number such that $A_{st} > A_{st,required}$ and also $A_{st} < A_{st,lim}$.

***Type 4:* To find the steel area for a given factored moment.**

(1) For a given ultimate moment (also known as factored moment) and assumed width of section, find out d from equation (6-7a)

$$d = \sqrt{\frac{M_u}{Q_{lim} \times b}}.$$

This is a balanced section and balanced steel area may be found out using equation (6-7c). Alternatively, $p_{t,lim}$ may be obtained from table 6-4.

(2) For a given factored moment, width and depth of section.

Obtain $M_{u,lim} = Q_{lim}\ bd^2$.

If $M_u < M_{u,lim}$: design as under-reinforced as explained in type 3.

If $M_u = M_{u,lim}$: design as balanced section as explained in (1) above.

If $M_u > M_{u,lim}$: redesign the section either increasing the dimensions of section or design as doubly-reinforced beam.

For under-reinforced section, the steel area can be obtained by using equation (6-7c), (6-7d) or (6-10).

Example 6-1.

A rectangular beam 230 mm wide and 520 mm effective depth is reinforced with 4 no. 16 mm diameter bars. Find out the depth of neutral axis and specify the type of beam. The materials are M20 grade concrete and HYSD reinforcement of grade Fe 415. Also find out the depth of neutral axis if the reinforcement is increased to 4 no. 20 mm diameter bars.

Solution:

Case 1: $A_{st} = 4 \times 201 = 804\ \text{mm}^2$.

Total compression $= 0.36\ f_{ck}\ bx_u = 0.36 \times 20 \times 230\ x_u = 1656\ x_u$.

Total tension $= 0.87\ f_y\ A_{st} = 0.87 \times 415 \times 804 = 290284$ N.

Equating $1656\ x_u = 290284$

$\therefore$ $x_u = 175.3$ mm.

Limiting value of neutral axis

$x_{u,max} = 0.48\ d = 0.48 \times 520 = 250$ mm

$x_u < x_{u,max}$.

$\therefore$ Section is under-reinforced

and $x_u = 175.3$ mm.

Case 2: $A_{st} = 4 \times 314 = 1256\ \text{mm}^2$.

Total compression $= 0.36 \times 20 \times 230\ x_u = 1656\ x_u$.

Total tension $= 0.87 \times 415 \times 1256 = 453479$ N.

Equating $\quad 1656\ x_u = 453479 \Rightarrow x_u = 273.8$ mm.

Here $\quad x_u > x_{u,max}$, i.e., over-reinforced section

$\therefore \quad x_u = x_{u,max} = 250$ mm.

Example 6-2.

A singly reinforced rectangular beam of width 230 mm and 460 mm effective depth is reinforced with 3 no. 20 mm diameter bars. Find out the factored moment of resistance of the section. The materials are M20 grade concrete and HYSD reinforcement of grade Fe 415. Also find out the factored moment of resistance if it is reinforced with 5 no. 20 mm diameter bars.

Solution:

Case 1: $\quad A_{st} = 3 \times 314 = 942\ mm^2.$

$$\text{Total compression} = 0.36\ f_{ck}\ b\ x_u = 0.36 \times 20 \times 230\ x_u = 1656\ x_u.$$

$$\text{Total tension} = 0.87\ f_y\ A_{st} = 0.87 \times 415 \times 942 = 340109\ \text{N}.$$

$$\text{Total compression} = \text{total tension}$$

$$1656\ x_u = 340109 \Rightarrow x_u = 205.4\ \text{mm}$$

$$x_{u,max} = 0.48\ d = 0.48 \times 460 = 220.8\ \text{mm}.$$

$$x_u < x_{u,max}.$$

The section is under-reinforced and $x_u = 205.4$ mm.

$$M_u = 0.87\ f_y\ A_{st}\ (d - 0.42\ x_u)$$

$$= 0.87 \times 415 \times 942\ (460 - 0.42 \times 205.4) \times 10^{-6}\ \text{kNm}$$

$$= 127.12\ \text{kNm}.$$

Alternatively, $\quad M_u = 0.36\ f_{ck}\ b\ x_u\ (d - 0.42\ x_u)$

$$= 0.36 \times 20 \times 230 \times 205.4\ (460 - 0.42 \times 205.4) \times 10^{-6}$$

$$= 127.12\ \text{kNm}.$$

Case 2: $\quad A_{st} = 4 \times 314 = 1256\ mm^2.$

$$\text{Total compression} = 0.36\ f_{ck}\ b\ x_u = 0.36 \times 20 \times 230\ x_u = 1656\ x_u.$$

$$\text{Total tension} = 0.87\ f_y\ A_{st} = 0.87 \times 415 \times 1256 = 453479\ \text{N}.$$

$$\text{Total compression} = \text{total tension}$$

$$1656\ x_u = 453479 \Rightarrow x_u = 273.8\ \text{mm}$$

$$x_{u,max} = 220.8\ \text{mm}$$

$$x_u > x_{u,max}.$$

$\therefore$ Section is over-reinforced. Use $x_u = 220.8$ mm.

Lever arm $\quad z = d - 0.42\ x_u = 460 - 0.42 \times 220.8 = 367.2$ mm.

$$M_u = 0.36\ f_{ck}\ b\ x_u\ z$$

$$= 0.36 \times 20 \times 230 \times 220.8 \times 367.2 \times 10^{-6}\ \text{kNm}$$

$$= 134.2\ \text{kNm}.$$

The actual stress in steel, f_s, when the section is subjected to $M_{u,lim}$ is determi[ned] by equating total tension and total compression.

Then $\quad 1256\ f_s = 1656 \times 220.8$

i.e. $\quad f_s = 291.1\ N/mm^2$, as against $0.87\ f_y = 0.87 \times 415 = 361\ N/m[m^2]$

Example 6-8.

A simply supported rectangular beam of 8 m span carries a uniformly distributed load of 23 kN/m, inclusive of its self-weight. Determine the reinforcement for flexure. The materials are M30 grade concrete and TMT bars of grade Fe 415.

Solution:

To determine the design constants, use design indices which depend on grade of steel and not on grade of concrete.

From table 6-5, for Fe 415 grade reinforcement

$$\frac{M_{u,lim}}{f_{ck}\ b d^2} = 0.138$$

and

$$\frac{p_{t,lim}\ f_y}{f_{ck}} = 19.86.$$

For M30 grade concrete

$$Q_{lim} = 0.138 \times 30 = 4.14;$$

$$p_{t,lim} = \frac{19.86 \times 30}{415} = 1.44.$$

For the present case

$$M = 23 \times \frac{8^2}{8} = 184 \text{ kNm}$$

$$M_u = 1.5 \times 184 = 276 \text{ kNm}.$$

Adopt $b = 250$ mm.

$$\text{Balanced depth } d = \sqrt{\frac{276 \times 10^6}{4.14 \times 250}} = 516.4 \text{ mm}.$$

Assume 10% larger depth and 60 mm effective cover

$$D = 1.1 \times 516.4 + 60 = 628 \text{ mm}.$$

Use

$$D = 650 \text{ mm},\ d = 650 - 60 = 590 \text{ mm}.$$

$$\frac{M_u}{b d^2} = \frac{276 \times 10^6}{250 \times 590 \times 590} = 3.17$$

$$p_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{30} \times 3.17}}{415 / 30}\right] = 1.02$$

$$A_{st} = \frac{1.02}{100} \times 250 \times 590 = 1505 \text{ mm}^2$$

$$A_{st,lim} = \frac{1.44}{100} \times 250 \times 590 = 2124 \text{ mm}^2$$

Provide 5-20 # = 5 × 314 = 1570 mm².

$$A_{st,required} < A_{st,provided} < A_{st,lim} \ldots\ldots$$

Example 6-9.

A simply supported rectangular beam of 6 m span carries a characteristic load of 24 kN/m inclusive of its self-weight. The beam section is 230 mm × 600 mm overall. Design the beam. The materials are M 20 grade concrete and HYSD reinforcement of grade Fe 415. The beam is resting on R.C.C. columns.

Solution:

Factored load = 1.5 × 24 = 36 kN/m.

$$M_u = \frac{36 \times 6^2}{8} = 162 \text{ kNm.}$$

$$V_u = \frac{36 \times 6}{2} = 108 \text{ kN.}$$

(a) *Moment steel:*

Assuming 20 mm diameter bars in one layer

$$d = 600 - 30 - 10 = 560 \text{ mm}$$

$$\frac{M_u}{b\,d^2} = \frac{162 \times 10^6}{230 \times 560^2} = 2.25 < 2.76.$$

∴ The section is singly reinforced.

$$p_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{b\,d^2}}}{f_y/f_{ck}} \right] = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{20} \times 2.25}}{415/20} \right]$$

$$= 0.74$$

$$A_{st} = \frac{0.74}{100} \times 230 \times 560 = 953 \text{ mm}^2.$$

Minimum steel, $A_s = \frac{0.205}{100} \times 230 \times 560 = 264 \text{ mm}^2$

$$A_{st,lim} = \frac{0.96}{100} \times 230 \times 560 = 1236 \text{ mm}^2.$$

Provide 2-20 # + 2-16 # = 2 (314 + 201) = 1030 mm^2

Let 2-16 # bars are bent at 1.25 D = 1.25 × 600 = 750 mm, from the face of the support.

The remaining bars should extend within the support for a distance of $\frac{L_d}{3} = \frac{1}{3} \times 47 \times 20 = 313$ mm.

(b) *Check for development length:*

(1) A bar can be bent up at a distance greater than L_d = 47 # from the centre of the support, i.e., 47 × 16 = 752 mm.

In this case, this distance is (3000 – 750) = 2250 mm ....(Safe)

(2) For the remaining bars, $A_{st} = 2 \times 314 = 628\ mm^2$

$$M_{u1} = 0.87\ f_y\ A_{st}\, d \left(1 - \frac{f_y\ A_{st}}{f_{ck}\ b\, d}\right)$$

$$= 0.87 \times 415 \times 628 \times 560 \left(1 - \frac{415 \times 628}{20 \times 230 \times 560}\right) \times 10^{-6}$$

$$= 114.12\ kNm$$

$$V_u = 108\ kN,\ L_0 = 12\ \#\ (\text{assume})$$

As the reinforcement is confined by compressive reaction

$$1.3\ \frac{M_{u1}}{V_u} + L_0 \geq L_d$$

$$\therefore \quad 1.3 \times \frac{114.12 \times 10^6}{108 \times 10^3} + 12\ \# \geq 47\ \#$$

$$\therefore \quad 39.2 \geq \#$$

$$\#_{provided} = 20\ mm \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{Safe})$$

The remaining bars should extend within the support for a distance of $\frac{L_d}{3} = \frac{1}{3} \times 47 \times 20 = 313$ mm. If support width is 300 mm, the bars extends for $150 + L_0 = 150 + 12 \times 20 = 390$ mm within the support. Note that this requirement is only for one-third of the positive moment bars. In most cases this requirement is satisfied and will not be checked again in the examples that follows:

(c) *Check for shear*:

At support, $V_u = 108$ kN.

As the ends of the reinforcement are confined with compressive reaction, shear at distance d will be used for checking shear at support.

$$V_u \text{ at } d = 108 - 0.565 \times 36 = 87.66\ kN$$

$$\tau_v = \frac{108 \times 10^3}{230 \times 560} = 0.839\ N/mm^2 < 2.8\ N/mm^2$$

$$\frac{100\ A_s}{b\,d} = \frac{100 \times 2 \times 314}{230 \times 560} = 0.487$$

$$\tau_c = 0.472\ N/mm^2 < \tau_v.\ (\text{from table 19, IS : 456})$$

$\therefore$ Shear design is necessary.

At support $V_{us} = V_u - \tau_c\ b\,d$

$$= 87.66 - 0.472 \times 230 \times 560 \times 10^{-3}$$

$$= 87.66 - 60.8 = 26.86\ kN.$$

Capacity of bent bars to resist shear

$$= 2 \times 201 \times 0.87 \times 415 \times \sin 45° \times 10^{-3} = 102.6\ kN.$$

Bent bars share 50% = 13.43 kN

Stirrups share 50% = 13.43 kN.

At distance d, where bent bars not availabe

$$V_{us} = 26.86\ kN$$

$\therefore$ Design stirrups for $V_{us} = 26.86$ kN.

Using 6 mm ϕ (mild steel) stirrups, $A_{sv} = 56$ mm^2.

$$s_v = \frac{0.87\, f_y\, A_{sv}\, d}{V_{us}} = \frac{0.87 \times 250 \times 56 \times 560}{26.86 \times 10^3} = 253.9 \text{ mm.}$$

Spacing of minimum shear reinforcement using 6 ϕ stirrups

$$= \frac{0.87\, A_{sv}\, f_y}{0.4\, b} = \frac{0.87 \times 56 \times 250}{0.4 \times 230} = 132.4 \text{ mm.}$$

Spacing should not exceed

(i) 300 mm

(ii) $0.75\, d = 0.75 \times 560 = 420$ mm

(iii) 132.4 mm (minimum)

(iv) 253.9 mm (designed)

Provide 6 mm ϕ two-legged stirrups @ 130 mm c/c.

Shear of $\frac{\tau_c\, bd}{2} = \frac{60.8}{2} = 30.4$ kN occurs at $x = \frac{108 - 30.4}{36} = 2.16$ m from support.

No. of 6 mm ϕ stirrups with 130 mm spacing $= \frac{2160}{130} + 1 \simeq 18$.

In central portion, provide larger spacing which should not exceed 300 mm or $0.75\, d = 420$ mm, i.e., 300 mm.

Provide 6 mm ϕ @ 300 mm c/c in central portion.

(d) *Check for deflection:*

Basic $\frac{\text{span}}{d}$ ratio = 20

$$\text{service stress} = 0.58\, f_y \times \frac{A_{st,req}}{A_{st,pro}}$$

$$= 0.58 \times 415 \times \frac{936}{1030} = 219 \text{ N/mm}^2$$

$$\frac{100\, A_{st}}{b\, d} = \frac{100 \times 1030}{230 \times 560} = 0.80$$

Modification factor = 1.15

$\frac{\text{span}}{d}$ permissible $= 20 \times 1.15 = 23$

Actual $\frac{\text{span}}{d} = \frac{6000}{565} = 10.62$ ....(Safe)

(e) *Spacing of bars:*

Clear distance between bars

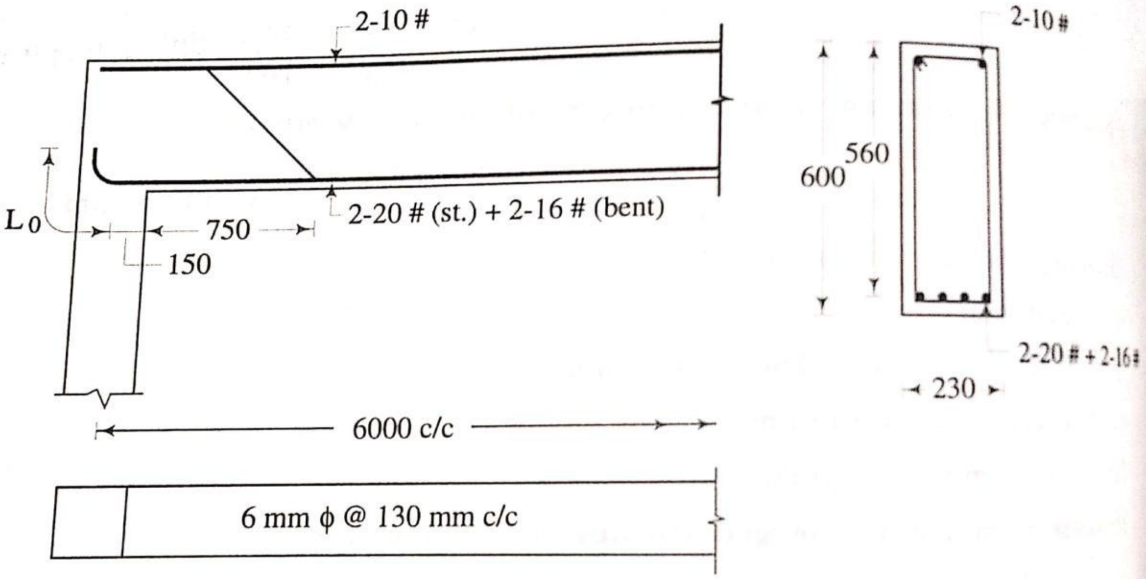
$= 230 - 60 - 2 \times 20 = 130$ mm.

Minimum clear distance permitted

$= 20 + 5 = 25$ mm or 16 mm (ϕ of bar), i.e., 25 mm.

Maximum clear distance permitted

= 180 mm (cracking – table 8-1) ....(Safe)



(a) Beam elevation (b) Section at centre

A simply supported rectangular beam of clear span 5 mm carries a uniformly distributed super-imposed load of 27 kN/m. The beam section is 230 mm × 500 mm overall. Design the beam. The beam rests on 350 mm thick brick walls running perpendicular to the axis of the beam. The materials are M 20 grade concrete and HYSD reinforcement of grade Fe 415.

Solution:

Effective span (1) 5.0 + 0.65 (effective depth) = 5.65 m

(2) 5.0 + 0.35 (c/c supports) = 5.35 m

Adopt l = 5.35 m.

Self-weight of beam = 0.23 × 0.5 × 25 = 3 kN/m

Super-imposed load = 27 kN/m

Total 30 kN/m

Factored load = 1.5 × 30 = 45 kN/m

$$M_{u,max} = 45 \times \frac{5.35^2}{8} = 161 \text{ kNm}$$

$$V_{u,max} = 45 \times \frac{5}{2} = 112.5 \text{ kN.}$$

Note that clear span is considered for calculating shear force.

If clear span is not given, shear and moment both may be calculated from effective span. But if actual geometry is known from the problem, clear span may be considered for shear calculation to get economy.

(a) *Moment steel:*

The section is 230 mm × 500 mm overall. Assuming one layer of 20 mm diameter bars, the effective depth is

$$d = 500 - 30 \text{ (cover)} - 10 = 460 \text{ mm.}$$

Depth required for singly reinforced section

$$= \sqrt{\frac{161 \times 10^6}{2.76 \times 230}} = 504 \text{ mm} > 460 \text{ mm}$$

∴ Design as doubly reinforced beam.

$$M_u = 161 \text{ kNm}$$
$$b = 230 \text{ mm}; \; d = 460 \text{ mm}.$$
$$\frac{M_u}{b d^2} = \frac{161 \times 10^6}{230 \times 460 \times 460} = 3.31$$

Assume $d' = 40$ mm $\frac{d'}{d} = 0.1.$

$$f_{sc} = 353 \text{ N/mm}^2.$$

From table 6-8

$$p_t = 1.125, \; p_c = 0.177$$
$$A_{st} = \frac{1.125 \times 230 \times 460}{100} = 1190 \text{ mm}^2.$$
$$A_{sc} = \frac{0.155 \times 230 \times 460}{100} = 164 \text{ mm}^2.$$

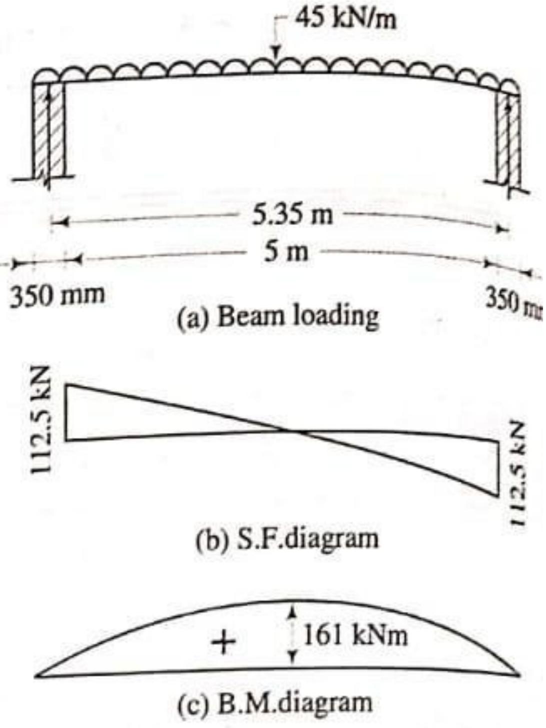


FIG. 9-8

Provide 2-12 # top = 426 mm², and
4-20 # bottom = 1256 mm²

For designed section C = T

$$0.36 f_{ck}\, b\, x_u + A_{sc} f_{sc} = 0.87 f_y A_{st}$$
$$0.36 \times 20 \times 230\, x_u + 420 \times 353 = 0.87 \times 415 \times 1256$$
$$\therefore \quad x_u = 184.31 \text{ mm}$$
$$x_{u,max} = 0.48 \times 460 = 220.8 \text{ mm}$$
$$x_u < x_{u,max}$$

∴ under-reinforced section ....(O.K.)

(b) *Check for development length:*

No bar is bent up and all the bars are carried in the support. If 90° bend is provided at the end

$$L_0 = 175 - 30 \text{ (cover)} - 5\,\# + 8\,\#$$
$$= 145 + 3 \times 20 = 205 \text{ mm.}$$

Maximum $L_0 = 12\,\# = 12 \times 20 = 240$ mm or $d_{eff} = 460$ mm,

whichever is greater.

Consider $L_0 = 240$ mm.

$M_{u1} = 161$ kNm, full design moment may be considered.

As the ends of the reinforcement are confined with compressive reaction

$$1.3 \frac{M_{u1}}{V_u} + L_0 \geq L_d$$

i.e., $$1.3 \times \frac{161 \times 10^6}{112.5 \times 10^3} + 240 \geq 47\,\#$$

Which gives # ≤ 44.68

$$1.3\frac{Mu}{V_u} + L_0 \geq L_d$$

$$1.3\times\frac{161\times10^6}{112\times10^3} + 240 \geq 47\# \text{ which gives } \leq 44.69\ mm$$

Provided diameter is 20 mm .... OK

(c) *Check for shear:*

Section at support can be designed for shear at distance d as the ends of reinforcement are confined with compressive reaction.

Shear at distance d = 112.5 − 0.460 × 45 = 91.8 kN.

$$\tau_v = \frac{91.8 \times 10^3}{230 \times 460} = 0.867 \text{ N/mm}^2$$

$$\frac{100\,A_s}{b\,d} = \frac{100 \times 1256}{230 \times 460} = 1.19$$

$$\tau_c = 0.658 \text{ N/mm}^2.$$

∴ The shear design is necessary.

Shear resistance of concrete

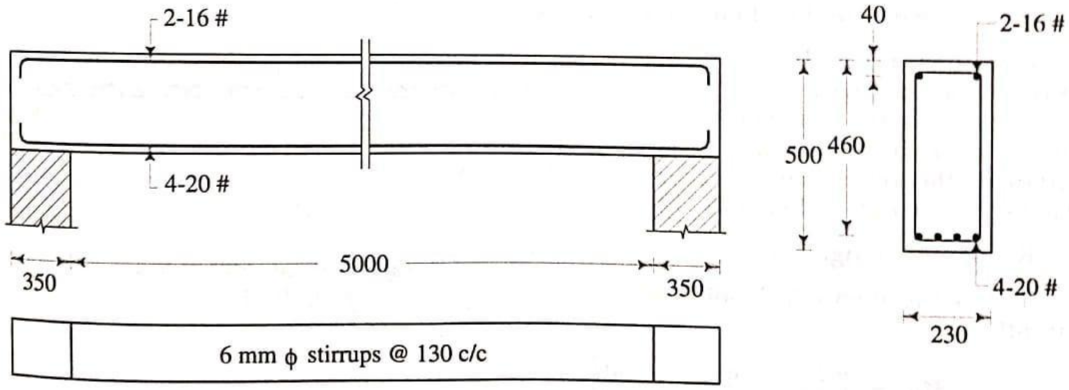
$$\tau_c\, b\, d = 0.658 \times 230 \times 460 \times 10^{-3} = 69.6 \text{ kN}.$$

$$V_{us} = V_u - \tau_c\, b\, d = 91.8 - 69.6 = 22.2 \text{ kN}.$$

Minimum shear reinforcement is sufficient. For 230 mm wide beam, from table 6-3, minimum shear reinforcement is 6 mm ϕ @ 130 mm c/c (mild steel).

Check for deflection and cracking may be done as usual.

The designed beam is shown in fig 9-9.



(a) Beam elevation

(b) Section

Shear Stress

The distribution of shear stress in reinforced concrete rectangular, *T* and *L*-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τv is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):

Figure 1.11: Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs. 1.11a and b):

$$r_v = \frac{V_u}{bd}$$

where *Vu* = shear force due to design loads,
b = breadth of rectangular beams and breadth of the web *bw* for flanged beams, and

d = effective depth.

In beams of varying depth:

$$r_v = \frac{V_u \pm \frac{M_u}{d} tan\beta}{bd}$$

re ηv, Vu, b or bw and d are the same as in

Mu = bending moment at the section, and
β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment *Mu* decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment *Mu* increases numerically in the same direction as the effect ive depth increases.

Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (*τc*) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value *τcmax* depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)
Table 19 of IS 456 stipulates the design shear strength of concrete *τc* for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when *τv* is less than *τc* given in Table 3.

Table 3 Design shear strength of concrete, *τc* in N/mm2

$\frac{100 A_s}{bd}$ (1)	Concrete grade M20 (2)	M25 (3)	M30 (4)
≤ 0.15	0.28	0.29	0.29
0.25	0.36	0.36	0.37
0.50	0.48	0.49	0.50
0.75	0.56	0.57	0.59
1.00	0.62	0.64	0.66
1.25	0.67	0.70	0.71
1.50	0.72	0.74	0.76

Note: The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to the code requirements.

Maximum shear stress *τcmax* with shear reinforcement (cls. 40.2.3, 40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress of reinforced concrete in beams *τcmax* as given below in Table 6.2. Under no circumstances, the nominal shear stress in beams *τv* shall exceed *τcmax* given in Table 6.2 for different grades of concrete.

Maximum shear stress, *τcmax* in N/mm2

MAXIMUM SHEAR STRESS $\tau_{c,max}$ N/mm^2

Concrete grade	M20	M25	M30
$\tau_{c,max}$ N/mm^2	2.8	3.1	3.5

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

For deformed bars conforming to IS 1786, these values shall be increased by 60 per cent. For bars in compression, the values of bond stress in tension shall be increased by 25 per cent.

Minimum shear reinforcement has to be provided even when τv is less than τc given in Table 3 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.
The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{b_{sv}} \geq \frac{0.4}{0.87f_y}$$

where *Asv* = total cross-sectional area of stirrup legs effective in shear,
sv = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam *bw*, and
fy = characteristic strength of the stirrup reinforcement in N/mm2 which shall not be taken greater than 415 N/mm2.

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.
The minimum shear reinforcement is provided for the following:
(i) Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.
(ii) Brittle shear failure is arrested which would have occurred without shear reinforcement.
(iii) Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.
(iv) To hold the reinforcement in place when concrete is poured.
(v) Section becomes effective with the tie effect of the compression steel.
Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 *d* for vertical stirrups and *d* for inclined stirrups at 45o, where *d* is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

Development Length

When a reinforcing bar is embedded in concrete, the concrete adheresto its surface and resist any force that tries to cause slippage of bar relative to its surrounding concrete, by a phenomenon called Bond.

Bond between steel and concrete should be perfect at service loads. Bond transfers stress from one material to the other, by strain compatibility.

Development length

The reinforcement bar must extend the concrete sufficiently so that it can develop the required stress. The extended length of the bar force the concrete is known as development length.

$$L_d = \frac{\emptyset \sigma_s}{4r_{bd}}$$

Where r_{bd}= Anchorage bond stress

$\sigma_s = stress$ in reinforcement bar

Ø= diameter of reinforcement bar

Grade of concrete	M15	M20	M25	M30	M35	M40
r_{bd} MPa	1.0	1.2	1.4	1.5	1.7	1.9

Note: In case of tor steel increases the above value by 60.

In case of compression, the values can be further increased by 25%

Checking of Development Lengths of Bars in Tension

The following are the stipulation of cl. 26.2.3.3 of IS 456.

(i) At least one-third of the positive moment reinforcement in simple members and one-fourth of the positive moment reinforcement in continuous members shall be extended along the same face of the member into the support, to a length equal to *Ld/*3.

(ii) **(ii)** Such reinforcements of (i) above shall also be anchored to develop its design stress in tension at the face of the support, when such member is part of the primary lateral load resisting system.

(iii) The diameter of the positive moment reinforcement shall be limited to a diameter such that the *Ld* computed for *ζs* = *fd* in Eq. (20) does not exceed the following:

$$L_d \leq \frac{M1}{V} + L_0$$

IS 456 recommends that the value of $\frac{M1}{V}$ be increased by 30% when the ends of reinforcements are contained by compressive reactionsuch as at simply supported end.

Bundled bars

The development length required for each bar in the bundle shall be increased by 10% for 2 bars in contact,20% for three bars in a group and 33% for 4 bars in the bundle.

Anchorage length for main reinforcement:

The anchorage value of a hook or a bend of longitudinal reinforcement is the equivalent length of straight bar. The anchorage value of each 45° bend is equal to 4Ø and subjected to a maximum value of 16Ø.

Angle of bending	45	90	135	180
Anchorage value	4Ø	8Ø	12Ø	16Ø

Splicing of bars:

1. Splicing of bars should be avoided at a section where the bendig moment is 50% of moment of resistance of the section.
2. Not more than 50% of bars should be spliced at a section.
3. The lap length for bars in bending tension should not be more than L_d *or* 30Ø.
4. The lap length in compression is not less than L_d *or* 24Ø whichever is greater.

When a beam requires compression reinforcement, in case of doubly reinforced section for example, the arrangement of shear reinforcement shall be as follows to ensure the effective lateral restraint to the compression bars.

(i) $\phi_s \nless \dfrac{\phi_{main}}{4}$ or 6 mm

(ii) $s_v \ngtr b$ (the width of beam); 16 times the diameter of smallest main bar; and 300 mm.

Here ϕ_s is the diameter of stirrup, and ϕ_{main} is the diameter of largest main bar.

(3) ***Minimum shear reinforcement:*** Minimum shear reinforcement in the form of vertical stirrups shall be provided such that

$$\frac{A_{sv}}{b\, s_v} \geq \frac{0.4}{0.87\, f_y} \qquad \text{(7-12a)}$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear

s_v = stirrup spacing along the length of the member in mm

b = width of the beam or width of rib of flanged beam

f_y = characteristic strength of the stirrup reinforcement in N/mm² which shall not be taken greater than 415 N/mm².

However, where the maximum shear stress calculated is less than half the permissible value and in the members of minor structural importance such as lintels, this provision need not be complied with.

The above provision of minimum shear reinforcement provides a shear resistance of 0.4 N/mm². From equation (7-12a), to provide the minimum shear reinforcement, the spacing of stirrups shall not exceed

$$s_v \leq \frac{0.87\, f_y\, A_{sv}}{0.4\, b} \qquad \text{(7-12b)}$$

It can be seen that for given type of steel and selected diameter of stirrups, the spacing that provides minimum shear resistance is inversely proportional to the width b of the member. In case of tee or ell beams the b shall be equal to b_w, the width of web.

The instant a shear crack is formed, the tension carried by concrete is transferred to the shear reinforcement. In other words, the shear reinforcement restrains the growth of shear crack and increases the ductility of the beam.

In case of over-loading, this provides the warning before sudden failure. To ensure that stirrups will have sufficient strength to absorb the diagonal tension in concrete, the minimum shear reinforcement is required. Many times this provision governs the shear design.

Example 7-1.

A tee beam section having 230 mm width of web × 460 mm effective depth is reinforced with 5 no. 16 mm diameter bars as tension reinforcement, which continue for a distance greater than effective depth, past the section. The section is subjected to a factored shear of 52.5 kN. Check the shear stresses and design the shear reinforcement. The materials are M20 grade concrete and HYSD reinforcement of grade Fe 415. For stirrups mild steel bars may be used.

Solution:

Here b = 230 mm. $\quad V_u$ = 52.5 kN.

d = 460 mm. $\quad A_s = A_{st} = 5 \times 201 = 1005$ mm²

Nominal shear stress $\tau_v = \dfrac{52.5 \times 10^3}{230 \times 460} = 0.496$ N/mm²

$$\frac{100\, A_s}{b\, d} = \frac{100 \times 1005}{230 \times 460} = 0.95.$$

Note that A_{st} denotes steel area provided for maximum bending moment while A_s denotes the area of steel which continues at least one effective depth beyond the section being considered for checking the shear.

For $\frac{100\, A_s}{b\, d} = 0.95$, from table 7-1, τ_c shall be calculated by interpolation.

For $\frac{100\, A_s}{b\, d} = 0.75$, $\tau_c = 0.56$ N/mm²

For $\frac{100\, A_s}{b\, d} = 1.0$, $\tau_c = 0.62$ N/mm²

For $\frac{100\, A_s}{b\, d} = 0.95$

$$\therefore \quad \tau_c = 0.56 + \frac{0.62 - 0.56}{1 - 0.75}(0.95 - 0.75) = 0.608 \text{ N/mm}^2$$

Now, $\tau_v < \tau_c$, therefore only nominal shear reinforcement is required.

Select 6 mm diameter M.S. bars for stirrups.

$$A_{sv} = 2 \times 28 = 56 \text{ mm}^2 \text{ for two-legged stirrups.}$$

For minimum shear reinforcement

$$\frac{A_{sv}}{b\, s_v} \geq \frac{0.4}{0.87\, f_y}$$

$$\frac{56}{230\, s_v} \geq \frac{0.4}{0.87 \times 250}$$

$$s_v \leq \frac{56 \times 0.87 \times 250}{230 \times 0.4}$$

$$\therefore \quad s_v \leq 132.4 \text{ mm.}$$

The spacing shall not exceed

(a) $0.75\, d = 0.75 \times 460 = 345$ mm, (b) 300 mm and (c) 132.4 mm as calculated above.

Provide 6 mm diameter two-legged stirrups about 130 mm c/c.

Alternatively minimum shear reinforcement may be selected from table 7-4. This is explained in art. 7-10.

Example 7-2.

If the factored shear of the above section is increased to 90 kN, check the shear stresses and find the spacing of 6 mm diameter mild steel stirrups.

Solution:

$$V_u = 90 \text{ kN},\ b = 230 \text{ mm},\ d = 460 \text{ mm.}$$

Nominal shear stress

$$\tau_v = \frac{90 \times 10^3}{230 \times 460} = 0.85 \text{ N/mm}^2 < 2.8 \text{ N/mm}^2$$

where 2.8 N/mm² is the maximum nominal shear stress for M20 mix.

$\tau_c = 0.608$ N/mm² as calculated in Example 7-1.

Now, $\tau_v > \tau_c$, therefore shear reinforcement shall be designed.

Shear resistance of concrete

$$V_{uc} = \tau_c\, b\, d = 0.608 \times 230 \times 460 \times 10^{-3} = 64.3 \text{ kN.}$$

Calculate the anchorage length in tension and compression for

(a) a single mild steel bar of diameter ϕ in concrete of grade M20.

(b) an HYSD bar of grade Fe 415 of diameter # in concrete of grade M20.

Solution:

(a) **M.S. Bar:**

(1) *Tension:*

Design stress for M.S. $\sigma_s = 0.87\ f_y = 0.87 \times 250 = 217.5\ \text{N/mm}^2$

$\tau_{bd} = 1.2\ \text{N/mm}^2$.

Anchorage length = development length

$$= \frac{\phi \times 0.87\ f_y}{4\ \tau_{bd}} = \frac{\phi \times 217.5}{4 \times 1.2}$$

$= 45.3\ \phi$, say $46\ \phi$

(2) *Compression:*

(i) *Flexural compression (Beam bars)*

Design stress for M.S. $\sigma_s = 0.87\ f_y = 217.5\ \text{N/mm}^2$

$\tau_{bd} = 1.2 \times 1.25$ (for compression) $= 1.5\ \text{N/mm}^2$

$$L_d = \frac{\phi \times 217.5}{4 \times 1.5} = 36.3\ \phi, \text{ say } 37\ \phi$$

Development length for different types of bars in M20 grade of concrete is tabulated in table 7-6.

TABLE 7-6
DEVELOPMENT LENGTH FOR SINGLE BARS IN CONCRETE OF GRADE M20

f_y N/mm²	Tension bars	Compression bars	
		In Beams	In Columns
250	46 ϕ	37 ϕ	32 ϕ
415	47 #	38 #	33 #
500	57 #	46 #	39 #

Art. 7.18]

(ii) *Direct compression (column bars)*

Design stress for M.S. $= 0.75\ f_y = 0.75 \times 250 = 187.5\ \text{N/mm}^2$

$$\tau_{bd} = 1.2 \times 1.25 \text{ (for compression)} = 1.5\ \text{N/mm}^2$$

$$L_d = \frac{\phi \times 187\cdot5}{4 \times 1\cdot5} = 31.3\ \phi, \text{ say } 32\ \phi$$

(b) HYSD bar of grade Fe 415:

(1) *Tension:*

Design stress $\sigma_s = 0.87\ f_y = 0.87 \times 415 = 361\ \text{N/mm}^2$

$$\tau_{bd} = 1.2 \times 1.6 \text{ (for HYSD bars)} = 1.92\ \text{N/mm}^2$$

Anchorage length = Development length

$$= \frac{\# \times 361}{4 \times 1.92} = 47\ \#$$

(2) *Compression:*

(i) *Flexural compression (Beam bars)*

Design stress, $\sigma_s = 0.87\ f_y = 0.87 \times 415 = 361\ \text{N/mm}^2$

$$\tau_{bd} = 1.2 \times 1.6 \text{ (HYSD bar)} \times 1.25 \text{ (compression)}$$

$$= 2.4\ \text{N/mm}^2$$

$$L_d = \frac{\# \times 361}{4 \times 2.4} = 37.6\ \#, \text{ say } 38\ \#$$

(ii) *Direct compression (column bars)*

Design stress $\sigma_s = 0.75\ f_y = 0.75 \times 415 = 311.2\ \text{N/mm}^2$

$$L_d = \frac{\# \times 311.2}{4 \times 2.4} = 32.4\ \#, \text{ say } 33\ \#$$

LIMIT STATE OF COLLAPSE:

TORSION- If the longitudinal axis of a structural member and loading axis are perpendicular to each other, the structural member will be subjected to twisting called Torsion.

CLASSIFICATION OF TORSION:

Statically determinate or equivalent or primary torsion:- A torsion that develops to maintain static equilibrium in the structural assemblage, it arrises as a result of primary action that can be external load has no alternationto being resistedbut by Torsion.

Ex: Torsion in a canopy beam in horizontal plane are statically determinate or equilibrium torsion.

*Statically indeterminate or compatibility or Secondary Torsion:*A torsion that arises as a result of secondary action from the requirement of continuity. The magnitude of this torsion in a member it self in relation to the stiffness of interconnecting members.

Approach of Design for Combined Bending, Shear and Torsion as per IS 456

As per the stipulations of IS 456, the longitudinal and transverse reinforcements are determined taking into account the combined effects of bending moment, shear force and torsional moment. Two impirical relations of equivalent shear and equivalent bending moment are given. These fictitious shear force and bending moment, designated as equivalent shear and equivalent bending moment, are separate functions of actual shear and torsion, and actual

bending moment and torsion, respectively. The total vertical reinforcement is designed to resist the equivalent shear Ve and the longitudinal reinforcement is designed to resist the equivalent bending moment Me1 and Me2, as explained in secs. 6.16.6 and 6.16.7, respectively. These design rules are applicable to beams of solid rectangular cross-section. However, they may be applied to flanged beams by substituting bw for b. IS 456 further suggests to refer to specialist literature for the flanged beams as the design adopting the code procedure is generally conservative.

Critical Section (cl. 41.2 of IS 456)

As per cl. 41.2 of IS 456, sections located less than a distance d from the face of the support is to be designed for the same torsion as computed at a distance d, where d is the effective depth of the beam.

Shear and Torsion

The equivalent shear, a function of the actual shear and torsional moment is determined from the following impirical relation:

$$Ve = Vu + 1.6(Tu/b)$$

where Ve = equivalent shear,

Vu = actual shear,

Tu = actual torsional moment,

b = breadth of beam.

The equivalent nominal shear stress r_{ve} is determined from: $r_{ve} = \frac{V_e}{bd}$

However, ve τ shall not exceed c max τ given in Table 20 of IS 456

Minimum shear reinforcement is to be provided as per cl. 26.5.1.6 of IS 456, if the equivalent nominal shear stress r_{ve} obtained from Eq.6.23 does not exceed r_c given in Table 19 of IS 456

$\frac{A_{sv}}{bS_v} = \frac{0.4}{0.87f_y}$

If r_{ve} is greater thanr_c Both longitudinal and transverse reinforcement shall be provided.

Reinforcement in Members subjected to Torsion

Reinforcement for torsion shall consist of longitudinal and transverse reinforcement as mentioned in sec. 6.16.6(d).

The longitudinal flexural tension reinforcement shall be determined to resist an equivalent bending moment Me1 as given below:

$$M_{e1} = M_u + M_t$$

where Mu = bending moment at the cross-section, and

Mt = (Tu/1.7) {1 + (D/b)}

where Tu = torsional moment,

D = overall depth of the beam, and

b = breadth of the beam.

The Me2 will be considered as acting in the opposite sense to the moment Mu.

The transverse reinforcement consisting of two legged closed loops (Fig.6.16.2) enclosing the corner longitudinal bars shall be provided having an area of cross-section Asv given below:

$A_{sv} = \frac{T_u S_v}{b_1 d_1 0.87 f_y} + \frac{V_u S_v}{2{,}5 d_1 (0.87 f_y)}$ or

$A_{sv} = \frac{(c_{ve-} b S_v}{0.87 f_y}$ whichever is more

where Tu = torsional moment, Vu = shear force, sv = spacing of the stirrup reinforcement, b1 = centre to centre distance between corner bars in the direction of the width, d1 = centre to centre distance between corner bars,

b = breadth of the member, fy = characteristic strength of the stirrup reinforcement, r_{ve}= equivalent shear stress and r_c= shear strength of concrete a

Requirements of Reinforcement

Tension Reinforcement

The minimum area of tension reinforcement should be governed by

As /(bd) = 0.85/fy

where As = minimum area of tension reinforcement, b = breadth of rectangular beam or breadth of web of T-beam, d = effective depth of beam, fy = characteristic strength of reinforcement in N/mm2

The maximum area of tension reinforcement shall not exceed 0.04 bD, where D is the overall depth of the beam.

Compression reinforcement

The maximum area of compression reinforcement shall not exceed 0.04 bD. They shall be enclosed by stirrups for effective lateral restraint.

Side face reinforcement

Beams exceeding the depth of 750 mm and subjected to bending moment and shear shall have side face reinforcement. However, if the beams are having torsional moment also, the side face reinforcement shall be provided for the overall depth exceeding 450 mm. The total area of side face reinforcement shall be at least 0.1 per cent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness, whichever is less.

Maximum spacing of shear reinforcement

The centre to centre spacing of shear reinforcement shall not be more than 0.75 d for vertical stirrups and d for inclined stirrups at 45o , but not exceeding 300 mm, where d is the effective depth of the section

.

Reinforced Concrete Slab Design

Slabs spanning in one direction: supported at two opposite ends
Slabs supporting on all four sides: these are further classified into two types based on aspect ratio.

One way slab :if $(\frac{l_y}{l_x}) > 2$

Two way slab :if $(\frac{l_y}{l_x}) < 2$

Effective span of slab:
Effective span of slab shall be lesser of the two

1. L = clear span + d (effective depth)
2. L = Center to center distance between the support

General notes on design of slabs:
Basic values of span to effective depth ratios for spans upto 10 mt

Cantilever -7
Simply supported - 20
Contineous - 26

For two way slabs for small spans upto 3.5 mt with mild steel span to over all depth ratios for loading class upto 3kn/m2 are

Simply supported slabs -35

Contineous - 40

Depth of slab:
The depth of slab depends on bending moment and deflection criterion. the trail depth can be obtained using:

- Effective depth d= Span /($(L/d)_{Basic}$ x modification factor)
- For obtaining modification factor, the percentage of steel for slab can be assumed from 0.2 to 0.5%.
- The effective depth d of two way slabs can also be assumed using cl.24.1,IS 456 provided short span is <3.5m and loading class is <3.5KN/m^2

Or, the following thumb rules can be used:

- One way slab d=(L/22) to (L/28).
- Two way simply supported slab d=(L/20) to (L/30)
- Two way restrained slab d=(L/30) to (L/32)

c) Load on slab:
The load on slab comprises of Dead load, floor finish and live load. The loads are calculated per unit area (load/m^2).
Dead load = D x 25 kN/m^2 (Where D is thickness of slab in m)
Floor finish (Assumed as)= 1 to 2 kN/m^2

Live load (Assumed as) = 3 to 5 kN/m^2 (depending on the occupancy of the building)

Detailing Requirements of Reinforced Concrete Slab as per IS456: 2000

a) Nominal Cover:

For Mild exposure – 20 mm

For Moderate exposure – 30 mm

However, if the diameter of bar do not exceed 12 mm, or cover may be reduced by 5 mm. Thus for main reinforcement up to 12 mm diameter bar and for mild exposure, the nominal cover is 15 mm.

Minimum reinforcement:

The reinforcement in either direction in slab shall not be less than

- 0.15% of the total cross sectional area for Fe-250 steel
- 0.12% of the total cross-sectional area for Fe-415 & Fe-500 steel.

c) Spacing of bars:

The maximum spacing of bars shall not exceed

- Main Steel – 3d or 300 mm whichever is smaller
- Distribution steel –5d or 450 mm whichever is smaller Where, 'd' is the effective depth of slab. Note: The minimum clear spacing of bars is not kept less than 75 mm (Preferably 100 mm) though code do not recommend any value.

d) Maximum diameter of bar:

The maximum diameter of bar in slab, shall not exceed D/8, where D is the total thickness of slab.

Torsion reinforcement: Shall be provided where the slab is simply supported on both edges meeting at that corner. It consists of top and bottom reinforcement at corner extending from edges a minimum distance of one-fifth of shorter span. The area of reinforcement in each of these four layers shall be three quarters of the required for the maximum mid span moments in the slab.

- Torsion reinforcement need not be provided at a corner continued by edges over both of which the slab is continuous.

- Half the torsion reinforcement is required at a cover with one edge discontinuous and the other continuous.

Simply supported slabs: When simply supported slabs do not have adequate provision to resist torsion at corners and to prevent the corners from lifting, the maximum moment per unit width are given by the following equation

$$M_x = \alpha_x w l_x^2$$
$$M = \alpha_y w l_y^2$$

At lease 50% of tension reinforcement is provided at mid span should extend to the supports.

Numerical Problems:-

A simply supported one-way slab of effective span 4 m is supported on massonary walls of 230 mm thickness. Design the slab. Take live load equal to kN/m2. And floor finish equal to 1 kN/m2. The materials are M20 gradeconcrete and HYSD reinforcement of grade Fe 415.

Solution:

Assuming 0.35 per cent steel, a trial depth can be found out by using deflection criteria.

$$\text{Service stress} = 0.58 f_y = 0.58 \times 415 = 240 \text{ N/mm}^2.$$

Modification factor from fig. 8-1 is 1.4.

$$\text{permissible } \frac{\text{span}}{d} \text{ ratio} = 1.4 \times 20 = 28$$

$$d_{\text{required}} = \frac{4000}{28} = 142.9 \text{ mm.}$$

$$D = 142.9 + 15 \text{ (cover)} + 5 \text{ (assume 10 \# bar)}$$
$$= 162.9 \text{ mm.}$$

Assume an overall depth = 170 mm.

Self weight = 0.17 × 25 = 4.25 kN/m^2

Floor finish = 1.00 kN/m^2

Live load = 2.50 kN/m^2

Total 7.75 kN/m^2

$$\text{Factored load} = 7.75 \times 1.5 = 11.6 \text{ kN/m}$$

$$\text{Maximum moment} = 11.6 \times \frac{4^2}{8} = 23.2 \text{ kNm.}$$

$$\text{Maximum shear} = 11.6 \times \frac{4}{2} = 23.2 \text{ kN.}$$

Design for flexure:

$$d = 170 - 15 - 5 = 150 \text{ mm.}$$

$$\frac{M_u}{b d^2} = \frac{23.2 \times 10^6}{1000 \times 150 \times 150} = 1.03$$

$$p_t = 0.299$$

$$A_{st} = \frac{0.299 \times 1000 \times 150}{100} = 449 \text{ mm}^2.$$

Provide 10 mm # @ 170 mm c/c = 462 mm^2.

Half the bars are bent at 0.1 l = 400 mm, and remaining bars provide 231 mm^2 area.

$$\frac{100 A_s}{b D} = \frac{100 \times 231}{1000 \times 170} = 0.136 > 0.12$$

i.e., remaining bars provide minimum steel. Thus, half the bars may be bent up.

$$\text{Distribution steel} = \frac{0.12}{100} \times 1000 \times 170 = 204 \text{ mm}^2.$$

Maximum spacing = 5 × 160 = 800 or 450 mm, i.e., 450 mm.

Provide 8 mm # @ 230 mm c/c = 217 mm^2.

Check for shear:

For bars at support, $d = 150$ mm

$$A_s = 231 \text{ mm}^2$$

$$\frac{100 A_s}{b D} = \frac{100 \times 231}{1000 \times 150} = 0.154.$$

$$\tau_c = 0.28 \text{ N/mm}^2$$

For 170 mm thick slab

$$k = 1.26$$

$$k\tau_c = 1.26 \times 0.28 = 0.353 \text{ N/mm}^2.$$

$$\text{Actual shear stress} = \frac{24 \times 10^3}{1000 \times 150} = 0.16 \text{ N/m}^2 < k\tau_c \text{ .... (Safe)}$$

Check for development length:

$$\text{Consider } L_0 = 8 \text{ \# for continuing bars}$$

$$A_s = 231 \text{ mm}^2$$

Assume,

$$M_{u1} = 0.87 \, f_y \, A_{st} \, (d - 0.42 \, x_u)$$

where

$$x_u = x_{u,max} = 0.48 \, d.$$

$$M_{u1} = 0.87 \times 415 \times 231 \times (150 - 0.42 \times 0.53 \times 150) \times 10^{-6}$$

$$= 9.73 \text{ kNm}$$

Note: Different formulae are used for calculations M_{u1} in different worked examples. Note that the lever arm of balanced section is assumed as actual lever arm. This is conservative, however used for speedy calculations. If the check is not satisfied, one may exactly find out the value of M_{u1}. Refer to example 10-3.

$$V_u = 24 \text{ kN}$$

$$1.3 \frac{M_{u1}}{V_u} + L_0 \geq L_d$$

i.e.

$$1.3 \times \frac{9.73 \times 10^6}{24 \times 10^3} + 8 \text{ \#} \geq 47 \text{ \#}$$

$$39 \text{ \#} \geq 527$$

$$\therefore \quad \text{\#} \leq 13.51 \text{ mm .... (O.K.)}$$

Check for deflection:

$$\text{Basic } \frac{\text{span}}{d} \text{ ratio} = 20$$

$$p_t = \frac{100 \times 462}{1000 \times 150} = 0.308$$

$$\text{service stress} = 0.58 \times 415 \times \frac{449}{462} = 234 \text{ N/mm}^2$$

$$\text{modification factor} = 1.42$$

$$\text{permissible } \frac{\text{span}}{d} \text{ ratio} = 20 \times 1.42 = 28.4$$

$$\text{actual } \frac{\text{span}}{d} \text{ ratio} = \frac{4000}{150} = 26.66 < 28.4 \text{ .... (O.K.)}$$

Check for cracking:

Maximum spacing permitted for main reinforcement

$= 3 \times 160 = 480$ mm or 300 mm, i.e., 300 mm.

Actual spacing = 170 mm .... (O.K.)

Maximum spacing permitted for secondary reinforcement

$= 5 \times 160 = 800$ mm or 450 mm, i.e., 450 mm

Actual spacing = 230 mm .... (O.K.)

For tying the bent bars at top, provide 8 mm # @ 230 mm c/c.

Provide 8 mm # @ 230 mm c/cfor bent-up bars at top

Sketch: The cross-section of the slab is shown in fig. 10-8.

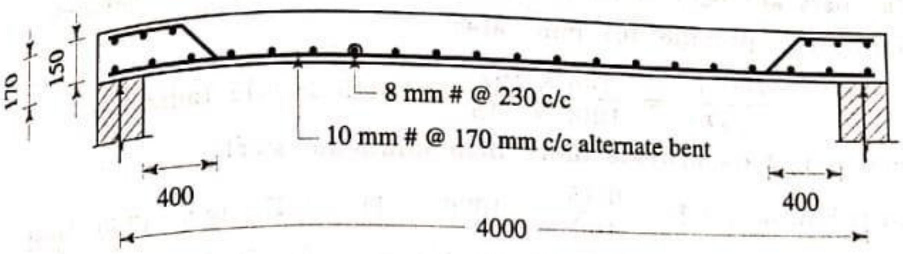


FIG. 10-8

Example 10-3.

A simply supported one-way slab of a corridor of an office building of a clear span 2.4 m is supported on beams of 230 mm width. Design the slab for a live load of 5 kN/m². The materials are M 20 grade concrete and HYSD reinforcement of grade Fe 415.

Solution:

Assume 115 mm thick slab with 10 mm diameter bars.

$$d = 115 - 15 \text{ (cover)} - 5 = 95 \text{ mm.}$$

Loads:

Self weight 0.115 × 25	= 2.88 kN/m²
Floor finish	= 1.00 kN/m²
Live load	= 5.00 kN/m²
Total	8.88 kN/m².

Factored load = 1.5 × 8.88 = 13.3 kN/m².

Span = (i) 2.4 + 0.10 = 2.5 m

(ii) 2.4 + 0.23 = 2.63 m c/c supports.

∴ Adopt 2.5 m span.

Consider 1 m length of slab

$$M_u = \frac{2.5^2}{8} \times 13.3 = 10.39 \text{ kNm}$$

$$V_u = \frac{2.4}{2} \times 13.3 = 15.96 \text{ kN (based on clear span)}$$

(a) *Moment steel:*

$$\frac{M_u}{b\,d^2} = \frac{10.39 \times 10^6}{1000 \times 95^2} = 1.15 < 2.76.$$

∴ Singly reinforced section.

$$p_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}} \times \frac{M_u}{b\,d^2}}}{f_y/f_{ck}} \right] = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{20} \times 1.15}}{415/20} \right] = 0.343$$

$$A_{st} = \frac{0.343}{100} \times 1000 \times 95 = 326 \text{ mm}^2/\text{m.}$$

Spacing $\not>$ 3 × 100, i.e., 300 mm.

Provide 8 mm # @ 150 mm c/c = 333 mm^2/m.

Half the bars are bent at 0.1 l = 240 mm from face of the support.

Remaining bars provide 167 mm^2 area.

$$\frac{100\,A_s}{b\,D} = \frac{100 \times 167}{1000 \times 115} = 0.145 > 0.12 \text{ (minimum)}$$

i.e., remaining bars provide more than minimum steel.

$$\text{Distribution steel} = \frac{0.15}{100} \times 1000 \times 115 = 173 \text{ mm}^2 \text{ (mild steel)}.$$

$$\text{Spacing} \not> 5 \times 100 \text{ or } 450 \text{ mm, i.e., } 450 \text{ mm.}$$

Provide 6 mm ϕ @ 160 mm c/c = 175 mm^2.

(b) *Check for development length:*

$$A_s = 167 \text{ mm}^2,\ L_0 = 8\ \#\ \text{(assume)},\ L_d = 47\ \#$$

At support

$$M_{u1} = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{f_y\, A_{st}}{b\, d\, f_{ck}}\right)$$

$$= 0.87 \times 415 \times 167 \times 95 \left(1 - \frac{415 \times 167}{1000 \times 95 \times 20}\right) \times 10^{-6}$$

$$= 5.52 \text{ kNm}$$

$$V_u = 15.96 \text{ kN}$$

$$1.3\ \frac{M_{u1}}{V_u} + L_0 \geq L_d$$

$$\therefore \quad 1.3 \times \frac{5.52 \times 10^6}{15.96 \times 10^3} + 8\ \# \geq 47\ \#,\ \text{assuming } L_o = 8\ \#$$

$$\therefore \quad 11.53 \geq \#$$

$$\#_{provided} = 8 \text{ mm} \ldots\ldots\ldots\ldots \text{(safe)}$$

(c) *Check for shear:*

$$\text{At support } \frac{100\,A_s}{b\,D} = \frac{100 \times 167}{1000 \times 95} = 0.176$$

$$\text{For } \frac{100\,A_s}{b\,D} = 0.15,\ \tau_c = 0.28 \text{ N/mm}^2$$

$$\text{and } \frac{100\,A_s}{b\,D} = 0.25,\ \tau_c = 0.36 \text{ N/mm}^2$$

$$\therefore \quad \text{for } \frac{100\,A_s}{b\,D} = 0.176,\ \tau_c = 0.30 \text{ N/mm}^2, \text{ by interpolation.}$$

For 115 mm depth, $k = 1.3$

$$\text{design shear strength} = k\,\tau_c = 1.3 \times 0.3 = 0.39 \text{ N/mm}^2$$

$$\tau_v = \frac{15.96 \times 10^3}{1000 \times 95} = 0.168 \text{ N/mm}^2 < k\,\tau_c \ldots\ldots \text{(safe)}$$

(d) *Deflection:*

$$\text{Basic } \frac{\text{span}}{d} \text{ ratio} = 20$$

$$p_t = \frac{100 \times 333}{1000 \times 95} = 0.35$$

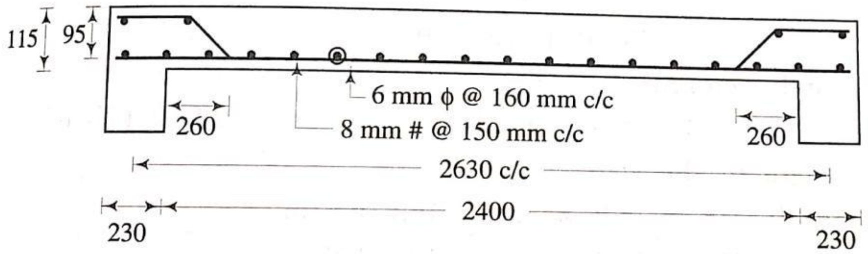
$$\text{service stress} = 0.58 \times 415 \times \frac{326}{335} = 234 \text{ N/mm}^2$$

modification factor = 1.48

$$\text{permissible } \frac{\text{span}}{d} = 1.48 \times 20 = 29.6$$

$$\text{actual } \frac{\text{span}}{d} = \frac{2500}{95} = 26.3 \text{ .... (safe)}$$

The designed section is shown in fig. 10-9.



LIMIT STATE OF COLLAPSE:(Design of column)

Definition:-

- *Column*- It is a compression member whose effective length is greater than 3 times the least dimension of the member .i.e. $(l_{eff}/b) > 3$.
- *Pedestal* - It is a compression member whose effective length is less than 3 times the least dimension of the member i.e. $l_{eff}/b) < 3$.

Minimum Eccentricity for axial load columns : All columns shall be designed for minimum eccentricity equal to

$$e_{min} = \frac{Unsupported\ length\ of\ column}{50} + \frac{leteral\ dimension}{30}$$

Subjected to a minimum of 20mm.

Short columns : $\frac{l_{eff}}{b} \leq 12$

Slenderness Ratio: It is the ratio of the length of a column and the least radius of gyration of its cross-section.

Assumptions in the Design of Compression Members by Limit State of Collapse:

It is thus seen that reinforced concrete columns have different classifications depending on the types of reinforcement, loadings and slenderness ratios. Detailed designs of all the different classes are beyond the scope here. Tied and helically reinforced short and slender columns subjected to axial loadings with or without the combined effects of uniaxial or biaxial bending will be taken up. However, the basic assumptions of the design of any of the columns under different classifications are the same.

I. The maximum compressive strain in concrete in axial compression is taken as 0.002.

II. The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. Following are the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc: The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.

(a) The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.

(b) Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.

(c) The diameter of the longitudinal bars should be at least 12 mm.

(d) Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.

(e) The bars shall be spaced not exceeding 300 mm along the periphery of the column.

(f) The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

Transverse Reinforcement

Transverse reinforcing bars are provided in forms of circular rings, polygonal links (lateral ties) with internal angles not exceeding 135° or helical reinforcement. The transverse reinforcing bars are provided to ensure that every longitudinal bar nearest to the compression face has effective lateral support against buckling. The salient points are:

- Transverse reinforcement shall only go round corner and alternate bars if the longitudinal bars are not spaced more than 75 mm on either side.

- Longitudinal bars spaced at a maximum distance of 48 times the diameter of the tie shall be tied by single tie and additional open ties for in between longitudinal bars.

- For longitudinal bars placed in more than one row (i) transverse reinforcement is provided for the outer-most row in accordance with (a) above, and (ii) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.

- For longitudinal bars arranged in a group such that they are not in contact and each group is adequately tied as per (a), (b) or (c) above appropriate, the transverse reinforcement for the compression member as a whole may be provided assuming that each group is a single longitudinal bar for determining the pitch and diameter of the transverse reinforcement . The diameter of such transverse reinforcement should not, however, exceed 20 mm .

Pitch and Diameter of Lateral Ties

(a) Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:

a. the least lateral dimension of the compression members;

b. sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and

c. 300 mm.

(b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

Helical Reinforcement

a) Pitch: Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and a half extra turns of the spiral bar. The pitch of helical reinforcement shall be determined as given for all cases except where an increased load on the column is allowed for on the strength of the helical reinforcement. In such cases only,

- The maximum pitch shall be the lesser of 75 mm
- One-sixth of the core diameter of the column, and the minimum pitch shall be the lesser of 25 mm and
- Three times the diameter of the steel bar forming the helix.

Short columns with helical reinforcement:

The permissible load for columns with helical reinforcement shall be 1.05 times the permissible load for similar members with ties.

$$\text{i.e. } P_u = 1.05\ (0.4 f_{ck} A_c + 0.67\ f_y A_{sc}$$

Where,

P_u = axial load on the member

f_{ck} = characteristic strength of concrete

A_c = area of concrete

f_y = characteristic strength of compression reinforcement, 415 /mm2

A_{sc} = area of longitudinal reinforcement for columns

A_g = gross area

Short column subjected to uni-axial bending

Short columns with uni axial moment:

The maximum strain in concrete at the outermost compression fibre is 0.0035 when the N.A lies within the section and

- In the limiting case when the N.A lies along the edge of the section, in the latter case strain carries from 0.0035 at the highly compressed edge to zero at the opposite edge.

- For purely axial compression, the strain is assumed to be uniform and equal to 0.002 across the section.

Slenderness limits for columns :

- With bolt end restrained: Unsupported length should not be greater than 60 times lateral dimension.

- If one end of column is unrestrained, unsupported length should not be greater than $100b^2/D$.

Numerical Problems

A short column of size 230 mm × 350 mm is subjected to a factored load of 1500 kN. If the unsupported length of the column is 3.2 m, find out design moments due to minimum eccentricity. Can we apply simplified formula of design in this case?

Solution:

Minimum eccentricity about x axis e_x, and that about y axis e_y are calculated and compared with respective sides D and b of the column as follows:

$$e_x = \frac{3200}{500} + \frac{350}{30} = 6.4 + 11.66$$

$$= 18.06 \text{ mm} < 20 \text{ mm}$$

$$\therefore \quad e_x = 20 \text{ mm}$$

$$0.05\,D = 0.05 \times 350 = 17.5 \text{ mm} < e_x$$

$$e_y = \frac{3200}{500} + \frac{230}{30} = 6.4 + 7.66$$

$$= 14.06 \text{ mm} < 20 \text{ mm}$$

$$e_y = 20 \text{ mm}$$

$$0.05\,b = 0.05 \times 230 = 11.5 \text{ mm} < e_y$$

$$M_{ux} = M_{uy} = 1500 \times 0.02 = 30 \text{ kNm.}$$

Design the column for

$$P_u = 1500 \text{ kN}$$

$$M_{ux} = 30 \text{ kNm}$$

$$M_{uy} = 30 \text{ kNm.}$$

As $e_x > 0.05\ D$ and $e_y > 0.05\ b$, simplied formula of design cannot be applied.

Note: The dimension of column in respect of major axis in D and that in respect of minor axis is b. However, it is usual to denote their dimension as D and its value is substituted as in respect of the axis selected for calculation. This will be clear from examples to follow.

Example 17-2.

A short column of size 300 mm × 450 mm is subjected to a factored load of 2000 kN. If the unsupported length of the column is 4.2 m, find out (a) The design moments due to minimum eccentricity.

If the above column is subjected to (b) M_{ux} = 100 kNm or (c) M_{uy} = 30 kNm or (d) M_{ux} = 30 kNm, and M_{uy} = 30 kNm, find design loads and moments.

Solution:

(a)
$$e_x = \frac{4200}{500} + \frac{450}{30}$$
$$= 8.4 + 15 = 23.4 \text{ mm} > 20 \text{ mm}$$
$$\therefore \quad e_x = 23.4 \text{ mm}$$
$$e_y = \frac{4200}{500} + \frac{300}{30}$$
$$= 8.4 + 10 = 18.4 \text{ mm} < 20 \text{ mm}$$
$$\therefore \quad e_y = 20 \text{ mm}$$
$$M_{ux} = \frac{23.4}{1000} \times 2000 = 46.8 \text{ kNm}$$
$$M_{uy} = \frac{20}{1000} \times 2000 = 40 \text{ kNm.}$$

(b) When the column is subjected to M_{ux} = 100 kNm which is more than moment due to minimum eccentricity, the minimum eccentricity is neglected. It is sufficient to ensure that eccentricity exceeds the minimum about one axis at a time. Design the column for

$$P_u = 2000 \text{ kN} \quad \text{and} \quad M_{ux} = 100 \text{ kNm}$$

(c) When the column is subjected to M_{uy} = 30 kNm, which is less than moment due to minimum eccentricity, the minimum eccentricity is not neglected. It is sufficient to ensure that eccentricity exceeds the minimum about one axis at a time. Design the column for

$$P_u = 2000 \text{ kN} \quad \text{and} \quad M_{uy} = 40 \text{ kNm.}$$

(d) The column is subjected to biaxial moment. It is sufficient to ensure that eccentricity exceeds the minimum about one axis at a time. Design the column for case I and case II as below:

Case I

$$P_u = 2000 \text{ kN}$$
M_{ux} = 46.8 kNm (since applied eccentricity is less than minimum)
$$M_{uy} = 30 \text{ kNm.}$$

Case II

$$P_u = 2000 \text{ kN} \quad \text{and} \quad M_{ux} = 30 \text{ kNm}$$
M_{uy} = 40 kNm (since the applied eccentricity is less than minimum).

A column with size 400 mm × 500 mm carries a factored axial load of 3000 kN. The column is short and having a minimum eccentricity $e_{min} < 0.05\ D$. Design the column. The materials are M 30 grade concrete and HYSD reinforcement of grade Fe 415.

Solution:

Longitudinal steel:

$$A_c = 400 \times 500 - A_{sc}$$

$$P_u = 0.4\ f_{ck}\ A_c + 0.67\ f_y\ A_{sc}$$

$$\therefore \quad 3000 \times 10^3 = 0.4 \times 30\ (400 \times 500) - A_{sc} + 0.67 \times 415 \times A_{sc}$$

$$= 2400 \times 10^3 + 266.05\ A_{sc}$$

$$\therefore \quad A_{sc} = 2255\ \text{mm}^2.$$

Provide 8 no. 20 mm diameter bars with $A_{sc} = 8 \times 314 = 2512\ \text{mm}^2$.

Lateral ties:

$$\phi_{tr} = \frac{\phi}{4} = \frac{20}{4} = 5\ \text{mm}$$

$$\nless 6\ \text{mm}$$

Use 6 mm ϕ M.S. ties.

Spacing should not exceed

(i) 400 mm (minimum column dimension)

(ii) $16 \times 20 = 320$ mm

(16 times ϕ of main bars)

(iii) 300 mm.

i.e. 300 mm.

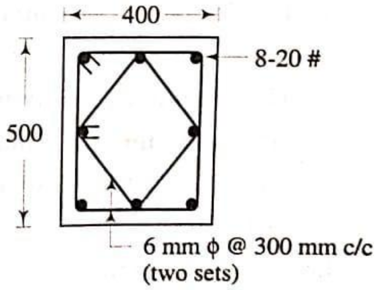


FIG. 17-9

Provide 6 mm ϕ ties about 300 mm c/c. The designed section is shown in fig. 17-9.

Note that the distance between corner bars on both the faces is more than 48 ϕ_{tr}. On shorter face, the distance $= 400 - 80 - 20 = 300$ mm $\ngtr 48 \times 6$. Therefore two sets of closed ties shall be used.

Example 17-3.

A short column 400 mm × 400 mm is reinforced with 4 no. 25 mm diameter bars. Find the ultimate load carrying capacity of the column if the minimum eccentricity is less than 0.05 times the lateral dimensions. The materials are M 20 grade concrete and HYSD reinforcement of grade Fe 415.

Solution:

$$A_{sc} = 4 \times 491 = 1964 \text{ mm}^2$$

$$A_c = 400 \times 400 - 1964 = 158036 \text{ mm}^2$$

$$P_u = 0.4\ f_{ck}\ A_c + 0.67\ f_y\ A_{sc}$$

$$= [0.4 \times 20 \times 158036 + 0.67 \times 415 \times 1964] \times 10^{-3}$$

$$= 1264.29 + 546.09 = 1810.4 \text{ kN.}$$

Example 17-4.

A short R.C.C. column is to carry a factored load of 1900 kN. If the column is to be a square, design the column. Assume $e_{min} < 0.05\ D$. The materials are M 20 grade concrete and mild steel.

Solution:

Here $e_{min} < 0.05\ D$. But $e_{min} = 20$ mm. Therefore the side of the column shall be minimum $\dfrac{20}{0.05} = 400$ mm, i.e., 400 mm × 400 mm.

As there is no restriction on size of the column, we may assume minimum percentage of steel to have economy in section. Assume 0.8 percent steel.

Then
$$A_{sc} = 0.008\ A_g$$

$$A_c = A_g - A_{sc} = 0.992\ A_g$$

$$P_u = 0.4\ f_{ck}\ A_c + 0.67\ f_y\ A_{sc}$$

$$1900 \times 10^3 = 0.4 \times 20 \times 0.992\ A_g + 0.67 \times 250 \times 0.008\ A_g$$

$$= 7.936\ A_g + 1.34\ A_g = 9.276\ A_g$$

∴
$$A_g = 204830 \text{ mm}^2.$$

If the column is to be a square, the side of column = 453 mm.

Adopt 450 mm × 450 mm size column. Then

$$1900 \times 10^3 = 0.4 \times 20 \times (450 \times 450 - A_{sc}) + 0.67 \times 250 \times A_{sc}$$

$$= 1620000 - 8\ A_{sc} + 167.5\ A_{sc}$$

i.e.
$$159.5\ A_{sc} = 280000$$

∴
$$A_{sc} = 1756 \text{ mm}^2.$$

Provide 6 no. 20 mm diameter bars giving $A_{sc} = 6 \times 314 = 1884 \text{ mm}^2$. The section is shown in fig. 17-7(a).

Note that the distance between the bars exceeds 300 mm on two parallel sides. Therefore, the arrangement of reinforcement should be changed.

Provide 4 no. 20 mm diameter bars plus 4 no. 16 mm diameter bars giving $A_{sc} = 4\ (314 + 201) = 2060 \text{ mm}^2$.

Lateral ties:

Use 6 mm φ lateral ties.

Spacing should not exceed:

(i) 450 mm (minimum column dimension)

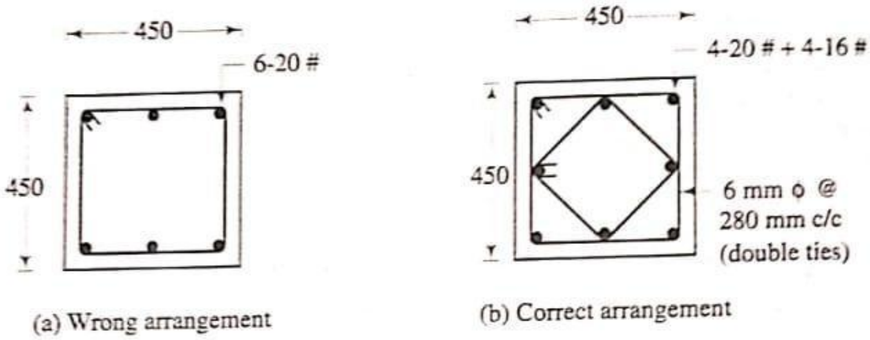
(ii) $16\ \phi = 16 \times 16 = 256$ mm

(iii) 300 mm.

i.e. 256 mm.

Provide 6 mm ϕ ties about 250 mm c/c. This is shown in fig. 17-7(b).

Note that distance between corner bars in one face is equal to $450 - 80 - 20 = 350$ mm. This is more than $48\ \phi_{tr}$ (i.e., $48 \times 6 = 288$ mm). Therefore two sets of closed ties shall be used. Note that if this distance would be less than (48×6) mm, open ties for internal bars would be sufficient.



(a) Wrong arrangement

(b) Correct arrangement

FIG. 17-7

Example 17-5.

A short R.C.C. column of size 450 mm × 450 mm has to carry an axial factored load of 1500 kN. Assume $e_{min} < 0.05\ D$.

Design the column using M 20 grade concrete and HYSD reinforcement of grade Fe 415.

Solution:

$$A_c = 450 \times 450 - A_{sc}$$

$$1500 \times 10^3 = 0.4 \times 20\ (450 \times 450 - A_{sc}) + 0.67 \times 415 \times A_{sc}$$

$$-\ 120 \times 10^3 = 286.05\ A_{sc}$$

$$A_{sc} = -\ 420\ \text{mm}^2.$$

Negative value indicates that there is no need of providing reinforcement. However, minimum reinforcement should be provided. Here it is based on concrete area required for direct load.

Area of concrete required for direct load

$$= \frac{1500 \times 10^3}{0.4 \times 20} = 187500\ \text{mm}^2.$$

Minimum steel required

$$= \frac{0.8}{100} \times 187500 = 1500\ \text{mm}^2.$$

Provide 8-16# = 8 × 201 = 1608 mm².

Use 6 mm ϕ lateral ties.

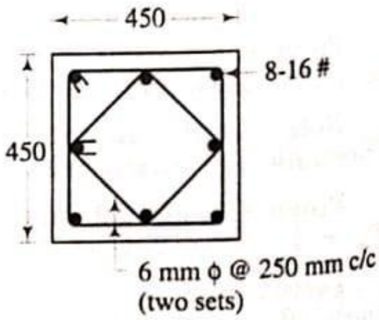


FIG. 17-8

Spacing should be lesser of

(i) 450 mm (minimum column dimension)

(ii) 16 × 16 = 256 mm (16 times φ of main bars)

(iii) 300 mm.

i.e. 256 mm.

Use 6 mm φ @ 250 mm c/c. Also, as the distance between corner bars exceeds 48 ϕ_{tr} closed double ties are used.

The designed section is shown in fig. 17-8.

Example 17-6.

A column with size 400 mm × 500 mm carries a factored axial load of 3000 kN. The column is short and having a minimum eccentricity $e_{min} < 0.05\ D$. Design the column. The materials are M 30 grade concrete and HYSD reinforcement of grade Fe 415.

Solution:

Longitudinal steel:

$$A_c = 400 \times 500 - A_{sc}$$

$$P_u = 0.4\ f_{ck}\ A_c + 0.67\ f_y\ A_{sc}$$

$$\therefore \quad 3000 \times 10^3 = 0.4 \times 30\ (400 \times 500) - A_{sc} + 0.67 \times 415 \times A_{sc}$$

$$= 2400 \times 10^3 + 266.05\ A_{sc}$$

$$\therefore \quad A_{sc} = 2255\ \text{mm}^2.$$

Provide 8 no. 20 mm diameter bars with $A_{sc} = 8 \times 314 = 2512\ \text{mm}^2$.

Lateral ties:

$$\phi_{tr} = \frac{\phi}{4} = \frac{20}{4} = 5\ \text{mm}$$

$$\not< 6\ \text{mm}$$

Use 6 mm φ M.S. ties.

Spacing should not exceed

(i) 400 mm (minimum column dimension)

(ii) 16 × 20 = 320 mm (16 times φ of main bars)

(iii) 300 mm.

i.e. 300 mm.

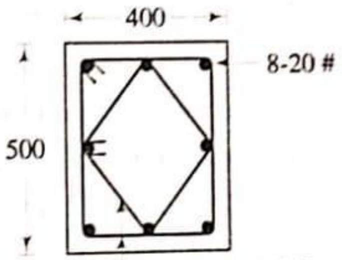


FIG. 17-9

Provide 6 mm φ ties about 300 mm c/c. The designed section is shown in fig. 17-9.

Note that the distance between corner bars on both the faces is more than 48 ϕ_{tr}. On shorter face, the distance = 400 − 80 − 20 = 300 mm $\not>$ 48 × 6. Therefore two sets of closed ties shall be used.

Limit state of serviceability: deflection

The serviceability requirement for the deflection should be such that neither the efficiency nor appearance of a structure should be affected by the deflection which will occur during its life.

Short- and Long-term Deflections

As per IS:456-2000

The following factors influence the short-term deflection of structures:

(i) magnitude and distribution of live loads

(ii) span and type of end supports,

(iii) cross-sectional area of the members,

(iv) amount of steel reinforcement and the stress developed in the reinforcement,

(v) characteristic strengths of concrete and steel, and

(vi) amount and extent of cracking.

The long-term deflection is almost two to three times of the short-term deflection. The following are the major factors influencing the long-term deflection of the structures.

(i) humidity and temperature ranges during curing,

(ii) age of concrete at the time of loading, and

(iii) Type and size of aggregates, water-cement ratio, amount of compression reinforcement, size of members etc., which influence the creep and shrinkage of concrete.

Control of Deflection:

- The maximum final deflection should not normally exceed span/250 due to all loads including the effects of temperatures, creep and shrinkage and measured from the as-cast level of the supports of floors, roof and all other horizontal members.

- The maximum deflection should not normally exceed the lesser of span/350 or 20 mm including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes.

It is essential that both the requirements are to be fulfilled for every structure.

Selection of Preliminary Dimensions

For the deflection requirements

Different basic values of span to effective depth ratios for three different support conditions are prescribed for spans up to 10 m, which should be modified under any or all of the four different situations:

(i) for spans above 10 m,
(ii) (ii) depending on the amount and the stress of tension steel reinforcement,
(iii) (iii) depending on the amount of compression reinforcement, and
(iv) (iv) for flanged beams.

Basic values of span to effective depth ratios for spans upto 10 mt

Cantilever -7

Simply supported - 20

Contineous - 26

For lateral stability

The lateral stability of beams depends upon the slenderness ratio and the support conditions.

- For simply supported and continuous beams, the clear distance between the lateral restraints shall not exceed the lesser of 60b or 250b2 /d, where d is the effective depth and b is the breadth of the compression face midway between the lateral restraints.
- For cantilever beams, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed the lesser of 25b or 100b2 /d.

Deflection Due to Creep

Concrete creep is defined as: deformation of structure under sustained load. Basically, long term pressure or stress on concrete can make it change shape. This deformation usually occurs in the direction the force is being applied. Like a concrete column getting more compressed, or a beam bending. Creep does not necessarily cause concrete to fail or break apart. When a load is applied to concrete, it experiences an instantaneous elastic strain which develops into creep strain if the load is sustained.

Long term deflection in concrete is calculated by

$$E_{ce} = \frac{E_c}{1+\theta}$$

Where θ ,being the creep coefficient,

Concrete Shrinkage : The volumetric changes of **concrete** structures due to the loss of moisture by evaporation is known as **concrete shrinkage** or **shrinkage of concrete**. It is a time-dependent deformation which reduces the volume of **concrete** without the impact of external forces.

Concrete Footing:-

INTRODUCTION

Foundation is that part of a structure which transfers the load of the structure to soil on which it rests. This term includes the portion of the structure below ground level (also known as sub-structure) which provides a base for the structure above the ground (also known as super-structure) as well as the extra provisions made to transmit the loads on the structure including its self wt. to the soil below.

It is often misunderstood that the foundation is provided to support the load of the structure. In fact, it is a media to transmit the load of the structure to the sub-soil. The objectives of foundation are:

- To distribute the weight of the structure over larger area so as to avoid over-loading of the soil beneath.
- To load the sub – structure evenly and thus prevent unequal settlement.
- To provide a level surface for building operations.
- To take the sub-structure deep into the ground and thus increase its stability preventing overturning.

Footings are an important part of foundation construction. They are typically made of concrete with rebar reinforcement that has been poured into an excavated trench. The purpose of footings is to support the foundation and prevent settling. Footings are especially important in areas with troublesome soils.

- *Thickness at the edge of footing:* In reinforced and plain concrete footing,the thivkness at the edge shall be not less than 150 mmfor footings on soil and not less than 300 mm for footing on piles.
- The depth of foundation minimum of 500 mm.
- *Minimum % of steel*: Footing is to be treated as an inverted slab. As per IS:456-2000 the minimum % of steel is 0.12% of gross area with HYSD bars and 0.15% of gross area with plain bars of mild steel.
- *Minimum clear cover*: 50 mm. For any tipe of exposure condition.

TYPES OF FOUNDATIONS

Foundations can be broadly classified into two types:

(i) Deep foundations

(ii) Shallow foundations.

Deep foundations: When the foundations are placed considerably below the lowest part of the super-structure it is termed as deep foundations. Pile foundations, pier foundation, well foundation, cassions etc. fall in the category of deep foundation.

(ii) Shallow foundations: When the foundation is placed immediately beneath the lowest part of the super-structure it is termed as shallow foundation. Shallow foundations can be broadly divided in the following groups:

(1) Spread footings

(2) Combined footings

(3) Mat or raft foundation.

Spread Footings

As the name suggest, in case of spread footings the base of the member (a column or a wall) transmitting the load is made wider so as to distribute the load over a larger area. A footing that supports a single column is known as isolated column footing. In case of a wall, the footing has to be a continuous one and hence it is known as wall footing or a continuous footing.

It is seen that square footing works out to be economical for square and circular columns. Under rectangular column, rectangular footings are considered to be more appropriate. In case the load on column is not large or the size of footing works out to be small requiring small depth of footing it is desirable to keep the thickness of footing uniform. In case the depth of the footing works out to be more, it is common practice to gradually reduce the depth of the footings towards the edges to achieve economy. The footing in such a case is termed as sloped footings.

Combined Footings

A common footing provided for two or more columns in a row is known as combined footing.

Mat or Raft Foundation

It is consist of thick reinforced concrete slab covering the entire area under a supporting several columns and walls. It is the type of shallow foundation. It is used in conditions where the soil bearing capacity is poor, and the loading to be supported are from high columns or wall loads.

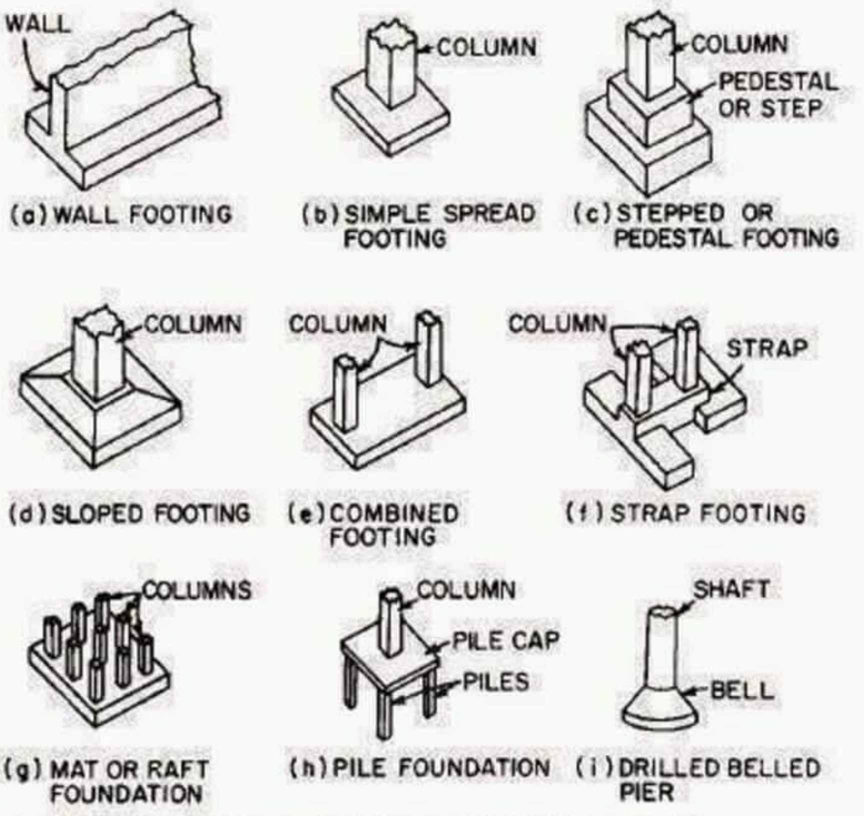


FIGURE 9.41 Common types of foundations for buildings.

DEPTH OF FOUNDATION

For all important buildings it is necessary to get the soil investigation of the site carried out by specialist agency. The test report should contain details regarding the type of sub-soil strata at various depths, depth of water table, and recommendation regarding the bearing capacity of soil at different depths. For normal buildings the depth of foundation below ground level is commonly calculated by the Rankine's formula.

According to Rankine's formula the minimum depth of foundation is given thus:

$$D_f = \frac{P_0}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

Where $D_f = minimum\ depth\ of\ foundation\ in\ metres$

$P_0 = bearing\ capacity\ of\ the\ soil\ in\ kN/m^3$

$\gamma = density\ of\ soil\ or\ the\ unit\ weight\ of\ soil\ in\ kN/m^3$

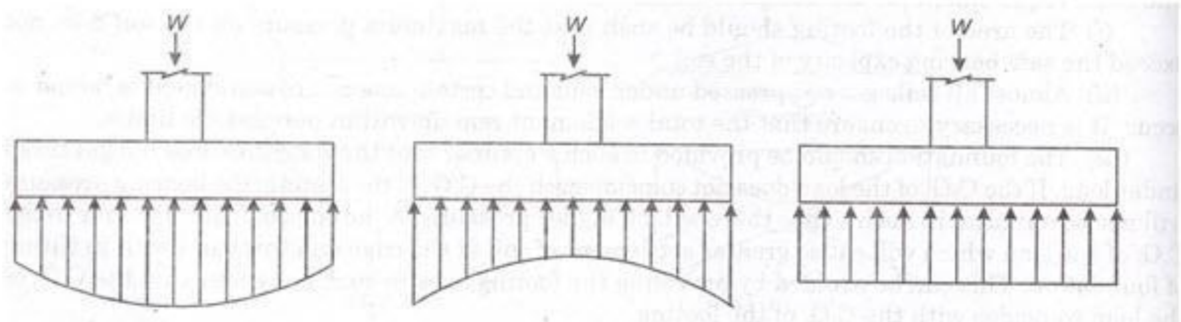
$\phi = the\ angle\ of\ repose\ of\ the\ soil.$

PRESSURE DISTRIBUTION UNDER FOOTINGS

The theory of elasticity analysis as well as the actual observations indicates that the pressure distribution under symmetrically loaded footings is not uniform. The actual stress distribution depends upon the nature of subsoil strata and the rigidity of the footings.

When a rigid footing is placed on loose cohesion-less soil, due to the load transmitted by the footing the soil grains at the edges having no lateral restraint displace laterally and in the centre the soil remain relatively confined.

On the other hand in case of rigid footing on cohesive soils, the load transmitted by the footing causes very large pressure at the edges and the parabola pressure distribution under the footing.



(a) Cohesion less or sandy soil (b) Cohesive less or clayey soil (c) Uniform pressure (assumed condition)

ANALYSIS AND DESIGN OF FOOTINGS

The analysis and design of footings can be broadly divided in the following steps.

(a) Determination of the area of footing.

(b) Determination of bending moments and shears at critical section and fixing the depth of footing.

(c) Determination of the area of reinforcement.

(d) Check for development length at critical section.

The area of the footing is worked out based on the load on the member including self wt. of footing and the bearing capacity of the soil. The calculations for bending moment, shear force, development length etc. are made based on provision in IS code. The various recommendations made in IS: 456-1978 for design of footing are given below.

1. General. (i) Footings shall be designed to sustain the applied loads, moments and forces and the induced reactions and to ensure that any settlement which may occur will be as nearly uniform as possible and the safe bearing capacity of the soil is not exceed.

(ii) Thickness at the edge of footing: In reinforced and plain concrete footings, thickness at the edges shall be not less than 150 mm for footings on the soils, nor less than 300 mm above the tops of piles for footings on piles.

2. Moments and forces. (i) In the case of footings on piles, computation for moments and shears may be based on the assumption that the reaction from any pile is concentrated at the centre of the pile.

(ii) For the purpose of computing stresses in footing which support a round or octagonal concrete column or pedestal, the face of the column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal.

3. Bending moment. (i) The bending moment at any section shall be determined by passing through the section a vertical plane which extends completely across the footing and computing the moments of the forces acting over the entire area of the footing on one side of the said plane.

(ii) The greatest bending moment to be used in the design of an isolated concrete footing which supports a column, pedestal or walls shall be the moment computed in the manner prescribed in Art. 3(i) at sections located as follows:

(a) At the face of the column, pedestal or wall for footings supporting a concrete column, pedestal or wall.

(b) Half way between the centre line and the edge of the wall, for footing under masonry walls.

(c) Half way between the face of the column or pedestal and the edges of the gusseted base for footings under gusseted bases.

4. Shear and bond. (i) The shear strength of footings is governed by the more severe of the following two conditions.

(a) The footing acting essentially as a wide beam, with a potential diagonal crack extending in a plane across the entire width; the critical section for the condition shall be assumed as a vertical section located from the face of the column pedestal or wall at a distance equal to the effective depth of the footing in case of footing on soils, and a distance equal to the half the effective depth of footing for footings on piles.

(b) Two-way action of the footing with potential diagonal cracking along the surface of truncated cone or pyramid around the concentrated load, in this case the footing shall be designed for shear in accordance with appropriate provision specified.

(ii) The critical section for checking the development length in a footing shall be assumed at the same plane as those described for bending moment in Art. 3 and also at all other vertical planes where abrupt changes of section occur. If the reinforcement is curtailed, the anchorage requirement shall be checked in accordance with provision.

5. Tensile reinforcement. The total reinforcement at any section shall provide a moment of resistance at least equal to the bending moment on the section calculated in accordance with Art. 3.

(i) In one-way reinforced footing the reinforcement shall be distributed uniformly across the full width of the footing.

(ii) In two-way reinforced square footing the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing.

(iii) In two-way reinforced rectangular footing, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing. For reinforcement in the short direction, a central band equal to the width of the footing shall be marked along the length of the footing and portion of the reinforcement determined in accordance with equation given below shall be uniformly distributed across the central band:

$$\frac{Reinforcement\ in\ central\ band\ width}{Total\ reinforcment\ in\ short\ direction} = \frac{2}{\beta + 1}$$

where β is the ratio of the long side to the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portions of the footing.

6. Transfer of load at the base of column. The compressive stress in concrete at the base of a column or pedestal shall be considered as being transferred by bearing to the top of the supporting pedestal or footing. The bearing pressure on the loaded area shall not exceed the permissible bearing stress in direct compression multiplied by a value = $\sqrt{\frac{A_1}{A_2}}$ but not greater than 2.

Where A_1 supporting area for bearing of footing, which in sloped or stepped footing may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal

A_2 = Loaded area at the column base

For working stress method of design the permissible bearing stress \[({\sigma _{cbc}})\] on full area of concrete shall be taken as 0.25 f_{ck} .

Hence the permissible bearing stress in concrete $(\sigma_{cbr}) = 0.25\, f_{ck}$

The actual bearing pressure or bearing stress $= \frac{W}{a \times b}$

It has to be ensured that $\frac{W}{a \times b}$ should not exceed , (a) and (b) being the dimensions of the column.

(i) Where the permissible bearing stress on the concrete in the supporting or supported member would be exceeded, reinforcement shall be provided for developing the excess force, either by extending the longitudinal bars into the supporting member of by dowels.

(ii) Where transfer of force is accomplished by reinforcement, the development length of the reinforcement shall be sufficient to transfer the compression or tension to the supporting member.

(iii) Extended longitudinal reinforcement or dowels of at least 0.5 per cent of cross-sectional area of the supported column or pedestal and a minimum of four bars shall be provided. Where dowels are used their diameter shall not exceed the diameter of the column bars by more than 3 mm.

(iv) Column bars of diameter larger than 36 mm, in compression only can be dowelled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel.

Example   Design a R.C.C. footing for a 300 mm thick brick wall carrying a load of 120 kN per metre length of the wall. The safe bearing capacity of soil is 90 kN/m^2. Use M 15 grade of concrete and using HYSD reinforcement.

Solution Design constants : B = width of footing in metre= 300 mt

b = width of wall in metre

p_0 = safe bearing capacity of soil in kN/m2=90 kN/m2

W = Load from wall in kN/m and

W_1 = weight of the footing in kN/m

For $\sigma_{cbc} = 5\ N/mm^2,\ m = 19$

and $\sigma_{st} = 230\ N/mm^2$

$k = 0.292, j = 0.903, R = 0.659$

Load per metre consist of the following:

(i) Load carried by wall (W) = 120 kN

(ii) Self wt. of footing @ 10% of (W) assumed = 12 kN

Total = 132 kN

Width of footing or (B) = $\frac{132}{p_o} = \frac{132}{90} = 1.467\ m\ say\ 1.5\ m$

Net upward pressure (p) = $\frac{120}{1.5} = 80\ kN/m^2$

Max. B.M. : The section for max. B.M. is considered to be located midway between centre of wall and the edge of the wall. Its value is given by

$$M = \frac{p}{8}(B-b)\left(B - \frac{b}{4}\right)$$

$$= \frac{80}{8}(1.5-0.3)\left(1.5 - \frac{0.3}{4}\right) = 17.1\ kNm = 17.1 \times 10^6\ Nmm$$

Required effective depth of footing is given by

$$d = \sqrt{\frac{M}{R \times 1000}} = \sqrt{\frac{17.1 \times 10^6}{0.659 \times 1000}} = 161\ mm$$

Using 10mmϕ HYSD bars and a clear cover of 50 mm

Overall depth of slab = $161 + \frac{10}{2} + 50 = 216\ mm\ say\ 220\ mm$

∴ Available effective depth $d = 220 - 50 - 5 = 165\ mm$

Area of tensile reinforcement $A_{st} = \frac{M}{j.d.\sigma_{st}} = \frac{17.1 \times 10^6}{0.903 \times 165 \times 230} = 499\ mm^2$

c/c spacing using 10mm ϕ HYSD bars ($A_\phi = \frac{\pi}{4}(10)^2 = 78.5\ mm^2$)

$$= \frac{78.5 \times 1000}{499} = 157\ mm\ say\ 150\ mm\ c/c$$

$$A_{st}\ provided = \frac{78.5 \times 1000}{150} = 523\ mm^2$$

Percentage reinforcement = $\frac{100 \times 523}{1000 \times 165} = 0.32$

Longitudinal reinforcement:

@ 0.12% of the area of concrete = $\frac{0.12}{100} \times 1000 \times 220 = 264\ mm^2$

c/c spacing using 8mm ϕ bars ($A_\phi = \frac{\pi}{4}(8)^2 = 50.26\ mm^2$)

$$= \frac{50.26 \times 1000}{264} = 190.4\ mm\ say\ 190\ mm\ c/c$$

Check for shear : Critical section for shear lies at a distance of d from the face of the wall.

Shear force at the critical section

or $V = 80 \times (0.6 - 0.165) = 34.8\ kN = 34.8 \times 10^3\ N$

Nominal shear stress, $\tau_v = \frac{34.8 \times 10^3}{1000 \times 165} = 0.21\ N/mm^2$

which is less than value of τ_c corresponding to even 0.25% reinforcement, hence safe.

Check for development length: $L_d = \frac{\phi \sigma_s}{4\tau_{bd}}$

In the case of HYSD bar, the value of τ_{bd} should be increased by 40%.

∴ $L_d = \frac{\phi \times 230}{4\left(0.6 + \frac{40}{100} \times 0.6\right)} = 68.45\ \phi = 68.45 \times 10 = 684\ mm$

Providing a side cover of 50 mm the available straight length of bar beyond critical section for bending is

$$= \frac{1}{2}(B-b) - 50 = \frac{1}{2}[1500 - 300] - 50 = 550\ mm$$

Since it works out to be less than L_d, it will be necessary to bend the bar at 90° up at the edge for a length of 6ϕ i.e., $6 \times 12 = 72\ mm$

THANK YOU